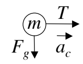
Example 1:

How much tension is in a string connected to a 0.25 kg ball that is being swung in horizontal circle with a radius of 1.5 meters, if the ball makes two revolutions per second?

Example 1:

$$m = 0.25\ \text{kg},\ r = 1.5\ \text{m},\ \text{and}\ \omega = 2\ \frac{\text{rev}}{\text{s}},\ T = ?$$

$$v = 2\ \frac{\text{rev}}{\text{s}}\left(\frac{2\pi(1.5\ \text{m})}{1\ \text{rev}}\right) = 18.85\ \frac{\text{m}}{\text{s}}$$



$$F_{net} = ma$$

$$T = m\frac{v^2}{r}$$

$$T = (0.25\ \text{kg})\frac{\left(18.85\ \frac{\text{m}}{\text{s}}\right)^2}{1.5\ \text{m}}$$

$$\boxed{T = 59.2\ \text{N}}$$

Example 2:

A 2.0 kg rock is attached to the end of a 1.5 m cord with negligible mass and swung in a vertical circle at an angular speed of 1.50 rev/s.

a.) Find the centripetal acceleration of the rock.

b.) Find the highest and lowest values of the tension in the cord.

c.) What is the minimum speed needed in order for the rock to complete a circular path?

Example 2:

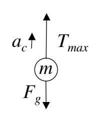
$$m = 2.0\ \text{kg},\ r = 1.5\ \text{m},\ \text{and}\ \omega = 1.5\ \frac{\text{rev}}{\text{s}}$$

a.) $a_c = ?$

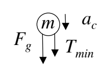
$$a_c = \frac{v^2}{r}\ \text{and}\ v = 1.5\ \frac{\text{rev}}{\text{s}}\left(\frac{2\pi(1.5\ \text{m})}{1\ \text{rev}}\right) = 14.14\ \frac{\text{m}}{\text{s}}$$

$$a_c = \frac{\left(14.14\ \frac{\text{m}}{\text{s}}\right)^2}{1.5\ \text{m}} = \boxed{133.2\ \frac{\text{m}}{\text{s}^2}}$$

b.) $T_{max} = ?$ and $T_{min} = ?$



$$\vec{F}_{net} = m\vec{a}$$
$$T_{max} - F_g = ma_c$$
$$T_{max} = ma_c + F_g$$
$$T_{max} = ma_c + mg$$
$$T_{max} = m(a_c + g)$$



$$\vec{F}_{net} = m\vec{a}$$
$$T_{min} + F_g = ma_c$$
$$T_{min} = ma_c - F_g$$
$$T_{min} = ma_c - mg$$
$$T_{min} = m(a_c - g)$$

$$T_{max} = (2.0\ \text{kg})\left(133.2\ \frac{\text{m}}{\text{s}^2} + 9.8\ \frac{\text{m}}{\text{s}^2}\right)$$

$$\boxed{T_{max} = 286\ \text{N}}$$

$$T_{min} = (2.0\ \text{kg})\left(133.2\ \frac{\text{m}}{\text{s}^2} - 9.8\ \frac{\text{m}}{\text{s}^2}\right)$$

$$\boxed{T_{min} = 247\ \text{N}}$$

Example 2:

$$m = 2.0\ \text{kg},\ r = 1.5\ \text{m},\ \text{and}\ \omega = 1.5\ \frac{\text{rev}}{\text{s}}$$

c.) minimum speed to complete circle $v = ?$

The minimum speed v is when $T = 0$.

The happens at the top where:

$$T = m(a_c - g)$$

$$0 = m\left(\frac{v^2}{r} - g\right)$$

$$\frac{v^2}{r} - g = 0$$

$$v = \sqrt{gr}$$

$$v = \sqrt{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(1.5\ \text{m})}$$

$$\boxed{v = 3.83\ \frac{\text{m}}{\text{s}}}$$

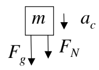
Example 3:

A girl has a mass of 65 kg and is riding on The Demon at Great America. She experiences an acceleration of 15.0 m/s^2 at the top of the loop. What is the normal force? (i.e., How much force is pushing her rear end into the seat?)

Example 3:
(Roller-coaster loop)

$m = 65 \text{ kg}$ and $a_c = 15 \ \frac{\text{m}}{\text{s}^2}$

at the top of the loop $F_N = \ ?$



$$\vec{F}_{net} = m\vec{a}$$
$$F_N + F_g = ma_c$$
$$F_N = ma_c - F_g$$
$$F_N = ma_c - mg$$
$$F_N = m(a_c - g)$$
$$F_N = (65 \text{ kg})\left(15 \ \frac{\text{m}}{\text{s}^2} - 9.8 \ \frac{\text{m}}{\text{s}^2}\right)$$
$$\boxed{F_N = 338 \text{ N}}$$

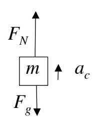
Example 4:

How much force would you feel against your derrière at the bottom of a roller-coaster loop with a diameter of 30 meters if your mass is 75 kg and the roller coaster car is traveling at 25 m/s?

