HO 25 Solutions

1.) harmonic oscillator $m = 0.300\text{ kg}$ with an ideal spring $T_s = 0.200\text{ s}$

$$T_s = 2\pi\sqrt{\frac{m}{k}} \text{ so } k = m\left(\frac{2\pi}{T_s}\right)^2 = (0.300\text{ kg})\left(\frac{2\pi}{0.200\text{ s}}\right)^2 = 296\ \frac{\text{kg}}{\text{s}^2} = \boxed{296\ \frac{\text{N}}{\text{m}}}$$

2.) harmonic oscillator $m = 0.200\text{ kg}$ and ideal spring $k = 140\ \frac{\text{N}}{\text{m}}$

$F = -kx = ma = m\frac{d^2x}{dt^2}$ so the displacement is a solution to the differential equation $\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$

the solution is $x(t) = A\cos(\omega t)$ where $\omega = \sqrt{\frac{k}{m}}$

or using $T_s = 2\pi\sqrt{\frac{m}{k}}$ and $T = \frac{2\pi}{\omega}$ or $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{m}{k}}} = \sqrt{\frac{k}{m}}$

a.) $$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{140\ \frac{\text{N}}{\text{m}}}{0.200\text{ kg}}} = 26.5\sqrt{\frac{\text{N}}{\text{m}\cdot\text{kg}}} = 26.5\sqrt{\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{\text{m}\cdot\text{kg}}} = \boxed{26.5\ \frac{\text{rad}}{\text{s}}}$$

b.) $$T = \frac{2\pi}{\omega} = \frac{2\pi}{26.5\ \frac{\text{rad}}{\text{s}}} = \boxed{0.24\text{ s}}$$ alternatively $$T_s = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.200\text{ kg}}{140\ \frac{\text{N}}{\text{m}}}} = \boxed{0.24\text{ s}}$$

3.) simple harmonic motion with amplitude $A = 0.18\text{ m}$ and frequency $f = 6.00\text{ Hz}$

$x(t) = A\cos(\omega t)$ and $\omega = 2\pi f$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left(A\cos(\omega t)\right) = -\omega A\sin(\omega t) \text{ so } v_{max} = \omega A = 2\pi f A = 2\pi\left(6.00\text{ s}^{-1}\right)(0.180\text{ m}) = \boxed{6.79\ \frac{\text{m}}{\text{s}}}$$

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(-\omega A\sin(\omega t)\right) = -\omega^2 A\cos(\omega t) \text{ so } a_{max} = \omega^2 A = (2\pi f)^2 A = \left(2\pi\left(6.00\text{ s}^{-1}\right)\right)^2(0.180\text{ m}) = \boxed{256\ \frac{\text{m}}{\text{s}^2}}$$

4.) harmonic oscillator amplitude A and angular frequency ω, $x(t) = A\cos(\omega t)$ and $v(t) = -\omega A\sin(\omega t)$

a.) kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\omega A\sin(\omega t)\right)^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t)$

potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(A\cos(\omega t)\right)^2 = \frac{1}{2}kA^2\cos^2(\omega t)$

$K = U$ when $\frac{1}{2}m\omega^2A^2\sin^2(\omega t) = \frac{1}{2}kA^2\cos^2(\omega t)$ also $\omega = \sqrt{\frac{k}{m}}$ so $\omega^2 = \frac{k}{m}$ or $k = m\omega^2$

so $\frac{1}{2}m\frac{k}{m}A^2\sin^2(\omega t) = \frac{1}{2}kA^2\sin^2(\omega t) = \frac{1}{2}kA^2\cos^2(\omega t)$ are equal when $\omega t = (2n+1)\frac{\pi}{4}$ (odd multiples of $\frac{\pi}{4}$)

so $x = A\cos\left((2n+1)\frac{\pi}{4}\right) = \boxed{\pm A\frac{\sqrt{2}}{2}}$ and $v = -\omega A\sin\left((2n+1)\frac{\pi}{4}\right) = \boxed{\mp\omega A\frac{\sqrt{2}}{2}}$

4.) (continued)

b.) occurs four times each cycle when $\boxed{\omega t = \frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\text{ and } \frac{7\pi}{4}}$

c.) time between occurrences is when $\omega \Delta t = \frac{\pi}{2}$ or $\Delta t = \frac{\pi}{2\omega}$ and since $\omega = \frac{2\pi}{T}$ $\Delta t = \frac{\pi}{2\left(\frac{2\pi}{T}\right)} = \boxed{\frac{T}{4}}$

