1.) $\alpha = 0.450\ \frac{\text{rad}}{\text{s}^2}$

a.) $\omega_o = 0$ and $\omega = 8.00\ \frac{\text{rad}}{\text{s}}$

$$\omega = \alpha t + \omega_o \text{ so } t = \frac{\omega - \omega_o}{\alpha} = \frac{8.00\ \frac{\text{rad}}{\text{s}} - 0}{0.450\ \frac{\text{rad}}{\text{s}^2}} = \boxed{17.8\text{ s}}$$

b.) $$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta \text{ so } \Delta\theta = \frac{\omega^2 - \omega_o^2}{2\alpha} = \frac{\left(8.00\ \frac{\text{rad}}{\text{s}}\right)^2 - 0}{2\left(0.450\ \frac{\text{rad}}{\text{s}^2}\right)} = 71.11\text{ rad or } \Delta\theta = 71.11\text{ rad}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{11.3\text{ rev}}$$

2.) $\omega_o = 1.50\ \frac{\text{rad}}{\text{s}}$, $\alpha = 0.300\ \frac{\text{rad}}{\text{s}^2}$, $\Delta\theta = 3.50\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 7\pi\text{ rad}$

$$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta \text{ so } \omega = \sqrt{\omega_o^2 + 2\alpha\Delta\theta} = \sqrt{\left(1.50\ \frac{\text{rad}}{\text{s}}\right)^2 + 2\left(0.300\ \frac{\text{rad}}{\text{s}^2}\right)(7\pi\text{ rad})} = \boxed{3.93\ \frac{\text{rad}}{\text{s}}}$$

3.) $\theta(t) = a + bt^2 + ct^3$

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}\left(a + bt^2 + ct^3\right) = 2bt + 3ct^2 \text{ and } \alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(2bt + 3ct^2\right) = \boxed{2b + 6ct}$$

4.) $\alpha = 2.50\ \frac{\text{rad}}{\text{s}^2}$, $\Delta\theta = 80.0\text{ rad}$, $t = 5.00\text{ s}$

$$\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_o t \text{ so } \omega_o = \frac{2\Delta\theta - \alpha t^2}{2t} = \frac{2(80.0\text{ rad}) - \left(2.50\ \frac{\text{rad}}{\text{s}^2}\right)(5.00\text{ s})^2}{2(5.00\text{ s})} = \boxed{9.75\ \frac{\text{rad}}{\text{s}}}$$

5.) $D = 0.200\text{ m}$, $\omega_o = 0$, $\omega = 140\ \frac{\text{rad}}{\text{s}}$, $t = 8.00\text{ s}$

$$\omega = \alpha t + \omega_o \text{ so } \alpha = \frac{\omega - \omega_o}{t} = \frac{140\ \frac{\text{rad}}{\text{s}} - 0}{8.00\text{ s}} = \boxed{17.5\ \frac{\text{rad}}{\text{s}^2}}$$

$$\Delta\theta = \frac{(\omega + \omega_o)}{2}t = \frac{\left(140\ \frac{\text{rad}}{\text{s}} + 0\right)}{2}(8.00\text{ s}) = \boxed{560\text{ rad}}$$

6.) $t = 3.00\text{ s}$, $\Delta\theta = 162\text{ rad}$, $\omega = 108\ \frac{\text{rad}}{\text{s}}$

a.) $$\Delta\theta = \frac{(\omega + \omega_o)}{2}t \text{ so } \omega_o = \frac{2\Delta\theta}{t} - \omega = \frac{2(162\text{ rad})}{3.00\text{ s}} - 108\ \frac{\text{rad}}{\text{s}} = \boxed{0}$$

b.) $$\omega = \alpha t + \omega_o \text{ so } \alpha = \frac{\omega - \omega_o}{t} = \frac{108\ \frac{\text{rad}}{\text{s}} - 0}{3.00\text{ s}} = \boxed{36\ \frac{\text{rad}}{\text{s}^2}}$$

7.) $\Delta\theta = \frac{(\omega + \omega_o)}{2} t$ so (1) $\omega_o = \frac{2\Delta\theta}{t} - \omega$ and (2) $\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_o t$

substituting (1) into (2) $\Delta\theta = \frac{1}{2}\alpha t^2 + \left(\frac{2\Delta\theta}{t} - \omega\right)t = \frac{1}{2}\alpha t^2 + 2\Delta\theta - \omega t$ and $\boxed{\Delta\theta = \omega t - \frac{1}{2}\alpha t^2}$

8.) $\omega_o = 24.0\ \frac{\text{rad}}{\text{s}}$, $\alpha_1 = 60\ \frac{\text{rad}}{\text{s}^2}$, $t = 0$ to $t_1 = 2.0\text{ s}$ then slows down to $\omega = 0$ as $\Delta\theta_2 = 432\text{ rad}$

a.) during the period of acceleration

$$\Delta\theta_1 = \frac{1}{2}\alpha_1 t_1^2 + \omega_o t_1 = \frac{1}{2}\left(60\ \frac{\text{rad}}{\text{s}^2}\right)(2.0\text{ s})^2 + \left(24.0\ \frac{\text{rad}}{\text{s}}\right)(2.0\text{ s}) = 168\text{ rad}$$

$$\Delta\theta_{total} = \Delta\theta_1 + \Delta\theta_2 = 168\text{ rad} + 432\text{ rad} = \boxed{600\text{ rad}}$$

b.) after 2.0 s the angular velocity is $\omega = \alpha_1 t_1 + \omega_o = \left(60\ \frac{\text{rad}}{\text{s}^2}\right)(2.0\text{ s}) + 24.0\ \frac{\text{rad}}{\text{s}} = 144.0\ \frac{\text{rad}}{\text{s}}$

so $\omega_o = 144.0\ \frac{\text{rad}}{\text{s}}$ for the period of deceleration

$\Delta\theta_2 = \frac{(\omega + \omega_o)}{2} t_2$ so $t_2 = \frac{2\Delta\theta_2}{\omega + \omega_o} = \frac{2(432\text{ rad})}{0 + 144.0\ \frac{\text{rad}}{\text{s}}} = 6.0\text{ s}$ and $t_{total} = t_1 + t_2 = 2.0\text{ s} + 6.0\text{ s} = \boxed{8.0\text{ s}}$

c.) $\omega = \alpha_2 t_2 + \omega_o$ so $\alpha_2 = \frac{\omega - \omega_o}{t_2} = \frac{0 - 144\ \frac{\text{rad}}{\text{s}}}{6.0\text{ s}} = \boxed{-24\ \frac{\text{rad}}{\text{s}^2}}$

9.) $\Delta\theta = 1\text{ rev}$ when blade slows down from ω_1 to 0

$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$ so $0 = \omega_1^2 + 2\alpha\Delta\theta$ and $\alpha = \frac{-\omega_1^2}{2\Delta\theta} = \frac{-\omega_1^2}{2(1\text{ rev})}$

$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$ so $0 = (2\omega_1)^2 + 2\alpha\Delta\theta$ and $\Delta\theta = \frac{-(2\omega_1)^2}{2\alpha} = \frac{-4\omega_1^2}{2\left(\frac{-\omega_1^2}{2(1\text{ rev})}\right)} = \boxed{4\text{ rev}}$

10.) $\Delta\theta = 65\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 130\pi\text{ rad}$, $v_1 = 28.0\ \frac{\text{m}}{\text{s}}$, $v_2 = 14.0\ \frac{\text{m}}{\text{s}}$, $D = 0.80\text{ m}$ so $r = 0.40\text{ m}$