Example 4:
(Roller-coaster loop)

$m = 75 \text{ kg}, \ v = 25 \ \frac{\text{m}}{\text{s}}, \ D = 30 \text{ m so } r = 15 \text{ m}$

at the bottom of the loop $F_N = \ ?$



$$\vec{F}_{net} = m\vec{a}$$
$$F_N - F_g = ma_c$$
$$F_N = ma_c + F_g$$
$$F_N = m\frac{v^2}{r} + mg$$
$$F_N = m\left(\frac{v^2}{r} + g\right) \quad \text{and} \quad F_N = (75 \text{ kg})\left(\frac{\left(25 \ \frac{\text{m}}{\text{s}}\right)^2}{15 \text{ m}} + 9.8 \ \frac{\text{m}}{\text{s}^2}\right)$$
$$\boxed{F_N = 3860 \text{ N}}$$

Example 5:

a.) How much force would you feel against your derrière at the top of a Ferris wheel with a radius of 30 meters if your mass is 80 kg and the Ferris wheel's speed is 15 m/s?

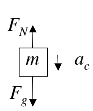
b.) Assuming the same conditions as in (a), what force would you feel at the bottom of the ride?

c.) What is the maximum speed for the Ferris wheel?

Example 5:
(Ferris Wheel)

$m = 80 \text{ kg}, \ v = 15 \ \frac{\text{m}}{\text{s}}, \text{ and } \ r = 30 \text{ m}$

a.) at the top of the ride $F_N = \ ?$

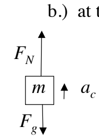


$$\vec{F}_{net} = m\vec{a}$$
$$F_g - F_N = ma_c$$
$$F_N = F_g - ma_c$$
$$F_N = mg - m\frac{v^2}{r}$$
$$F_N = m\left(g - \frac{v^2}{r}\right) \quad \text{and} \quad F_N = (80 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2} - \frac{\left(15 \ \frac{\text{m}}{\text{s}}\right)^2}{30 \text{ m}}\right)$$
$$\boxed{F_N = 184 \text{ N}}$$

Example 5:
(Ferris Wheel)

$m = 80 \text{ kg}, \ v = 15 \ \frac{\text{m}}{\text{s}}, \text{ and } \ r = 30 \text{ m}$

b.) at the bottom of the ride $F_N = \ ?$



$$\vec{F}_{net} = m\vec{a}$$
$$F_N - F_g = ma_c$$
$$F_N = ma_c + F_g$$
$$F_N = m\frac{v^2}{r} + mg$$
$$F_N = m\left(\frac{v^2}{r} + g\right) \quad \text{and} \quad F_N = (80 \text{ kg})\left(\frac{\left(15 \ \frac{\text{m}}{\text{s}}\right)^2}{30 \text{ m}} + 9.8 \ \frac{\text{m}}{\text{s}^2}\right)$$
$$\boxed{F_N = 1384 \text{ N}}$$

Example 5:
(Ferris Wheel)

$m = 80\text{ kg},\ v = 15\ \frac{\text{m}}{\text{s}},\text{ and }\ r = 30\text{ m}$

c.) maximum safe speed $v = \ ?$

The maximum safe speed v is when $F_N = 0$.

The happens at the top where :

$$F_N = m\left(g - \frac{v^2}{r}\right)$$

$$0 = m\left(g - \frac{v^2}{r}\right)$$

$$g - \frac{v^2}{r} = 0$$

$$v = \sqrt{gr}$$

$$v = \sqrt{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(30\text{ m})}$$

$$\boxed{v = 17.15\ \frac{\text{m}}{\text{s}}}$$

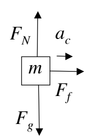
Example 6:

A flat curve on a highway has radius of 240 m. A car rounds the curve at a speed of 32 m/s. What is the minimum coefficient of friction that will prevent skidding.

Example 6:
(Curved Road)

$r = 240\text{ m and } v = 32\ \frac{\text{m}}{\text{s}}$

$\mu = \ ?$



$$\vec{F}_{net} = m\vec{a}$$

$$\sum F_y = ma \qquad \sum F_x = ma$$

$$F_N - F_g = 0 \qquad F_f = ma_c$$

$$F_N = F_g \qquad \mu F_N = ma_c$$

$$F_N = mg \qquad \mu mg = ma_c$$

$$\mu = \frac{a_c}{g}$$

$$\mu = \frac{\left(\frac{v^2}{r}\right)}{g} = \frac{v^2}{gr}$$

$$\mu = \frac{\left(32\ \frac{\text{m}}{\text{s}}\right)^2}{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(240\text{ m})} \quad \boxed{\text{and } \mu = 0.44}$$