5.) block $m = 3.00\text{ kg}$ suspended from ideal spring stretching it $\Delta x = 0.200\text{ m}$

a.) $F = k\Delta x$ and the force is provided by the weight of the block so $F_g = mg = k\Delta x$

$$\text{so } k = \frac{mg}{\Delta x} = \frac{(3.00\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)}{0.200\text{ m}} = \boxed{147\ \frac{\text{N}}{\text{m}}}$$

b.) for simple harmonic motion $T_s = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3.00\text{ kg}}{147\ \frac{\text{N}}{\text{m}}}} = \boxed{0.90\text{ s}}$

6.) simple pendulum makes 100 complete swings in 55.0 s where $g = 9.8\ \frac{\text{m}}{\text{s}^2}$

$$T = \frac{55.0\text{ s}}{100} = 0.55\text{ s and } T_p = 2\pi\sqrt{\frac{\ell}{g}} \text{ so } \ell = \frac{gT^2}{4\pi^2} = \frac{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.55\text{ s})^2}{4\pi^2} = \boxed{0.075\text{ m}}$$

7.) on Earth simple pendulum $T_E = 1.60\text{ s}$, and placed on Moon where $g_{Moon} = 1.62\ \frac{\text{m}}{\text{s}^2}$

on Earth $T_E = 2\pi\sqrt{\frac{\ell}{g_E}}$ so $\ell = \frac{g_E T_E^2}{4\pi^2}$

so on Moon $T_{Moon} = 2\pi\sqrt{\frac{\ell}{g_{Moon}}} = 2\pi\sqrt{\frac{\frac{g_E T_E^2}{4\pi^2}}{g_{Moon}}} = T_E\sqrt{\frac{g_E}{g_{Moon}}} = (1.60\text{ s})\sqrt{\frac{9.8\ \frac{\text{m}}{\text{s}^2}}{1.62\ \frac{\text{m}}{\text{s}^2}}} = \boxed{3.94\text{ s}}$

8.) simple pendulum $\ell = 0.55\text{ m}$ deflected 7°

highest speed is at the bottom of its swing or one-fourth of its period or when $\omega t = \frac{\pi}{2}$ and $t = \frac{\pi}{2\omega} = \frac{\pi}{2\left(\frac{2\pi}{T}\right)} = \frac{T}{4}$

$$T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{0.55\text{ m}}{9.8\ \frac{\text{m}}{\text{s}^2}}} = 1.49\text{ s and } t = \frac{T}{4} = \frac{1.49\text{ s}}{4} = \boxed{0.37\text{ s}}$$

9.) $m = 0.500\text{ kg}$ simple harmonic motion on a horizontal spring $k = 400\ \frac{\text{N}}{\text{m}}$ when $x = 0.012\text{ m}$, $v = 0.300\ \frac{\text{m}}{\text{s}}$

a.) $E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.500\text{ kg})\left(0.300\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}\left(400\ \frac{\text{N}}{\text{m}}\right)(0.012\text{ m})^2 = \boxed{0.0513\text{ J}}$

b.) displacement is a maximum when $v = 0$ and all of the energy is potential

$$E = U = \frac{1}{2}kx_{max}^2 \text{ and the maximum displacement is } x_{max} = \sqrt{\frac{2E}{k}} = A = \sqrt{\frac{2(0.0513\text{ J})}{400\ \frac{\text{N}}{\text{m}}}} = \boxed{0.016\text{ m}}$$

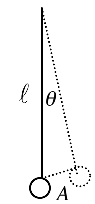
9.) (continued)

c.) $v(t) = -\omega A\sin(\omega t)$ so $v_{max} = \omega A = A\sqrt{\dfrac{k}{m}} = (0.016\text{ m})\sqrt{\dfrac{400\ \frac{\text{N}}{\text{m}}}{0.500\text{ kg}}} = \boxed{0.453\ \dfrac{\text{m}}{\text{s}}}$

alternatively when the speed is a maximum all energy is kinetic

$$E = K = \frac{1}{2}mv_{max}^2 \text{ and } v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.0513\text{ J})}{0.500\text{ kg}}} = \boxed{0.453\ \frac{\text{m}}{\text{s}}}$$