$\omega_1 = \frac{v_1}{r} = \frac{28.0\ \frac{\text{m}}{\text{s}}}{0.40\text{ m}} = 70.0\ \frac{\text{rad}}{\text{s}}$ and $\omega_2 = \frac{v_2}{r} = \frac{14.0\ \frac{\text{m}}{\text{s}}}{0.40\text{ m}} = 35.0\ \frac{\text{rad}}{\text{s}}$

a.) $\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta$ so $\alpha = \frac{\omega_2^2 - \omega_1^2}{2\Delta\theta} = \frac{\left(35.0\ \frac{\text{rad}}{\text{s}}\right)^2 - \left(70.0\ \frac{\text{rad}}{\text{s}}\right)^2}{2(130\pi\text{ rad})} = \boxed{-4.50\ \frac{\text{rad}}{\text{s}^2}}$

b.) $\omega_2 = \alpha t + \omega_1$ so $t = \frac{\omega_2 - \omega_1}{\alpha} = \frac{0 - 35.0\ \frac{\text{rad}}{\text{s}}}{-4.50\ \frac{\text{rad}}{\text{s}^2}} = \boxed{7.8\text{ s}}$

HO 20 Solutions

1.) $r_1 = 0.045\text{ m}$, $r_2 = 0.0025\text{ m}$, $v_2 = 2.00\ \frac{\text{m}}{\text{s}}$

The shaft and the disk have the same angular velocity $\omega = \frac{v_2}{r_2} = \frac{2.00\ \frac{\text{m}}{\text{s}}}{0.00250\text{ m}} = 800\ \frac{\text{rad}}{\text{s}}$

$$v_1 = r_1\omega = (0.0450\text{ m})\left(800\ \frac{\text{rad}}{\text{s}}\right) = \boxed{36\ \frac{\text{m}}{\text{s}}}$$

2.) $t = 3.00\text{ s}$, $r = 0.200\text{ m}$, $v = 40.0\ \frac{\text{m}}{\text{s}}$, $a = -10.0\ \frac{\text{m}}{\text{s}^2}$

a.) $$\alpha = \frac{a}{r} = \frac{-10.0\ \frac{\text{m}}{\text{s}^2}}{0.200\text{ m}} = \boxed{-50\ \frac{\text{rad}}{\text{s}^2}}$$

b.) $$\omega(3\text{ s}) = \frac{v(3\text{ s})}{r} = \frac{40.0\ \frac{\text{m}}{\text{s}}}{0.200\text{ m}} = \boxed{200\ \frac{\text{rad}}{\text{s}}}$$

$$\omega = \alpha t + \omega_o \text{ so } \omega_o = \omega - \alpha t = 200\ \frac{\text{rad}}{\text{s}} - \left(-50\ \frac{\text{rad}}{\text{s}^2}\right)(3.00\text{ s}) = \boxed{350\ \frac{\text{rad}}{\text{s}}}$$

c.) $$\Delta\theta = \frac{(\omega + \omega_o)}{2}t = \frac{\left(200\ \frac{\text{rad}}{\text{s}} + 350\ \frac{\text{rad}}{\text{s}}\right)}{2}(3.00\text{ s}) = \boxed{825\text{ rad}}$$

d.) $a_r = 9.8\ \frac{\text{m}}{\text{s}^2}$

$$a_r = \frac{v^2}{r} = \omega^2 r \text{ so } \omega = \sqrt{\frac{a_r}{r}} = \sqrt{\frac{9.8\ \frac{\text{m}}{\text{s}^2}}{0.200\text{ m}}} = 7.0\ \frac{\text{rad}}{\text{s}}$$

$$\omega = \alpha t + \omega_o \text{ so } t = \frac{\omega - \omega_o}{\alpha} = \frac{7.0\ \frac{\text{rad}}{\text{s}} - 350\ \frac{\text{rad}}{\text{s}}}{-50\ \frac{\text{rad}}{\text{s}^2}} = \boxed{6.86\text{ s}}$$

3.) $D = 0.850\text{ m}$ so $r = 0.425\text{ m}$, $\omega_o = 3.00\ \frac{\text{rev}}{\text{s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 6.00\pi\ \frac{\text{rad}}{\text{s}}$, $\alpha = 1.50\ \frac{\text{rev}}{\text{s}^2}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 3.00\pi\ \frac{\text{rad}}{\text{s}^2}$

a.) $$\omega = \alpha t + \omega_o = \left(3.00\pi\ \frac{\text{rad}}{\text{s}^2}\right)(1.00\text{ s}) + 6.00\pi\ \frac{\text{rad}}{\text{s}} = \boxed{28.3\ \frac{\text{rad}}{\text{s}}}$$

b.) $$\Delta\theta = \frac{(\omega + \omega_o)}{2}t = \frac{\left(28.3\ \frac{\text{rad}}{\text{s}} + 6.00\pi\ \frac{\text{rad}}{\text{s}}\right)}{2}(1.00\text{ s}) = 23.57\text{ rad}$$

$$\Delta\theta = 23.57\text{ rad}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{3.75\text{ rev}}$$

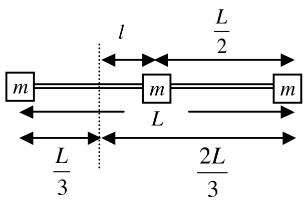
3.) (continued)

c.) $v = r\omega = (0.425 \text{ m})\left(28.3 \ \dfrac{\text{rad}}{\text{s}}\right) = \boxed{12.0 \ \dfrac{\text{m}}{\text{s}}}$

d.) $a_r = \omega^2 r = \left(28.3 \ \dfrac{\text{rad}}{\text{s}}\right)^2 (0.425 \text{ m}) = 340.4 \ \dfrac{\text{rad}}{\text{s}^2}$ and $a_t = r\alpha = (0.425 \text{ m})\left(3.00\pi \ \dfrac{\text{rad}}{\text{s}^2}\right) = 4.00 \ \dfrac{\text{m}}{\text{s}^2}$

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(4.00 \ \frac{\text{m}}{\text{s}^2}\right)^2 + \left(340.4 \ \frac{\text{m}}{\text{s}^2}\right)^2} = \boxed{340 \ \frac{\text{m}}{\text{s}^2}}$$

4.)

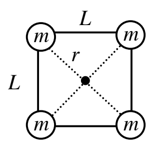


$$I = \sum m_i r_i^2 = m\left(\frac{L}{3}\right)^2 + m\left(\frac{2L}{3}\right)^2 + ml^2 = m\frac{L^2}{9} + m\frac{4L^2}{9} + ml^2$$

$$l = \frac{L}{2} - \frac{L}{3} = \frac{3L}{6} - \frac{2L}{6} = \frac{L}{6}$$

$$I = m\frac{L^2}{9} + m\frac{4L^2}{9} + m\left(\frac{L}{6}\right)^2 = m\left(\frac{L^2}{9} + \frac{4L^2}{9} + \frac{L^2}{36}\right) = m\left(\frac{4L^2}{36} + \frac{16L^2}{36} + \frac{L^2}{36}\right) = m\frac{21L^2}{36} = \boxed{\frac{7L^2}{12}m}$$

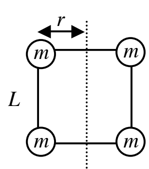
5.)



$m = 0.200$ kg, $L = 0.400$ m

a.) for all four spheres $r = \dfrac{\sqrt{2}}{2}L$

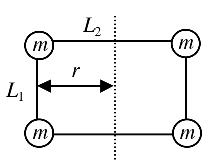
$$I = \sum m_i r_i^2 = 4m\left(\frac{\sqrt{2}}{2}L\right)^2 = 2mL^2 = 2(0.200 \text{ kg})(0.400 \text{ m})^2 = \boxed{0.064 \text{ kg}\cdot\text{m}^2}$$



b.) for all four spheres $r = \dfrac{L}{2}$

$$I = \sum m_i r_i^2 = 4m\left(\frac{L}{2}\right)^2 = mL^2 = (0.200 \text{ kg})(0.400 \text{ m})^2 = \boxed{0.032 \text{ kg}\cdot\text{m}^2}$$

6.)

