1.) $r_2 = 2r_1$

$$F_{G_1} = G\frac{m_1 m_2}{r_1^2} \text{ and } F_{G_2} = G\frac{m_1 m_2}{r_2^2} = G\frac{m_1 m_2}{(2r_1)^2} = G\frac{m_1 m_2}{4r_1^2} = \frac{1}{4}G\frac{m_1 m_2}{r_1^2} = \boxed{\frac{1}{4}F_{G_1}}$$

2.) the altitude is the distance above the surface of the earth and r is measured from the center of the planet so

so $r_2 = 2R_E + R_E = 3R_E$

at the surface of the earth $F_{G_E} = G\frac{mM_E}{R_E^2} = w$

the weight at r_2 is $F_{G_2} = G\frac{mM_E}{r_2^2} = G\frac{mM_E}{(3R_E)^2} = G\frac{mM_E}{9R_E^2} = \frac{1}{9}G\frac{mM_E}{R_E^2} = \boxed{\frac{1}{9}w}$

3.) The gravitational force on each mass due to other is the same.

4.) on Earth $g = G\frac{M_E}{R_E^2} = 9.8\ \frac{\text{m}}{\text{s}^2}$ and for Pluto $M_p = \frac{1}{500}M_E$ and $R_P = \frac{1}{15}R_E$ so

$$g_P = G\frac{M_P}{R_P^2} = G\frac{\frac{1}{500}M_E}{\left(\frac{1}{15}R_E\right)^2} = G\frac{\frac{1}{500}M_E}{\frac{1}{225}R_E^2} = \frac{225}{500}G\frac{M_E}{R_E^2} = \frac{225}{500}g = \frac{225}{500}\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{4.41\ \frac{\text{m}}{\text{s}^2}}$$

5.) circular orbit radius R and kinetic energy K_1 moved to new orbit radius $2R$

$$F_{G_1} = G\frac{mM_E}{R^2} = ma = m\frac{v_1^2}{R} \text{ so } v_1 = \sqrt{\frac{GM_E}{R}} \text{ and } K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\left(\sqrt{\frac{GM_E}{R}}\right)^2 = \frac{1}{2}\frac{GmM_E}{R}$$

$$F_{G_2} = G\frac{mM_E}{(2R)^2} = ma = m\frac{v_2^2}{2R} \text{ so } v_2 = \sqrt{\frac{GM_E}{2R}} \text{ and } K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m\left(\sqrt{\frac{GM_E}{2R}}\right)^2 = \frac{1}{2}\frac{GmM_E}{2R} = \frac{1}{2}\left(\frac{1}{2}\frac{GmM_E}{R}\right) = \boxed{\frac{1}{2}K_1}$$

6.) moon of Jupiter with a circular orbit radius R and period T

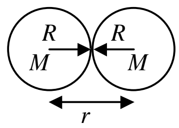
$$F_G = G\frac{mM_J}{R^2} = ma = m\frac{v^2}{R} \text{ so } v = \sqrt{\frac{GM_J}{R}} = \frac{2\pi R}{T} \text{ or } \frac{GM_J}{R} = \frac{4\pi^2 R^2}{T^2} \text{ and } M_J = \boxed{\frac{4\pi^2 R^3}{GT^2}}$$

7.) $R_s = 9R_E$

for Earth $F_{G_E} = G\frac{M_{Sun}M_E}{R^2} = M_E a = M_E\frac{v_E^2}{R}$ so $v_E = \sqrt{\frac{GM_{Sun}}{R}} = \frac{2\pi R}{T_E}$ and $T_E = 2\pi R\sqrt{\frac{R}{GM_{Sun}}} = 2\pi\sqrt{\frac{R^3}{GM_{Sun}}}$

for Saturn $F_{G_S} = G\frac{M_{Sun}M_S}{(9R)^2} = M_S a = M_S\frac{v_S^2}{9R}$ so $v_S = \sqrt{\frac{GM_{Sun}}{9R}} = \frac{2\pi(9R)}{T_S}$

$$T_S = 2\pi(9R)\sqrt{\frac{9R}{GM_{Sun}}} = 2\pi 9\sqrt{9}\sqrt{\frac{R^3}{GM_{Sun}}} = 27\left(2\pi\sqrt{\frac{R^3}{GM_{Sun}}}\right) = 27T_E \text{ or } \boxed{\text{27 Earth years}}$$

