Example 1:

A 2-kg object is attached to a horizontal spring of force constant 5 kN/m. The spring is stretched 10 cm from equilibrium and released. Find (*a*) the frequency, (*b*) the period, and (*c*) the amplitude of motion. (*d*) What is the maximum speed? (*e*) What is the maximum acceleration? (*f*) When does the object first reach equilibrium? What is its acceleration at this time?

Example 1:

$$m = 2\text{ kg},\ k = 5000\ \frac{\text{N}}{\text{m}},\ \text{and } A = 0.10\text{ m}$$

a.) $f = ?$

$f = \frac{2\pi}{\omega}$

$$F = ma$$
$$-kx = m\frac{d^2x}{dt^2}$$
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5000\text{ N}}{2\text{ kg}}}$$
$$\omega = 50\ \frac{\text{rad}}{\text{s}}$$
$$f = \frac{2\pi}{\omega} = \frac{2\pi}{\left(50\ \frac{\text{rad}}{\text{s}}\right)}$$
$$\boxed{f = 7.96\text{ Hz}}$$

Example 1:

$$m = 2\text{ kg},\ k = 5000\ \frac{\text{N}}{\text{m}},\ \text{and } A = 0.10\text{ m}$$

b.) $T = ?$

$$T = \frac{1}{f} = \frac{1}{7.96\text{ Hz}}$$
$$\boxed{T = 0.126\text{ s}}$$

b and a.) $f = ?$

$T = \frac{1}{f}$ so $f = \frac{1}{T}$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$
$$T = 2\pi\sqrt{\frac{(2\text{ kg})}{(5000\text{ N})}}$$
$$\boxed{T = 0.126\text{ s}}$$
$$f = \frac{1}{T} = \frac{1}{(0.126\text{ s})}$$
$$\boxed{f = 7.96\text{ Hz}}$$

Example 1:

$$m = 2\text{ kg},\ k = 5000\ \frac{\text{N}}{\text{m}},\ \text{and } A = 0.10\text{ m}$$

c.) $\boxed{A = 0.10\text{ m}}$

d.) $v_{\max} = ?$

$$x = x_{\max}\cos(\omega t + \phi)$$
$$x = A\cos(\omega t)$$
$$v = \frac{dx}{dt} = \frac{d(A\cos(\omega t))}{dt}$$
$$v = -\omega A\sin(\omega t) = -v_{\max}\sin(\omega t)$$
$$v_{\max} = \omega A = \left(50\ \frac{\text{rad}}{\text{s}}\right)(0.10\text{ m})$$
$$\boxed{v_{\max} = 5.0\ \frac{\text{m}}{\text{s}}}$$

Example 1:

$$m = 2\text{ kg},\ k = 5000\ \frac{\text{N}}{\text{m}},\ \text{and } A = 0.10\text{ m}$$

e.) $a_{\max} = ?$

$$v = -\omega A\sin(\omega t)$$
$$a = \frac{dv}{dt} = \frac{d(-\omega A\sin(\omega t))}{dt}$$
$$a = -\omega^2 A\cos(\omega t) = -a_{\max}\cos(\omega t)$$
$$a_{\max} = \omega^2 A = \left(50\ \frac{\text{rad}}{\text{s}}\right)^2(0.10\text{ m})$$
$$\boxed{a_{\max} = 250\ \frac{\text{m}}{\text{s}^2}}$$

f.) when $x = 0$, $t = ?$, $a = ?$ $\quad x = A\cos(\omega t) = 0$

$$\omega t = \frac{\pi}{2} \text{ so } t = \frac{\pi}{2\omega} = \frac{\pi}{2\left(\frac{2\pi}{T}\right)} = \frac{T}{4} = \frac{0.126\text{ s}}{4}$$
$$\boxed{t = 0.0314\text{ s}}$$
$$a = -\omega^2 A\cos(\omega t) = -\omega^2 A\cos\left(\frac{\pi}{2}\right)$$
$$\boxed{a = 0}$$

Example 2:

A 3-kg object oscillating on a spring of force constant 2 kN/m has a total energy of 0.9 J. (*a*) What is the amplitude of the motion? (*b*) What is the maximum speed?