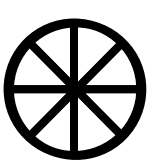


$m = 0.24$ kg, $L_1 = 2.0$ m, $L_2 = 3.0$ m

for all four spheres $r = \dfrac{L_2}{2}$

$$I = \sum m_i r_i^2 = 4m\left(\frac{L_2}{2}\right)^2 = mL_2^2 = (0.24 \text{ kg})(3.0 \text{ m})^2 = \boxed{2.16 \text{ kg}\cdot\text{m}^2}$$

7.)



$L = r = 0.300$ m, $m_{spoke} = 0.320$ kg, $m_{rim} = 1.60$ kg

treat the rim has a hollow cylinder $I = MR^2$ and spokes as rods rotated about their ends $I = \dfrac{1}{3}ML^2$

$$I = I_{rim} + 8I_{spoke} = m_{rim}r^2 + 8\left(\frac{1}{3}m_{spoke}L^2\right)$$

$$I = (1.60 \text{ kg})(0.300 \text{ m})^2 + 8\left(\frac{1}{3}(0.320 \text{ kg})(0.300 \text{ m})^2\right) = \boxed{0.221 \text{ kg}\cdot\text{m}^2}$$

8.)

a.) solid sphere $M = 5.0\text{ kg}$ and $R = 0.20\text{ m}$ about its central axis

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(5.0\text{ kg})(0.20\text{ m})^2 = \boxed{0.080\text{ kg}\cdot\text{m}^2}$$

b.) hoop $M = 10.0\text{ kg}$ and $R = 2.5\text{ m}$ about its central axis perpendicular to its diameter

$$I = MR^2 = (10.0\text{ kg})(2.5\text{ m})^2 = \boxed{62.5\text{ kg}\cdot\text{m}^2}$$

c.) hollow sphere $M = 15.0\text{ kg}$ and $R = 3.00\text{ m}$ about its central axis

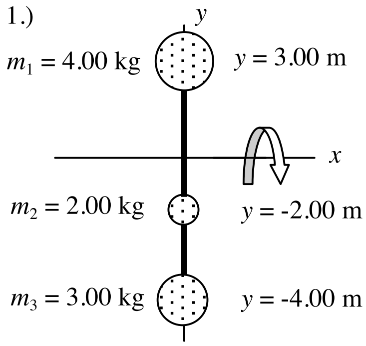
$$I = \frac{2}{3}MR^2 = \frac{2}{3}(15.0\text{ kg})(3.00\text{ m})^2 = \boxed{90\text{ kg}\cdot\text{m}^2}$$

d.) solid cylinder $M = 150\text{ kg}$ and $R = 1.5\text{ m}$ about its central axis perpendicular to its diameter

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(150\text{ kg})(1.5\text{ m})^2 = \boxed{169\text{ kg}\cdot\text{m}^2}$$

HO 21 Solutions

1.)



$$\omega = 2.00\ \frac{\text{rad}}{\text{s}}$$

a.)

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$I = (4.00\text{ kg})(3.00\text{ m})^2 + (2.00\text{ kg})(2.00\text{ m})^2 + (3.00\text{ kg})(4.00\text{ m})^2 = \boxed{92\text{ kg}\cdot\text{m}^2}$$

$$K = \frac{1}{2}I\omega^2 = \omega = \frac{1}{2}\left(92\text{ kg}\cdot\text{m}^2\right)\left(2.00\ \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{184\text{ J}}$$

b.)

$$v_1 = r_1\omega = (3.00\text{ m})\left(2.00\ \frac{\text{rad}}{\text{s}}\right) = \boxed{6.0\ \frac{\text{m}}{\text{s}}}$$

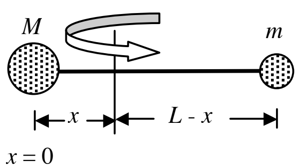
$$v_2 = r_2\omega = (2.00\text{ m})\left(2.00\ \frac{\text{rad}}{\text{s}}\right) = \boxed{4.0\ \frac{\text{m}}{\text{s}}}$$

$$v_3 = r_3\omega = (4.00\text{ m})\left(2.00\ \frac{\text{rad}}{\text{s}}\right) = \boxed{8.0\ \frac{\text{m}}{\text{s}}}$$

$$K = \sum \frac{1}{2}m_i v_i^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2$$

$$K = \frac{1}{2}(4.00\text{ kg})\left(6.0\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(2.00\text{ kg})\left(4.0\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(3.00\text{ kg})\left(8.0\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{184\text{ J}}$$

2.)



a.) $$I = \sum m_i r_i^2 = Mx^2 + m(L - x)^2$$

it is at a minimum when $\frac{dI}{dx} = 0$

$$\frac{dI}{dx} = \frac{d}{dx}\left(Mx^2 + m(L-x)^2\right) = 0$$

$$2Mx + 2m(L-x)(-1) = 2Mx - 2mL + 2mx = 0$$

$$Mx - mL + mx = 0$$

$$x(M+m) - mL = 0$$

$$x = \frac{mL}{M+m}$$

the center-of-mass is $x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{M(0) + mL}{M+m} = \frac{mL}{M+m}$ which is where I is a minimum

2.) (continued)

b.)

$$I = M\left(\frac{mL}{M+m}\right)^2 + m\left(L - \frac{mL}{M+m}\right)^2 = \frac{Mm^2L^2}{(M+m)^2} + m\left(L^2 - \frac{2mL^2}{M+m} + \left(\frac{mL}{M+m}\right)^2\right)$$

$$I = \frac{Mm^2L^2}{(M+m)^2} + m\left(L^2 - \frac{2mL^2}{M+m} + \frac{m^2L^2}{(M+m)^2}\right) = \frac{Mm^2L^2}{(M+m)^2} + mL^2 - \frac{2m^2L^2}{M+m} + \frac{m^3L^2}{(M+m)^2}$$

$$I = \frac{Mm^2L^2}{(M+m)^2} + \frac{mL^2(M+m)^2}{(M+m)^2} - \frac{2m^2L^2(M+m)}{(M+m)^2} + \frac{m^3L^2}{(M+m)^2}$$

$$I = \frac{Mm^2L^2}{(M+m)^2} + \frac{mL^2(M^2+2Mm+m^2)}{(M+m)^2} - \frac{2Mm^2L^2+2m^3L}{(M+m)^2} + \frac{m^3L^2}{(M+m)^2}$$

$$I = \frac{Mm^2L^2}{(M+m)^2} + \frac{M^2mL^2+2Mm^2L^2+m^3L^2}{(M+m)^2} - \frac{2Mm^2L^2+2m^3L}{(M+m)^2} + \frac{m^3L^2}{(M+m)^2}$$

$$I = \frac{Mm^2L^2 + M^2mL^2 + 2Mm^2L^2 + m^3L^2 - 2Mm^2L^2 - 2m^3L + m^3L^2}{(M+m)^2}$$

$$I = \frac{Mm^2L^2 + M^2mL^2}{(M+m)^2} = \frac{MmL^2(M+m)}{(M+m)^2} = \boxed{\frac{MmL^2}{M+m}}$$