8.) 

two spheres mass M and radius R

centers of mass are a distance of $2R$ from each other

$$F_G = G\frac{m_1 m_2}{r^2} = G\frac{MM}{(2R)^2} = \boxed{G\frac{M^2}{4R^2}}$$

9.) Moon mass M and radius R object dropped a distance of $3R$ from Moon's center

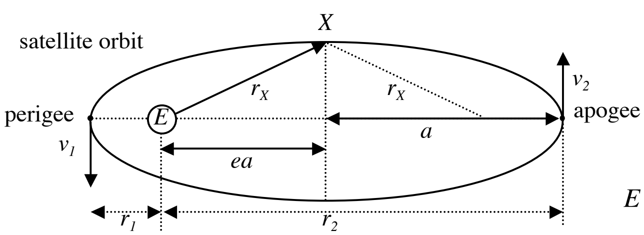
using Conservation of Energy $(r_1 = 3R \text{ and } r_2 = R)$

$$K_1 + U_{g1} = K_2 + U_{g2} \text{ and } v_1 = 0 \text{ so } U_{g1} = K_2 + U_{g2} \text{ or } \frac{-GMm}{r_1} = \frac{1}{2}mv_2^2 + \frac{-GMm}{r_2}$$

$$v_2 = \sqrt{\frac{2GM}{r_2} + \frac{-2GM}{r_1}} = \sqrt{2GM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{3R}\right)} = \sqrt{2GM\left(\frac{2}{3R}\right)} = \boxed{\sqrt{\frac{4GM}{3R}}}$$

10.) $M_C = 6M_E$ and $g_C = g_E$

for Earth $g_E = G\frac{M_E}{R_E^2}$ and for Cosmo $g_C = G\frac{M_C}{R_C^2} = G\frac{6M_E}{R_C^2}$ and $g_C = g_E$ so $G\frac{M_E}{R_E^2} = G\frac{6M_E}{R_C^2}$ and $R_C = \boxed{\sqrt{6}R_E}$

11.) 

For circular orbits the total energy is conserved

$$E = K + U = \frac{1}{2}mv^2 + \frac{-GMm}{r} \text{ and } v = \sqrt{\frac{GM}{r}}$$

$$E = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 - \frac{GMm}{r} = \frac{1}{2}m\frac{GM}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$$

for elliptical orbits energy is also conserved and r is replaced by a the semi-major axis of the ellipse so $E = -\frac{1}{2}\frac{GMm}{a}$

$$a = \frac{r_1 + r_2}{2} \text{ therefore at all points } E = -\frac{1}{2}\frac{GMm}{\left(\frac{r_1 + r_2}{2}\right)} = -\frac{GMm}{(r_1 + r_2)}$$

a.) at perigee

$$E = -\frac{GMm}{(r_1 + r_2)} = K + U = \frac{1}{2}mv_1^2 + \frac{-GMm}{r_1} \text{ so } v_1 = \sqrt{\frac{2GM}{r_1} - \frac{2GM}{(r_1 + r_2)}}$$

$$v_1 = \sqrt{\frac{2GM(r_1 + r_2)}{r_1(r_1 + r_2)} - \frac{2GMr_1}{r_1(r_1 + r_2)}} = \sqrt{\frac{2GMr_1 + 2GMr_2 - 2GMr_1}{r_1(r_1 + r_2)}} = \boxed{\sqrt{\frac{2GMr_2}{r_1(r_1 + r_2)}}}$$