10.) simple pendulum $m = 0.250$ kg and $\ell = 1.00$ m displace $\theta = 15°$



a.) $A = \ell\sin\theta = (1.00\text{ m})\sin 15° = \boxed{0.26\text{ m}}$

b.) $T = 2\pi\sqrt{\dfrac{\ell}{g}}$ and $\omega = \dfrac{2\pi}{T} = \sqrt{\dfrac{g}{\ell}}$

$$v(t) = -\omega A\sin(\omega t) \text{ so } v_{max} = \omega A = A\sqrt{\frac{g}{\ell}} = (0.26\text{ m})\sqrt{\frac{9.8\ \frac{\text{m}}{\text{s}^2}}{1.00\text{ m}}} = \boxed{0.81\ \frac{\text{m}}{\text{s}}}$$

c.) $a(t) = -\omega^2 A\cos(\omega t)$ so $a_{max} = \omega^2 A = A\dfrac{g}{\ell} = (0.26\text{ m})\left(\dfrac{9.8\ \frac{\text{m}}{\text{s}^2}}{1.00\text{ m}}\right) = 2.55\ \dfrac{\text{m}}{\text{s}^2}$ and

$$\alpha_{max} = \frac{a_{max}}{\ell} = \frac{2.55\ \frac{\text{m}}{\text{s}^2}}{1.00\text{ m}} = \boxed{2.55\ \frac{\text{rad}}{\text{s}^2}}$$

d.) using Newton's 2nd Law $F = ma = (0.250\text{ kg})\left(2.55\ \dfrac{\text{m}}{\text{s}^2}\right) = \boxed{0.64\text{ N}}$

11.) physical pendulum $f = 0.450$ Hz, $m = 2.20$ kg, $d = 0.350$ m

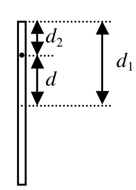
$\tau = -I\alpha = -I\dfrac{d^2\theta}{dt^2}$ and since $\tau = rF\sin\theta = dmg\sin\theta$ combining these $dmg\sin\theta = -I\dfrac{d^2\theta}{dt^2}$

for small angles $\sin\theta \approx \theta$ so $dmg\theta = -I\dfrac{d^2\theta}{dt^2}$ or $\dfrac{d^2\theta}{dt^2} = -\dfrac{mgd}{I}\theta = -\omega^2\theta$ and the motion is simple harmonic

therefore for a physical pendulum $\omega = \sqrt{\dfrac{mgd}{I}}$ and $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{I}{mgd}}$

since $\omega = 2\pi f$ it follows that $I = mgd\left(\dfrac{1}{2\pi f}\right)^2 = (2.20\text{ kg})\left(9.8\ \dfrac{\text{m}}{\text{s}^2}\right)(0.350\text{ m})\left(\dfrac{1}{2\pi(0.450\text{ s}^{-1})}\right)^2 = \boxed{0.944\text{ kg}\cdot\text{m}^2}$

12.) physical pendulum thin rod $L = 1.0$ m, $M = 0.40$ kg, axis is $d_2 = 0.20$ m from end



for a thin rod $I_{cm} = \dfrac{1}{12}ML^2$ distance from end to center of mass $d_1 = 0.5$ m

distance from axis to center of mass $d = 0.3$ m

using parallel axis theorem $I = I_{cm} + Md^2 = \dfrac{1}{12}ML^2 + Md^2 = M\left(\dfrac{1}{12}L^2 + d^2\right)$

12.) (continued)

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M\left(\frac{1}{12}L^2 + d^2\right)}{Mgd}} = 2\pi\sqrt{\frac{\left(\frac{1}{12}L^2 + d^2\right)}{gd}} = 2\pi\sqrt{\frac{\left(\frac{1}{12}(1.00\text{ m})^2 + (0.3\text{ m})^2\right)}{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.3\text{ m})}} = \boxed{1.53\text{ s}}$$

$$T = \frac{1}{f} \text{ so } f = \frac{1}{T} = \frac{1}{1.53\text{ s}} = \boxed{0.65\text{ Hz}}$$