3.) solid cylinder mass M and radius R rolls down an incline height h and $v_1 = 0$

using Conservation of Energy and the bottom of the incline as a reference $y_1 = h$ and $y_2 = 0$

$$K_1 + U_{g1} + W_{other} = K_2 + U_{g2} \text{ and } 0 + U_{g1} + 0 = K_2 + 0 \text{ or } U_{g1} = K_2$$

$$Mgy_1 = \frac{1}{2}Mv_2^2 + \frac{1}{2}I\omega_2^2 = \frac{1}{2}Mv_2^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_2}{R}\right)^2$$

$$Mgy_1 = \frac{1}{2}Mv_2^2 + \frac{1}{4}MR^2\left(\frac{v_2^2}{R^2}\right) = \frac{1}{2}Mv_2^2 + \frac{1}{4}Mv_2^2 = \frac{3}{4}Mv_2^2$$

$$v_2 = \sqrt{\frac{4}{3}gy_1} = \boxed{2\sqrt{\frac{gh}{3}}}$$

4.) solid sphere mass M and radius R rolls down an incline height h and $v_1 = 0$

using Conservation of Energy and the bottom of the incline as a reference $y_1 = h$ and $y_2 = 0$

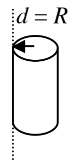
$$U_{g1} = K_2 \text{ so } Mgy_1 = \frac{1}{2}Mv_2^2 + \frac{1}{2}I\omega_2^2 = \frac{1}{2}Mv_2^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_2}{R}\right)^2$$

$$Mgy_1 = \frac{1}{2}Mv_2^2 + \frac{2}{10}MR^2\left(\frac{v_2^2}{R^2}\right) = \frac{1}{2}Mv_2^2 + \frac{2}{10}Mv_2^2 = \frac{7}{10}Mv_2^2$$

$$v_2 = \sqrt{\frac{10}{7}gy_1} = \boxed{\sqrt{\frac{10gh}{7}}}$$

5.)

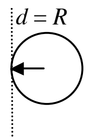
a.) solid cylinder about an axis parallel to its center of mass and passing through its edge



for a solid cylinder $I_{cm} = \frac{1}{2}MR^2$ and using parallel-axis theorem $I_p = I_{cm} + Md^2$

$$I_p = \frac{1}{2}MR^2 + MR^2 = \boxed{\frac{3}{2}MR^2}$$

b.) hollow sphere about an axis tangent to its surface



for a hollow sphere $I_{cm} = \frac{2}{3}MR^2$ and using parallel-axis theorem $I_p = I_{cm} + Md^2$

$$I_p = \frac{2}{3}MR^2 + MR^2 = \boxed{\frac{5}{3}MR^2}$$

6.) merry-go-round $M = 1640\text{ kg}$ and $R = 8.20\text{ m}$ $\omega_o = 0$ and $\omega = \frac{1\text{ rev}}{8\text{ s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \frac{\pi}{4}\ \frac{\text{rad}}{\text{s}}$

using the rotational equivalent of the Work-Energy Theorem

$$W = \Delta K = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_o^2 = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}MR^2\omega^2$$

$$W = \frac{1}{4}(1640\text{ kg})(8.20\text{ m})^2\left(\frac{\pi}{4}\ \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{17{,}000\text{ J}}$$

7.) hollow sphere $R = 0.200\text{ m}$ and $M = 1.20\text{ kg}$ rolling down an incline $L = 10.0\text{ m}$ and $\theta = 30°$

$v_1 = 0$ and using bottom of incline as a reference $y_1 = L\sin\theta$ and $y_2 = 0$

using Conservation of Energy

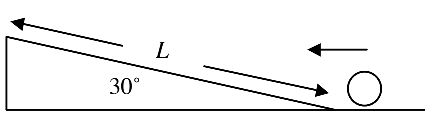
$$U_{g1} = K_2 \text{ so } Mgy_1 = \frac{1}{2}Mv_2^2 + \frac{1}{2}I\omega_2^2 = \frac{1}{2}Mv_2^2 + \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{v_2}{R}\right)^2$$

$$Mgy_1 = \frac{1}{2}Mv_2^2 + \frac{1}{3}MR^2\left(\frac{v_2^2}{R^2}\right) = \frac{1}{2}Mv_2^2 + \frac{1}{3}Mv_2^2 = \frac{5}{6}Mv_2^2$$

$$v_2 = \sqrt{\frac{6}{5}gy_1} = \sqrt{\frac{6gL\sin\theta}{5}} = \sqrt{\frac{6\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(10.0\text{ m})\sin 30°}{5}} = \boxed{7.67\ \frac{\text{m}}{\text{s}}}$$

$$\omega_2 = \frac{v_2}{R} = \frac{7.67\ \frac{\text{m}}{\text{s}}}{0.200\text{ m}} = \boxed{38.3\ \frac{\text{rad}}{\text{s}}}$$

8.)



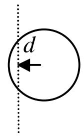
hollow cylinder $R = 0.40\text{ m}$ and $M = 1.0\text{ kg}$ rolling up an incline $\theta = 30°$ and $v_1 = 4.00\ \frac{\text{m}}{\text{s}}$ using bottom of the incline as a reference $y_1 = 0$ and $y_2 = L\sin\theta$

using Conservation of Energy

$$K_1 = U_{g2} \text{ so } \frac{1}{2}Mv_1^2 + \frac{1}{2}I\omega_1^2 = Mgy_2 \text{ or } \frac{1}{2}Mv_1^2 + \frac{1}{2}\left(MR^2\right)\left(\frac{v_1}{R}\right)^2 = \frac{1}{2}Mv_1^2 + \frac{1}{2}\left(MR^2\right)\left(\frac{v_1^2}{R^2}\right) = Mgy_2$$

$$Mv_1^2 = Mgy_2 \text{ and } y_2 = L\sin\theta = \frac{v_1^2}{g} \text{ so } L = \frac{v_1^2}{g\sin\theta} = \frac{\left(4.00\ \frac{\text{m}}{\text{s}}\right)^2}{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\sin 30°} = \boxed{3.3\text{ m}}$$

9.) for a hollow sphere $I_{cm} = \frac{2}{3}MR^2$ and for a solid sphere $I_{cm} = \frac{2}{5}MR^2$



using parallel-axis theorem $I_p = I_{cm} + Md^2$

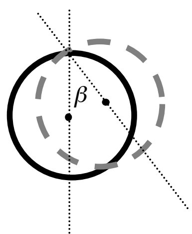
$$\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$$

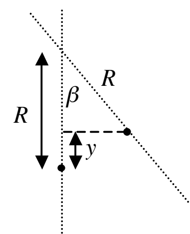
$$\frac{2}{3}R^2 - \frac{2}{5}R^2 = d^2$$

$$\frac{10}{15}R^2 - \frac{6}{15}R^2 = d^2$$

$$d = \sqrt{\frac{4}{15}R^2} = \boxed{2R\sqrt{\frac{1}{15}}}$$

10.)





the distance y that the center of mass falls is

$$y = R - R\cos\beta = R(1 - \cos\beta)$$

the moment of inertia of the hoop about its center of mass is

$$I_{cm} = MR^2$$

using parallel-axis theorem $I_p = I_{cm} + Md^2$, the moment of inertia about an axis passing through its edge is

$$I_p = MR^2 + MR^2 = 2MR^2$$

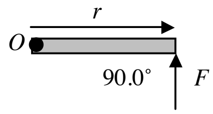
using Conservation of Energy

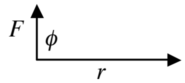
the potential energy comes from the fact that the center of mass falls a distance y and using the equilibrium position as a reference $y_1 = y$ and $y_2 = 0$ this is converted into rotational kinetic energy for the hoop which is rotating about its rim

$$U_{g1} = K_2 \text{ or } Mgy_1 = \frac{1}{2}I_p\omega_2^2 \text{ and } \omega_2 = \sqrt{\frac{2Mgy_1}{I_p}}$$