b.) at apogee

$$E = -\frac{GMm}{(r_1 + r_2)} = K + U = \frac{1}{2}mv_2^2 + \frac{-GMm}{r_2} \text{ so } v_2 = \sqrt{\frac{2GM}{r_2} - \frac{2GM}{(r_1 + r_2)}}$$

$$v_2 = \sqrt{\frac{2GM(r_1 + r_2)}{r_2(r_1 + r_2)} - \frac{2GMr_2}{r_2(r_1 + r_2)}} = \sqrt{\frac{2GMr_1 + 2GMr_2 - 2GMr_2}{r_2(r_1 + r_2)}} = \boxed{\sqrt{\frac{2GMr_1}{r_2(r_1 + r_2)}}}$$

11.) (continued)

c.) $$\frac{v_1}{v_2} = \frac{\sqrt{\dfrac{2GMr_2}{r_1(r_1+r_2)}}}{\sqrt{\dfrac{2GMr_1}{r_2(r_1+r_2)}}} = \frac{\sqrt{\dfrac{r_2}{r_1}}}{\sqrt{\dfrac{r_1}{r_2}}} = \sqrt{\frac{r_2^2}{r_1^2}} = \boxed{\frac{r_2}{r_1}}$$

d.) $$\vec{L}_2 = \vec{r}_2 \times \vec{p}_2 = mr_2v_2\sin\phi_2 = mr_2v_2\sin 90° = mr_2\sqrt{\frac{2GMr_1}{r_2(r_1+r_2)}} = \boxed{m\sqrt{\frac{2GMr_1r_2}{(r_1+r_2)}}}$$

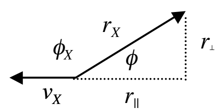
e.) since X is the same distance from each focal point $r_X + r_X = 2a$ and $r_X = a = \dfrac{r_1+r_2}{2}$

using Conservation of Energy

$$E = -\frac{GMm}{(r_1+r_2)} = K + U = \frac{1}{2}mv_X^2 + \frac{-GMm}{r_X}$$

$$v_X = \sqrt{\frac{2GM}{r_X} - \frac{2GM}{(r_1+r_2)}} = \sqrt{\frac{2GM}{\left(\dfrac{r_1+r_2}{2}\right)} - \frac{2GM}{(r_1+r_2)}} = \sqrt{\frac{4GM}{(r_1+r_2)} - \frac{2GM}{(r_1+r_2)}} = \boxed{\sqrt{\frac{2GM}{(r_1+r_2)}}}$$

angular momentum is also conserved so $L_X = L_1$ and $mv_Xr_X\sin\phi_X = m\sqrt{\dfrac{2GMr_1r_2}{(r_1+r_2)}}$ (1)



$$\sin\phi = \sin(180° - \phi_X) = \sin(180°)\cos\phi_X - \cos(180°)\sin\phi_X = \sin\phi_X$$

$r_\parallel^2 + r_\perp^2 = r_X^2$ and $r_\parallel = a - r_1 = \dfrac{r_1+r_2}{2} - r_1 = \dfrac{r_2-r_1}{2}$

$$r_\perp = \sqrt{r_X^2 - r_\parallel^2} = \sqrt{\left(\frac{r_1+r_2}{2}\right)^2 - \left(\frac{r_2-r_1}{2}\right)^2} = \sqrt{\frac{r_1^2+2r_1r_2+r_2^2}{4} - \frac{r_1^2-2r_1r_2+r_2^2}{4}} = \sqrt{\frac{4r_1r_2}{4}} = \sqrt{r_1r_2}$$

so $\sin\phi_X = \sin\phi = \dfrac{r_\perp}{r_X} = \dfrac{\sqrt{r_1r_2}}{r_X}$ and $mv_Xr_X\sin\phi_X = mv_Xr_X\dfrac{\sqrt{r_1r_2}}{r_X} = mv_X\sqrt{r_1r_2}$ and using (1)