13.) simple pendulum $\ell = 2.23$ m, $m = 6.74$ kg given an initial speed $v_{max} = 2.06\ \frac{\text{m}}{\text{s}}$ at its equilibrium point

a.) $$T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{2.23\text{ m}}{9.8\ \frac{\text{m}}{\text{s}^2}}} = \boxed{3.00\text{ s}}$$

b.) at equilibrium point all energy is kinetic so $E = K = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(6.74\text{ kg})\left(2.06\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{14.3\text{ J}}$

c.) $a(t) = -\omega^2 A\cos(\omega t)$ so $a_{max} = \omega^2 A$ and $v(t) = -\omega A\sin(\omega t)$ so $v_{max} = \omega A$

$$T = \frac{2\pi}{\omega} \text{ so } \omega = \frac{2\pi}{T} = \sqrt{\frac{g}{\ell}} \text{ and } a_{max} = \omega^2 A = (\omega A)\omega = v_{max}\omega = v_{max}\sqrt{\frac{g}{\ell}}$$

$$\text{so } a_{max} = \left(2.06\ \frac{\text{m}}{\text{s}}\right)\sqrt{\frac{9.8\ \frac{\text{m}}{\text{s}^2}}{2.23\text{ m}}} = \boxed{4.32\ \frac{\text{m}}{\text{s}^2}}$$

d.) $$v(t) = -\omega A\sin(\omega t) \text{ so } v_{max} = \omega A \text{ so } A = \frac{v_{max}}{\omega} = \frac{v_{max}}{\sqrt{\frac{g}{\ell}}} = v_{max}\sqrt{\frac{\ell}{g}} \text{ and } A = \ell\sin\theta_{max}$$

$$\theta_{max} = \sin^{-1}\left(\frac{A}{\ell}\right) = \sin^{-1}\left(\frac{v_{max}\sqrt{\frac{\ell}{g}}}{\ell}\right) = \sin^{-1}\left(\frac{v_{max}}{\sqrt{g\ell}}\right) = \boxed{26^\circ}$$

14.) vertical spring length $\ell_s = 0.050$ m when block $m = 0.20$ kg suspended from it spring length is $\ell = 0.060$ m so the spring is stretched $\Delta y = \ell - \ell_s = 0.060\text{ m} - 0.050\text{ m} = 0.010\text{ m}$

$$F_g = mg = k\Delta y \text{ so } k = \frac{mg}{\Delta y} = \frac{(0.20\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)}{0.010\text{ m}} = 196\ \frac{\text{N}}{\text{m}}$$

placed on horizontal surface and displaced so the spring is $\ell = 0.10$ m long and released undergoing simple harmonic motion with an amplitude $A = \ell - \ell_s = 0.10\text{ m} - 0.050\text{ m} = 0.050\text{ m}$

a.) $$\omega = 2\pi f \text{ so } f = \frac{\omega}{2\pi} = \frac{\sqrt{\frac{k}{m}}}{2\pi} = \frac{\sqrt{\frac{196\ \frac{\text{N}}{\text{m}}}{0.20\text{ kg}}}}{2\pi} = \boxed{4.98\text{ Hz}}$$

14.) (continued)

b.) when spring is $\ell = 0.050 \text{ m}$ long it is at its equilibrium point and the velocity $v = v_{max}$

$$v(t) = -\omega A\sin(\omega t) \text{ so } v_{max} = \omega A = A\sqrt{\frac{k}{m}} = (0.050 \text{ m})\sqrt{\frac{196 \ \frac{\text{N}}{\text{m}}}{0.20 \text{ kg}}} = \boxed{1.565 \ \frac{\text{m}}{\text{s}}}$$

c.) $$a(t) = -\omega^2 A\cos(\omega t) \text{ so } a_{max} = \omega^2 A = A\frac{k}{m} = (0.050 \text{ m})\left(\frac{196 \ \frac{\text{N}}{\text{m}}}{0.20 \text{ kg}}\right) = \boxed{49 \ \frac{\text{m}}{\text{s}^2}}$$

d.) energy is conserved and is all potential energy when displacement is a maximum

$$E = U = \frac{1}{2}kx_{max}^2 = \frac{1}{2}\left(196 \ \frac{\text{N}}{\text{m}}\right)(0.050 \text{ m})^2 = \boxed{0.245 \text{ J}}$$

alternatively, the energy is all kinetic energy when velocity is a maximum

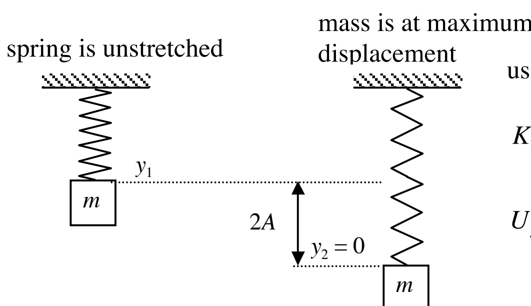
$$E = K = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.20 \text{ kg})\left(1.565 \ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{0.245 \text{ J}}$$