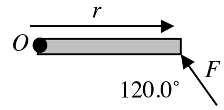
$$\omega_2 = \sqrt{\frac{2MgR(1 - \cos\beta)}{2MR^2}} = \boxed{\sqrt{\frac{g(1 - \cos\beta)}{R}}}$$

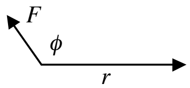
1.) $L = 4.00\text{ m}$, $F = 15.0\text{ N}$



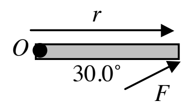


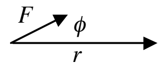
$\phi = 90°$ $\quad \vec{\tau} = \vec{r} \times \vec{F} = rF\sin\phi = (4.00\text{ m})(15.0\text{ N})\sin 90° = \boxed{60\text{ N}\cdot\text{m (ccw)}}$



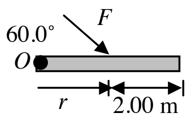


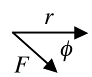
$\phi = 120°$ $\quad \vec{\tau} = \vec{r} \times \vec{F} = rF\sin\phi = (4.00\text{ m})(15.0\text{ N})\sin 120° = \boxed{52\text{ N}\cdot\text{m (ccw)}}$



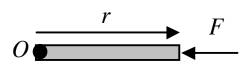


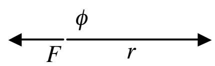
$\phi = 30°$ $\quad \vec{\tau} = \vec{r} \times \vec{F} = rF\sin\phi = (4.00\text{ m})(15.0\text{ N})\sin 30° = \boxed{30\text{ N}\cdot\text{m (ccw)}}$





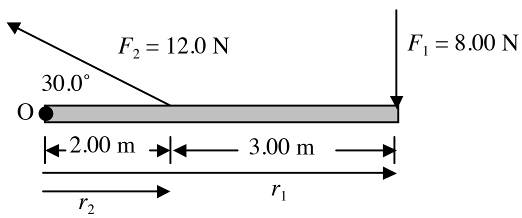
$\phi = -60°$ $\quad \vec{\tau} = \vec{r} \times \vec{F} = rF\sin\phi = (2.00\text{ m})(15.0\text{ N})\sin(-60°) = \boxed{-26\text{ N}\cdot\text{m (cw)}}$

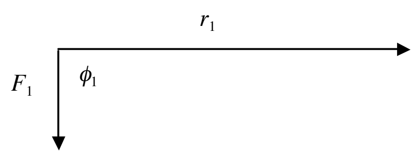




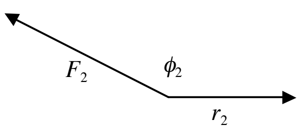
$\phi = 180°$ $\quad \vec{\tau} = \vec{r} \times \vec{F} = rF\sin\phi = (4.00\text{ m})(15.0\text{ N})\sin(180°) = \boxed{0}$

2.)





$\phi_1 = -90°$



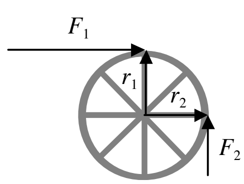
$\phi_2 = 150°$

$\tau_1 = r_1 F_1 \sin\phi_1 = (5.00\text{ m})(8.00\text{ N})\sin(-90°) = -40.0\text{ N}\cdot\text{m (cw)}$

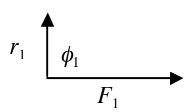
$\tau_2 = r_2 F_2 \sin\phi_2 = (2.00\text{ m})(12.00\text{ N})\sin 150° = 12.0\text{ N}\cdot\text{m (ccw)}$

$\tau_{net} = \tau_1 + \tau_2 = -40.0\text{ N}\cdot\text{m} + 12.0\text{ N}\cdot\text{m} = \boxed{-28.0\text{ N}\cdot\text{m (cw)}}$

3.)

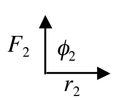


$F_1 = 8.60\text{ N}$, $F_2 = 4.30\text{ N}$, $r_1 = r_2 = 0.330\text{ m}$



$\phi_1 = -90°$

$\tau_1 = r_1 F_1 \sin\phi_1 = (0.330\text{ m})(8.60\text{ N})\sin(-90°) = -2.838\text{ N}\cdot\text{m (cw)}$



$\phi_2 = 90°$

$\tau_2 = r_2 F_2 \sin\phi_2 = (0.330\text{ m})(4.30\text{ N})\sin 90° = 1.419\text{ N}\cdot\text{m (ccw)}$

$\tau_{net} = \tau_1 + \tau_2 = -2.838\text{ N}\cdot\text{m} + 1.419\text{ N}\cdot\text{m} = \boxed{-1.419\text{ N}\cdot\text{m (cw)}}$

4.) $I = 3.50\text{ kg}\cdot\text{m}^2$

a.) $\omega_o = 0$, $\omega = 600\ \dfrac{\text{rev}}{\text{min}}\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 20\pi\ \dfrac{\text{rad}}{\text{s}}$

$$\omega = \alpha t + \omega_o \text{ so } \alpha = \frac{\omega - \omega_o}{t} = \frac{20\pi\ \frac{\text{rad}}{\text{s}} - 0}{8.00\text{ s}} = 7.85\ \frac{\text{rad}}{\text{s}^2}$$

4.) a.) (continued)

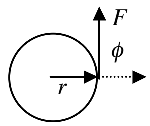
using Newton's 2nd Law for rotation $\tau_{net} = \sum \tau = I\alpha$

$$\tau = I\alpha = \left(3.50\ \text{kg}\cdot\text{m}^2\right)\left(7.85\ \frac{\text{rad}}{\text{s}^2}\right) = \boxed{27.5\ \text{N}\cdot\text{m}}$$

b.)

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(3.50\ \text{kg}\cdot\text{m}^2\right)\left(20\pi\ \frac{\text{rad}}{\text{s}}\right)^2 = \boxed{6910\ \text{J}}$$

5.)



$r = 0.300\ \text{m}$, $F = 50\ \text{N}$, $I = 4.00\ \text{kg}\cdot\text{m}^2$

$$\tau = rF\sin\phi = \left(0.300\ \text{m}\right)\left(50.0\ \text{N}\right)\sin 90° = 15\ \text{N}\cdot\text{m}\ (\text{ccw})$$

using Newton's 2nd Law for rotation $\tau_{net} = \sum \tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{15\ \text{N}\cdot\text{m}}{4.00\ \text{kg}\cdot\text{m}^2} = \boxed{3.75\ \frac{\text{rad}}{\text{s}^2}}$$

6.) two disks $D_1 = 0.075\ \text{m}$, $M = 0.050\ \text{kg}$ joined by cylindrical hub $D_2 = 0.010\ \text{m}$, $M = 0.0050\ \text{kg}$

for the disks $R_1 = \frac{D_1}{2} = 0.0375\ \text{m}$ and for the hub $R_2 = \frac{D_2}{2} = 0.0050\ \text{m}$

$$I = 2I_{disk} + I_{hub} = 2\left(\frac{1}{2}M_1R_1^2\right) + \frac{1}{2}M_2R_2^2$$

$$I = 2\left(\frac{1}{2}\left(0.050\ \text{kg}\right)\left(0.0375\ \text{m}\right)^2\right) + \frac{1}{2}\left(0.0050\ \text{kg}\right)\left(0.0050\ \text{m}\right)^2 = 7.0375 \text{ x } 10^{-5}\ \text{kg}\cdot\text{m}^2$$

a.) using the lowest point as a reference $y_1 = h = 1.0\ \text{m}$ and $y_2 = 0$ also $v_1 = \omega_1 = 0$

using Conservation of Energy

$$K_1 + U_{g1} + W_{other} = K_2 + U_{g2} \text{ or } 0 + U_{g1} + 0 = K_2 + 0 \text{ so } U_{g1} = K_2$$

$$mgy_1 = \frac{1}{2}I\omega_2^2 + \frac{1}{2}mv_2^2 \quad \text{where } m = 2M_1 + M_2 = 2\left(0.050\ \text{kg}\right) + 0.0050\ \text{kg} = 0.105\ \text{kg}$$

since the cord is attached to the hub $\omega_2 = \frac{v_2}{R_2}$ and $mgy_1 = \frac{1}{2}I\left(\frac{v_2}{R_2}\right)^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}I\left(\frac{v_2^2}{R_2^2}\right) + \frac{1}{2}mv_2^2 = \frac{1}{2}v_2^2\left(\frac{I}{R_2^2} + m\right)$