$$mv_X\sqrt{r_1r_2} = m\sqrt{\frac{2GMr_1r_2}{(r_1+r_2)}} \text{ and } v_X = \frac{1}{\sqrt{r_1r_2}}\sqrt{\frac{2GMr_1r_2}{(r_1+r_2)}} = \frac{\sqrt{r_1r_2}}{\sqrt{r_1r_2}}\sqrt{\frac{2GM}{(r_1+r_2)}} = \boxed{\sqrt{\frac{2GM}{(r_1+r_2)}}}$$

f.) for circular orbits $v = \sqrt{\dfrac{GM}{r}} = \dfrac{2\pi r}{T}$ and $T = 2\pi r\sqrt{\dfrac{r}{GM}} = 2\pi\sqrt{\dfrac{r^3}{GM}}$

for elliptical orbits replace r with a so $T = 2\pi\sqrt{\dfrac{a^3}{GM}} = 2\pi\sqrt{\dfrac{\left(\dfrac{r_1+r_2}{2}\right)^3}{GM}} = \boxed{2\pi\sqrt{\dfrac{(r_1+r_2)^3}{8GM}}}$

g.) $ea = r_\parallel = \dfrac{r_2-r_1}{2}$ and $a = \dfrac{r_1+r_2}{2}$ so $e = \dfrac{r_2-r_1}{2a} = \dfrac{r_2-r_1}{2\left(\dfrac{r_1+r_2}{2}\right)} = \boxed{\dfrac{r_2-r_1}{r_1+r_2}}$

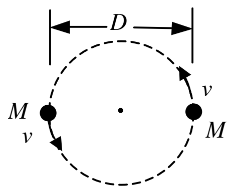
12.) $R_E = 6000$ km

at altitude of 300 km the orbit radius is $r = d + R_E = 300\text{ km} + 6000\text{ km} = 6300\text{ km}$

at the surface of the Earth $g = G\dfrac{M_E}{R_E^2}$ so $M_E = \dfrac{gR_E^2}{G}$

at an orbit radius of r $g_r = G\dfrac{M_E}{r^2} = G\dfrac{\frac{gR_E^2}{G}}{r^2} = \dfrac{gR_E^2}{r^2} = g\left(\dfrac{R_E}{r}\right)^2 = \left(9.8\ \dfrac{\text{m}}{\text{s}^2}\right)\left(\dfrac{6000\text{ km}}{6300\text{ km}}\right)^2 = \boxed{8.9\ \dfrac{\text{m}}{\text{s}^2}}$

13.)



the gravitational attraction between the stars is $F_G = G\dfrac{m_1 m_2}{r^2} = G\dfrac{MM}{D^2}$ (1)

looking at the circular motion of one star $F_G = Ma = M\dfrac{v^2}{r} = M\dfrac{v^2}{\frac{D}{2}} = 2M\dfrac{v^2}{D}$ (2)

equating (1) and (2) $G\dfrac{MM}{D^2} = 2M\dfrac{v^2}{D}$ and $v = \boxed{\sqrt{\dfrac{GM}{2D}}}$

14.) Planet X: $M_X = \dfrac{M_E}{6}$ and $D_X = \dfrac{D_E}{3}$ so $2R_X = \dfrac{2R_E}{3}$ and $R_X = \dfrac{R_E}{3}$

at the surface of the Earth, $g_E = G\dfrac{M_E}{R_E^2}$

at the surface of Planet X, $g_X = G\dfrac{M_X}{R_X^2} = G\dfrac{\frac{M_E}{6}}{\left(\frac{R_E}{3}\right)^2} = G\dfrac{\frac{M_E}{6}}{\frac{R_E^2}{9}} = G\dfrac{\frac{9}{6}M_E}{R_E^2} = \dfrac{3}{2}G\dfrac{M_E}{R_E^2} = \boxed{\dfrac{3}{2}g_E}$

so $g_X = \dfrac{3}{2}\left(9.8\ \dfrac{\text{m}}{\text{s}^2}\right) = \boxed{14.7\ \dfrac{\text{m}}{\text{s}^2}}$