15.) vertical spring stretches $\Delta y = 0.02 \text{ m}$ when mass $m = 0.40 \text{ kg}$ is hung from it

$$F_g = mg = k\Delta y \text{ so } k = \frac{mg}{\Delta y} = \frac{(0.40 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)}{0.020 \text{ m}} = 196 \ \frac{\text{N}}{\text{m}}$$

a mass $m = 0.20 \text{ kg}$ is attached and held at rest when spring is not stretched and then released and oscillates with simple harmonic motion

a.)



using Conservation of Energy ($v_1 = v_2 = 0$)

$$K_1 + U_{g1} + U_{e1} = K_2 + U_{g2} + U_{e2} \text{ so } 0 + U_{g1} + 0 = 0 + 0 + U_{e2}$$

$$U_{g1} = U_{e2} \text{ and } mgy_1 = \frac{1}{2}k\Delta y^2 = \frac{1}{2}k(y_1 - y_2)^2$$

$$mgy_1 = \frac{1}{2}ky_1^2 \text{ so } y_1 = \frac{2mg}{k} = 2A$$

$$A = \frac{mg}{k} = \frac{(0.20 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)}{196 \ \frac{\text{N}}{\text{m}}} = \boxed{0.010 \text{ m}}$$

so the amplitude is the amount that the mass would stretch the spring when hung from it

b.) $$v(t) = -\omega A\sin(\omega t) \text{ so } v_{max} = \omega A = A\sqrt{\frac{k}{m}} = (0.010 \text{ m})\sqrt{\frac{196 \ \frac{\text{N}}{\text{m}}}{0.20 \text{ kg}}} = \boxed{0.313 \ \frac{\text{m}}{\text{s}}}$$

16.) $m = 0.4$ kg has position $x = (0.4\text{ m})\cos\left(\frac{\pi}{3}t\right)$ where t is in seconds

by inspection of the equation for position and recalling that for simple harmonic motion $x(t) = A\cos(\omega t)$

$$A = 0.4\text{ m and } \omega = \frac{\pi}{3}\ \frac{\text{rad}}{\text{s}}$$

a.) $v(t) = -\omega A\sin(\omega t)$ so $v_{max} = \omega A = \left(\frac{\pi}{3}\ \frac{\text{rad}}{\text{s}}\right)(0.4\text{ m}) = \boxed{0.42\ \frac{\text{m}}{\text{s}}}$

b.) the energy is all kinetic energy when velocity is a maximum

$$E = K = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.4\text{ kg})\left(0.42\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{0.035\text{ J}}$$

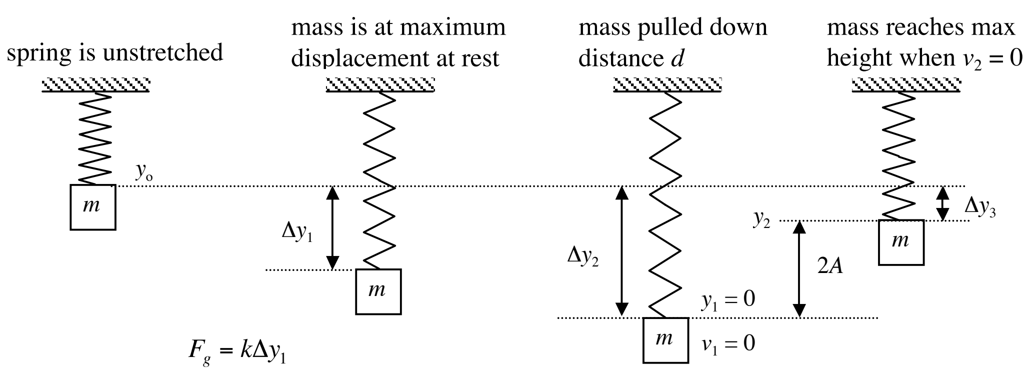
c.) $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\left(\frac{\pi}{3}\ \frac{\text{rad}}{\text{s}}\right)} = \boxed{6\text{ s}}$

d.) $a(t) = -\omega^2 A\cos(\omega t)$ so $a(1.5\text{ s}) = -\left(\frac{\pi}{3}\ \frac{\text{rad}}{\text{s}}\right)^2(0.4\text{ m})\cos\left(\left(\frac{\pi}{3}\ \frac{\text{rad}}{\text{s}}\right)(1.5\text{ s})\right) = \boxed{0}$

one could also notice that when $t = 1.5$ s, $t = \frac{T}{4}$ and the mass is moving through the equilibrium point where $v = v_{max}$ and $a = 0$