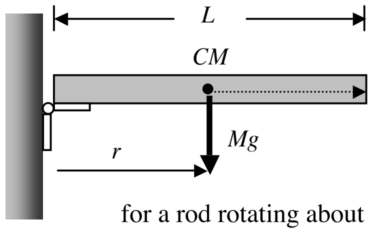
$$v_2 = \sqrt{\frac{2mgy_1}{\frac{I}{R_2^2} + m}} = \sqrt{\frac{2\left(0.105\ \text{kg}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\left(1.00\ \text{m}\right)}{\frac{\left(7.0375 \text{ x } 10^{-5}\ \text{kg}\cdot\text{m}^2\right)}{\left(0.0050\ \text{m}\right)^2} + 0.105\ \text{kg}}} = \boxed{0.84\ \frac{\text{m}}{\text{s}}}$$

b.) the rotational kinetic energy is $K_{rot} = \frac{1}{2}I\omega_2^2 = \frac{1}{2}I\left(\frac{v_2}{R_2}\right)^2 = \frac{1}{2}\left(7.0375 \text{ x } 10^{-5}\ \text{kg}\cdot\text{m}^2\right)\left(\frac{0.84\ \frac{\text{m}}{\text{s}}}{0.0050\ \text{m}}\right)^2 = 0.993\ \text{J}$

the translational kinetic energy is $K_{trans} = \frac{1}{2}mv_2^2 = \frac{1}{2}\left(0.105\ \text{kg}\right)\left(0.84\ \frac{\text{m}}{\text{s}}\right)^2 = 0.0370\ \text{J}$

the fraction of which is rotational is $\frac{K_{rot}}{K_{rot} + K_{trans}} = \frac{0.993\ \text{J}}{0.993\ \text{J} + 0.0370\ \text{J}} = \boxed{0.964}$ or 96.4 % rotational

7.)



a.) rotation results from the torque due to the weight of the rod

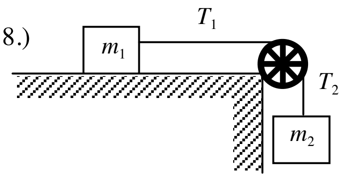
$$\tau = rF_g \sin\phi = \frac{L}{2} Mg \sin(-90°) = -\frac{LMg}{2} \text{ (cw)}$$

for a rod rotating about its end $I = \frac{1}{3}ML^2$ and using Newton's 2nd Law for rotation $\tau_{net} = \sum \tau = I\alpha$

$$\alpha = \frac{\tau}{I} = -\frac{\frac{LMg}{2}}{\frac{1}{3}ML^2} = -\frac{3}{2}\frac{g}{L} \text{ so } \alpha = \boxed{\frac{3}{2}\frac{g}{L} \text{ (cw)}}$$

b.) the tip is a distance $r = L$ from the axis of rotation so $a = r\alpha = L\frac{3}{2}\frac{g}{L} = \boxed{\frac{3}{2}g}$

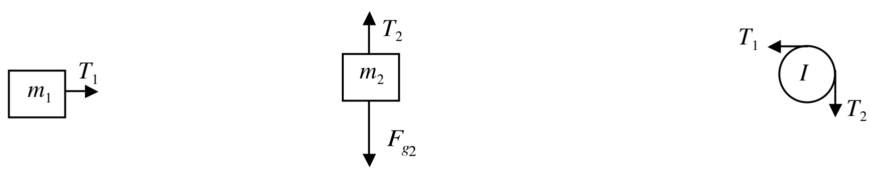
8.)



$m_1 = 2.0$ kg, $m_2 = 1.5$ kg and the pulley is hollow cylinder $M = 1.0$ kg, $R = 0.15$ m

for a hollow cylinder $I = MR^2$

Looking at forces in the direction of motion and applying Newton's 2nd Law and $a = R\alpha$



$\sum F = ma$	$\sum F = ma$	$\sum \tau = I\alpha$
(1) $T_1 = m_1 a$	$F_{g2} - T_2 = m_2 a$	$\tau_2 - \tau_1 = I\alpha$
	(2) $m_2 g - T_2 = m_2 a$	$RT_2 - RT_1 = MR^2\left(\frac{a}{R}\right)$
		(3) $T_2 - T_1 = Ma$

combining (1) and (2) and (3)

$$m_2 g = m_1 a + m_2 a + Ma = (m_1 + m_2 + M)a$$

$$a = \frac{m_2 g}{m_1 + m_2 + M} = \frac{(1.5 \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^2}\right)}{2.0 \text{ kg} + 1.5 \text{ kg} + 1.0 \text{ kg}} = \boxed{3.27 \frac{\text{m}}{\text{s}^2}}$$

$$\alpha = \frac{a}{R} = \frac{3.27 \frac{\text{m}}{\text{s}^2}}{0.15 \text{ m}} = \boxed{21.8 \frac{\text{rad}}{\text{s}^2}}$$

from (1)

$$T_1 = m_1 a = (2.0 \text{ kg})\left(3.27 \frac{\text{m}}{\text{s}^2}\right) = \boxed{6.54 \text{ N}}$$

from (2)

$$T_2 = m_2 g - m_2 a = m_2(g - a) = (1.5 \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^2} - 3.27 \frac{\text{m}}{\text{s}^2}\right) = \boxed{9.80 \text{ N}}$$

8.) (again)

could use energy conservation to find the speed of block m_2 (and m_1) after falling a distance h and relating it to the acceleration a using the kinematic relationship $v^2 = v_o^2 + 2a\Delta x$.

using the lowest point as a reference and starting from rest $y_1 = h$, $y_2 = 0$, and $v_1 = 0$

$$K_1 + U_{g1} + W_{other} = K_2 + U_{g2} \text{ or } 0 + U_{g1} + 0 = K_2 + 0 \text{ so } U_{g1} = K_2$$

the final kinetic energy includes the rotational kinetic energy of the pulley and the translational kinetic energy of the blocks

only block m_2 contributes to changes in gravitational energy

$$m_2 g y_1 = \frac{1}{2}(m_1 + m_2)v_2^2 + \frac{1}{2}I\omega_2^2 = \frac{1}{2}(m_1 + m_2)v_2^2 + \frac{1}{2}I\left(\frac{v_2}{R}\right)^2$$

$$m_2 g y_1 = \frac{1}{2}(m_1 + m_2)v_2^2 + \frac{1}{2}MR^2\left(\frac{v_2^2}{R^2}\right) = \frac{1}{2}(m_1 + m_2)v_2^2 + \frac{1}{2}Mv_2^2$$

$$2m_2 g y_1 = (m_1 + m_2 + M)v_2^2 \text{ and } v_2^2 = \frac{2m_2 g y_1}{m_1 + m_2 + M} = \frac{2m_2 g h}{m_1 + m_2 + M} \qquad (1)$$

$$v^2 = v_o^2 + 2a\Delta x \text{ so } v_2^2 = v_1^2 + 2a\Delta y \text{ or } v_2^2 = 0 + 2ah = 2ah \qquad (2)$$