Example 2:

$$m = 3\text{ kg},\ k = 2000\ \frac{\text{N}}{\text{m}},\text{ and } E = 0.9\text{ J}$$

a.) $A = ?$

$$E = K + U$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

when $x = A,\ v = 0$ and $E = 0 + \frac{1}{2}kA^2 = \frac{1}{2}kA^2$

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.9\text{ J})}{\left(2000\ \frac{\text{N}}{\text{m}}\right)}}$$

$$\boxed{A = 0.03\text{ m}}$$

b.) $v_{max} = ?$

$$v_{max} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{\left(2000\ \frac{\text{N}}{\text{m}}\right)}{3\text{ kg}}}(0.03\text{ m})$$

$$\boxed{v_{max} = 0.775\ \frac{\text{m}}{\text{s}}}$$

Example 2:

$$A\ [=]\ \sqrt{\frac{\text{J}}{\frac{\text{N}}{\text{m}}}}\ [=]\ \sqrt{\frac{\text{N}\cdot\text{m}}{\frac{\text{N}}{\text{m}}}}\ [=]\ \sqrt{\text{m}^2}\ [=]\ \text{m}$$

$$v_{max}\ [=]\ \sqrt{\frac{\frac{\text{N}}{\text{m}}}{\text{kg}}}\text{ m}\ [=]\ \sqrt{\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{\text{kg}}}\text{ m}\ [=]\ \sqrt{\frac{\text{m}^2}{\text{s}^2}}\ [=]\ \frac{\text{m}}{\text{s}}$$

Example 3:

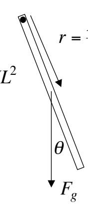
A pendulum is made using a meter stick with a mass of 94 g pivoting through an axis on one of its ends. What is the period for its motion for small angular displacements?

Example 3:

$$M = 94\text{ g},\ L = 1.0\text{ m, and } d = 0.5\text{ m}$$

$T = ?$

$I = \frac{1}{3}ML^2$



$$\tau = -I\alpha$$

$$rF_g\sin\theta = -I\frac{d^2\theta}{dt^2}$$

$$\frac{L}{2}Mg\theta = -\frac{1}{3}ML^2\frac{d^2\theta}{dt^2}\quad \text{(for small angles }\sin\theta \approx \theta\text{)}$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{3g}{2L}\right)\theta = -\omega^2\theta$$

$$\omega = \sqrt{\frac{3g}{2L}}$$

$$T = \frac{2\pi}{\omega}\quad \text{so } T = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(1\text{ m})}{3\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)}}$$

$$\boxed{T = 1.64\text{ s}}$$

Example 4:

The motion of a particle is described by the following equation.

$$x = (10\text{ m})\cos\left(5\frac{\text{rad}}{\text{s}}\cdot t\right)$$

Find:

a.) the amplitude

b.) the maximum displacement

c.) the angular velocity

d.) the maximum velocity

e.) the maximum acceleration

Example 4:

$$x = (10\text{ m})\cos\left(5\frac{\text{rad}}{\text{s}}\cdot t\right)$$

a.) $A = ?$

$$x = A\cos(\omega t)$$

$$\boxed{A = 10\text{ m}}$$

$$v = -\left(50\ \frac{\text{m}}{\text{s}}\right)\sin\left(5\frac{\text{rad}}{\text{s}}\cdot t\right)$$

b.) $x_{max} = ?$

$$x = x_{max}\cos(\omega t)$$

$$\boxed{x_{max} = 10\text{ m}}$$

$$a = -\left(250\ \frac{\text{m}}{\text{s}^2}\right)\cos\left(5\frac{\text{rad}}{\text{s}}\cdot t\right)$$

c.) $\omega = ?$

$$x = A\cos(\omega t)$$

$$\boxed{\omega = 5\ \frac{\text{rad}}{\text{s}}}$$

d.) $v_{max} = ?$

$$v_{max} = \omega A = \left(5\ \frac{\text{rad}}{\text{s}}\right)(10\text{ m})$$

$$\boxed{v_{max} = 50\ \frac{\text{m}}{\text{s}}}$$

e.) $a_{max} = ?$

$$a_{max} = \omega^2 A = \left(5\ \frac{\text{rad}}{\text{s}}\right)^2(10\text{ m})$$

$$\boxed{a_{max} = 250\ \frac{\text{m}}{\text{s}^2}}$$