17.) $m = 1.5$ kg attached to a vertical spring $k = 300\ \frac{\text{N}}{\text{m}}$

block is released and comes to rest and then pulled down a distance $d = 0.02$ m and released



$F_g = k\Delta y_1$

spring $\quad \Delta y_1 = \dfrac{mg}{k} \qquad \Delta y_2 = \Delta y_1 + d \qquad \Delta y_3 = \Delta y_2 - y_2$

using Energy Conservation between the time when the block is released and when it reaches its maximum height

$K_1 + U_{g1} + U_{e1} = K_2 + U_{g2} + U_{e2}$ and $0 + 0 + U_{e1} = 0 + U_{g2} + U_{e2}$ or $U_{e1} = U_{g2} + U_{e2}$

$\frac{1}{2}k\Delta y_2^2 = mgy_2 + \frac{1}{2}k\Delta y_2^2$ and $\frac{1}{2}k(\Delta y_1 + d)^2 = mgy_2 + \frac{1}{2}k(\Delta y_2 - y_2)^2$ or $\frac{1}{2}k\left(\frac{mg}{k} + d\right)^2 = mgy_2 + \frac{1}{2}k\left(\frac{mg}{k} + d - y_2\right)^2$

$$k\left(\left(\frac{mg}{k}\right)^2 + 2\frac{mg}{k}d + d^2\right) = 2mgy_2 + k\left(\left(\frac{mg}{k}\right)^2 + 2\frac{mgd}{k} - 2\frac{mgy_2}{k} - 2dy_2 + d^2 + y_2^2\right)$$

$$k\left(\frac{mg}{k}\right)^2 + 2mgd + kd^2 = 2mgy_2 + k\left(\frac{mg}{k}\right)^2 + 2mgd - 2mgy_2 - 2kdy_2 + kd^2 + ky_2^2$$

17.) (continued)

$0 = -2kdy_2 + ky_2^2$ so $0 = y_2(-2kd + ky_2)$ and the roots are $y_2 = 0$ and $y_2 = 2d$

so $y_2 = 2d = 2A$ and $A = d = 0.020$ m the amount the block was pulled down

a.) for simple harmonic motion $y(t) = A\cos(\omega t)$ and for spring oscillator $\omega = \sqrt{\frac{k}{m}}$

so $\omega = \sqrt{\dfrac{300\ \frac{\text{N}}{\text{m}}}{1.5\ \text{kg}}} = 14.1\ \frac{\text{rad}}{\text{s}}$ and $\boxed{y(t) = (0.020\ \text{m})\cos\left((14.14\ {}^{\text{rad}}\!/\!_{\text{s}})t\right)}$

b.) maximum amount of stretch is when block is at lowest point and spring is stretched $\Delta y_2 = \Delta y_1 + d$

or $\Delta y_2 = \dfrac{mg}{k} + d = \dfrac{(1.5\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)}{300\ \frac{\text{N}}{\text{m}}} + 0.020\ \text{m} = 0.049\ \text{m} + 0.020\ \text{m} = \boxed{0.069\ \text{m}}$

minimum amount of stretch is when block is at highest point and spring is stretched $\Delta y_3 = \Delta y_2 - y_2$

or $\Delta y_3 = \Delta y_1 + d - 2d = \dfrac{mg}{k} - d = 0.049\ \text{m} - 0.020\ \text{m} = \boxed{0.029\ \text{m}}$

so the block oscillates $\pm A$ from it equilibrium point which the amount the block stretches the spring after coming to rest

Some general comments on the two special cases for vertical spring oscillations

(1) In problem 15, the mass is released from an unstretched spring and the amplitude of the oscillation was found to be $A = \frac{mg}{k}$ which is the amount the mass stretches the spring when at rest. The range of values for the spring is bounded by $\Delta y = 0$ and $\Delta y = \frac{2mg}{k}$ during the oscillatory motion.

(2) In problem 17, the mass was pulled down a distance *d* from the point in which the mass stretches the spring when at rest. The amplitude of the oscillation in this case was the distance *d* that block was pulled down and the equilibrium point is when the block returns to point where the mass stretches the spring when at rest. The amount of stretch in the spring in this case is bounded by $\Delta y = \frac{mg}{k} \pm d$ during the oscillatory motion.