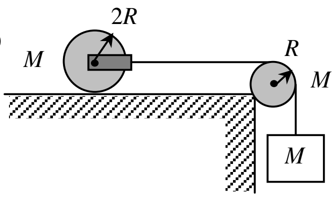
combining (1) and (2)

$$2ah = \frac{2m_2 g h}{m_1 + m_2 + M} \text{ or } a = \frac{m_2 g}{m_1 + m_2 + M} \text{ (the same result as previously obtained)}$$

α can be obtained from a and the tensions obtained by considering the forces on the blocks and applying Newton's 2nd Law to each block

HO 23 Solutions

1.)



for uniform cylinder $I = \frac{1}{2}MR^2$

no slipping so acceleration a of the block and cylinder are the same and the angular acceleration of the pulley is related to the acceleration by the relationship $a = R\alpha$ and the linear velocity of the block and the cylinder are the same

use energy conservation to find the speed of block M after falling a distance h and relating it to the acceleration a using the kinematic relationship $v^2 = v_o^2 + 2a\Delta y$.

using the lowest point as a reference and starting from rest $y_1 = h$, $y_2 = 0$, and $v_1 = 0$

$K_1 + U_{g1} + W_{other} = K_2 + U_{g2}$ or $0 + U_{g1} + 0 = K_2 + 0$ so $U_{g1} = K_2$

the final kinetic energy includes the rotational kinetic energy of the pulley and the cylinder and the translational kinetic energy of the block and cylinder

only block M contributes to changes in gravitational energy

$$Mgy_1 = \frac{1}{2}(M+M)v_2^2 + \frac{1}{2}I_{pulley}\omega_2^2 + \frac{1}{2}I_{cylinder}\omega_2^2 = \frac{1}{2}(2M)v_2^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_2^2 + \frac{1}{2}\left(\frac{1}{2}M(2R)^2\right)\omega_2^2$$

$$Mgy_1 = Mv_2^2 + \frac{1}{4}MR^2\omega_2^2 + \left(\frac{1}{4}M\left(4R^2\right)\right)\omega_2^2 = Mv_2^2 + \frac{1}{4}MR^2\left(\frac{v_2}{R}\right)^2 + MR^2\left(\frac{v_2}{2R}\right)^2$$

$$Mgy_1 = Mv_2^2 + \frac{1}{4}MR^2\left(\frac{v_2^2}{R^2}\right) + MR^2\left(\frac{v_2^2}{(2R)^2}\right) = Mv_2^2 + \frac{1}{4}MR^2\left(\frac{v_2^2}{R^2}\right) + MR^2\left(\frac{v_2^2}{4R^2}\right)$$

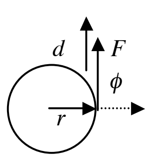
$$Mgy_1 = Mv_2^2 + \frac{1}{4}Mv_2^2 + \frac{1}{4}Mv_2^2 = Mv_2^2\left(1 + \frac{1}{4} + \frac{1}{4}\right) = Mv_2^2\left(\frac{3}{2}\right)$$

$$Mgy_1 = Mv_2^2\left(\frac{3}{2}\right) \text{ so } v_2^2 = \frac{2gy_1}{3} = \frac{2gh}{3} \quad (1)$$

$$v^2 = v_o^2 + 2a\Delta y \text{ so } v_2^2 = v_1^2 + 2a\Delta y \text{ or } v_2^2 = 0 + 2ah = 2ah \quad (2)$$

combining (1) and (2) $2ah = \frac{2gh}{3}$ or $a = \boxed{\frac{g}{3}}$

2.)



$D = 0.500 \text{ m}$, $F = 60 \text{ N}$. $v_o = \omega_o = 0$, $\Delta d = 3.00 \text{ m}$, and $t = 4.00 \text{ s}$

$r = \frac{D}{2} = 0.250 \text{ m}$ and $\Delta\theta = \frac{\Delta d}{r} = \frac{3.00 \text{ m}}{0.250 \text{ m}} = 12 \text{ rad}$

a.) $\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_o t = \frac{1}{2}\alpha t^2$ so $\alpha = \frac{2\Delta\theta}{t^2} = \frac{2(12 \text{ rad})}{(4.00 \text{ s})^2} = \boxed{1.5 \ \frac{\text{rad}}{\text{s}^2}}$

b.) $\Delta\theta = \frac{(\omega + \omega_o)}{2}t = \frac{\omega}{2}t$ so $\omega = \frac{2\Delta\theta}{t} = \frac{2(12 \text{ rad})}{4.00 \text{ s}} = \boxed{6.0 \ \frac{\text{rad}}{\text{s}}}$

c.) $W_{net} = \Delta K = K - K_o = K$ and $W_{net} = W_F = \vec{F}\cdot\vec{d} = Fd\cos\theta$ so $K = Fd\cos\theta = (60.0 \text{ N})(3.00 \text{ m})\cos 0 = \boxed{180 \text{ J}}$

d.) $\sum\tau = I\alpha$ so $I = \frac{\tau}{\alpha} = \frac{rF\sin\phi}{\alpha} = \frac{(0.250 \text{ m})(60.0 \text{ N})\sin 90°}{1.5 \ \frac{\text{rad}}{\text{s}^2}} = \boxed{10.0 \text{ kg}\cdot\text{m}^2}$

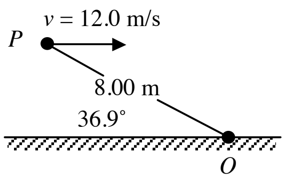
HO 23 Solutions

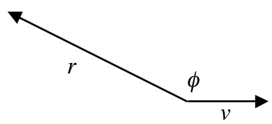
3.) $\ell = 0.250\text{ m},\ M = 0.0150\text{ kg}$, constant angular velocity $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{2\pi\text{ rad}}{60\text{ s}} = \dfrac{\pi}{30}\ \dfrac{\text{rad}}{\text{s}}$

for a slender rod rotating about one end $I = \dfrac{1}{3}M\ell^2$

$$L = I\omega = \frac{1}{3}M\ell^2\omega = \frac{1}{3}(0.0150\text{ kg})(0.250\text{ m})^2\left(\frac{\pi}{30}\ \frac{\text{rad}}{\text{s}}\right) = \boxed{3.27\text{ x }10^{-5}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

4.)





$m = 3.00\text{ kg},\ v = 12.0\ \dfrac{\text{m}}{\text{s}},\ r = 8.00\text{ m}$

$\phi = 143.1°$

$$\vec{L} = \vec{r}\times\vec{p} = mrv\sin\phi = (3.00\text{ kg})(8.00\text{ m})\left(12.0\ \frac{\text{m}}{\text{s}}\right)\sin 143.1° = \boxed{173\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

5.) $M_{arms} = 8.0\text{ kg},\ L_{arms} = 1.8\text{ m},\ R_{arms} = 0.25\text{ m},\ I_{body} = 0.40\text{ kg}\cdot\text{m}^2,\ \omega_1 = 0.60\ \dfrac{\text{rev}}{\text{s}}$

when arms are extended $I_1 = I_{body} + I_{arms} = I_{body} + \dfrac{1}{12}M_{arms}L_{arms}^2 = 0.40\text{ kg}\cdot\text{m}^2 + \dfrac{1}{12}(8.0\text{ kg})(1.8\text{ m})^2 = 2.56\text{ kg}\cdot\text{m}^2$

when arms are brought in $I_1 = I_{body} + I_{arms} = I_{body} + M_{arms}R_{arms}^2 = 0.40\text{ kg}\cdot\text{m}^2 + (8.0\text{ kg})(0.25\text{ m})^2 = 0.90\text{ kg}\cdot\text{m}^2$

using Conservation of Angular Momentum $(L_1 = L_2)$

$$I_1\omega_1 = I_2\omega_2 \text{ or } \omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{2.56\text{ kg}\cdot\text{m}^2}{0.90\text{ kg}\cdot\text{m}^2}\left(0.60\ \frac{\text{rev}}{\text{s}}\right) = \boxed{1.71\ \frac{\text{rev}}{\text{s}}}$$

6.) $\Delta\theta = 1\text{ rev},\ \Delta t = 6.00\text{ s},\ I_{mgr} = 1200\text{ kg}\cdot\text{m}^2,\ m_{man} = 80.0\text{ kg},\ r_1 = 0$, and $r_2 = 2.00\text{ m}$

with man at center $I_1 = I_{mgr} + I_{man} = I_{body} + m_{man}r_1^2 = 1200\text{ kg}\cdot\text{m}^2 + (80.0\text{ kg})(0)^2 = 1200\text{ kg}\cdot\text{m}^2$

after man moves $I_2 = I_{mgr} + I_{man} = I_{body} + m_{man}r_2^2 = 1200\text{ kg}\cdot\text{m}^2 + (80.0\text{ kg})(2.00)^2 = 1520\text{ kg}\cdot\text{m}^2$

$\omega_1 = \dfrac{\Delta\theta}{\Delta t} = \dfrac{1\text{ rev}}{6\text{ s}} = 0.167\ \dfrac{\text{rev}}{\text{s}}$ and using Conservation of Angular Momentum $(L_1 = L_2)$

$$I_1\omega_1 = I_2\omega_2 \text{ or } \omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{1200\text{ kg}\cdot\text{m}^2}{1520\text{ kg}\cdot\text{m}^2}\left(0.167\ \frac{\text{rev}}{\text{s}}\right) = \boxed{0.132\ \frac{\text{rev}}{\text{s}}}$$

7.) uniform disk $\omega_1 = 7.0\ \dfrac{\text{rev}}{\text{s}}$, M, R and uniform stick $M,\ L = 2R$

for the disk $I_1 = I_{disk} = \dfrac{1}{2}MR^2$

for the disk and stick $I_2 = I_{disk} + I_{stick} = \dfrac{1}{2}MR^2 + \dfrac{1}{12}ML^2 = \dfrac{1}{2}MR^2 + \dfrac{1}{12}M(2R)^2 = \dfrac{5}{6}MR^2$

using Conservation of Angular Momentum $(L_1 = L_2)$

$$I_1\omega_1 = I_2\omega_2 \text{ or } \omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{\frac{1}{2}MR^2}{\frac{5}{6}MR^2}\left(7.0\ \frac{\text{rev}}{\text{s}}\right) = \boxed{4.2\ \frac{\text{rev}}{\text{s}}}$$

8.) $I_1 = I$, $I_2 = 2I$, $\omega_1 = \omega$

using Conservation of Angular Momentum $(L_1 = L_2)$

$$I_1\omega_1 = I_2\omega_2 \text{ or } \omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{I}{2I}\omega = \boxed{\frac{1}{2}\omega}$$

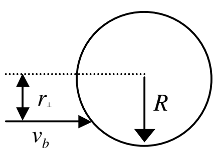
9.) merry-go-round $D = 4.2\text{ m}$, $I_{mgr} = 1760\text{ kg}\cdot\text{m}^2$, $\omega_1 = 0.80\ \frac{\text{rad}}{\text{s}}$ and four men $m = 65\text{ kg}$, $r_1 = 0$, $r_2 = \frac{D}{2} = 2.1\text{ m}$

$$I_1 = I_{mgr} = 1760\text{ kg}\cdot\text{m}^2 \text{ and } I_2 = I_{mgr} + I_{men} = I_{mgr} + 4mr^2 = 1760\text{ kg}\cdot\text{m}^2 + 4(65\text{ kg})(2.1\text{ m})^2 = 2907\text{ kg}\cdot\text{m}^2$$

using Conservation of Angular Momentum $(L_1 = L_2)$

$$I_1\omega_1 = I_2\omega_2 \text{ or } \omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{1760\text{ kg}\cdot\text{m}^2}{2907\text{ kg}\cdot\text{m}^2}\left(0.80\ \frac{\text{rad}}{\text{s}}\right) = \boxed{0.48\ \frac{\text{rad}}{\text{s}}}$$

10.)



$m_b = 0.00188\text{ kg}$, $v_b = 360\ \frac{\text{m}}{\text{s}}$, solid disk $M = 10.0\text{ kg}$, $R = 0.300\text{ m}$, $\omega_1 = 0$

the bullet becomes embedded along a line 0.25 m to the right of the center of the disk

before striking the disk the angular momentum of the bullet with respect to the axis of rotation of the disk is

$$L_1 = L_b = m_b v_b r_b \sin\phi = m_b v_b r_\perp$$

after striking the disk angular momentum of the disk and bullet is

$$I_2 = I_b + I_{disk} = m_b {R_b}^2 + \frac{1}{2}MR^2$$

using Conservation of Angular Momentum $(L_1 = L_2)$

$$\text{so} \quad L_1 = I_2\omega_2 \text{ or } \omega_2 = \frac{L_1}{I_2} = \frac{m_b v_b r_\perp}{m_b {R_b}^2 + \frac{1}{2}MR^2}$$

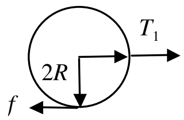
$$\omega_2 = \frac{(0.00188\text{ kg})\left(360\ \frac{\text{m}}{\text{s}}\right)(0.25\text{ m})}{\left(0.00188\text{ kg}(0.25\text{ m})^2 + \frac{1}{2}(10.0\text{ kg})(0.30\text{ m})^2\right)} = 0.376\ \frac{\text{rad}}{\text{s}}$$

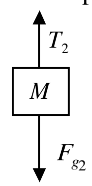
the time to make one complete revolution is the period T and $T = \frac{2\pi}{\omega} = \frac{2\pi}{0.376\ \frac{\text{rad}}{\text{s}}} = \boxed{16.7\text{ s}}$

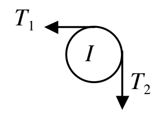
HO 23 Solutions

1.) (again) the rotation of the cylinder is due to the frictional force between the cylinder and the surface that it is in contact with and the tension in the cord does not contribute

Looking at forces in the direction of motion and applying Newton's 2nd Law and $a = R\alpha$







$\sum F = ma$

(1) $T_1 - f = Ma$

$\sum \tau = I\alpha$

$f2R = \frac{1}{2}M(2R)^2\alpha$

$f2R = \frac{1}{2}M(2R)^2\left(\frac{a}{2R}\right)$

(4) $f = \frac{1}{2}Ma$

$\sum F = ma$

$F_{g2} - T_2 = Ma$

(2) $Mg - T_2 = Ma$

$\sum \tau = I\alpha$

$\tau_2 - \tau_1 = I\alpha$

$RT_2 - RT_1 = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

(3) $T_2 - T_1 = \frac{1}{2}Ma$

combining (1) and (2) and (3) and (4) $Mg = Ma + Ma + \frac{1}{2}Ma + \frac{1}{2}Ma = 3Ma$ and $a = \frac{g}{3}$

also $f = \frac{1}{2}Ma = \frac{1}{2}M\left(\frac{g}{3}\right) = \frac{Mg}{6}$ and since $f = \mu F_N = \mu Mg$ it follows that $\mu = \frac{f}{Mg} = \frac{\frac{Mg}{6}}{Mg} = \frac{1}{6} = 0.167$