HO 16 Solutions

1.) $m_1 = 80.0\text{ kg}$, $m_2 = 0.160\text{ kg}$, $v_{1_i} = v_{2_i} = 0$, $v_{2_f} = 30.0\ \dfrac{\text{m}}{\text{s}}$

using Conservation of Momentum

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f}$$

$$v_{2_f} = \frac{m_1 v_{1_i} + m_2 v_{2_i} - m_2 v_{2_f}}{m_1} = \frac{0 + 0 - (0.160\text{ kg})\left(30.0\ \frac{\text{m}}{\text{s}}\right)}{80.0\text{ kg}} = \boxed{-0.06\ \frac{\text{m}}{\text{s}}}$$

2.) $m = 0.145\text{ kg}$, $v_i = 30.0\ \dfrac{\text{m}}{\text{s}}$, and $v_f = -45.0\ \dfrac{\text{m}}{\text{s}}$

a.)

$$\Delta p = mv_f - mv_i = m(v_f - v_i) \text{ so } \Delta p = (0.145\text{ kg})\left(-45.0\ \frac{\text{m}}{\text{s}} - 30.0\ \frac{\text{m}}{\text{s}}\right) = \boxed{-10.875\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

using Impulse-Momentum Theorem $J = \Delta p = \boxed{-10.875\ \dfrac{\text{kg}\cdot\text{m}}{\text{s}}}$

b.) $\Delta t = 2.00 \text{ x } 10^{-3}\text{ s}$

$$J = F\Delta t \text{ so } F = \frac{J}{\Delta t} = \frac{10.875\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}{2.00 \text{ x } 10^{-3}\text{ s}} = \boxed{5437.5\text{ N}}$$

3.) $F(t) = A + Bt^2$

a.) $$J = \int_{t_1}^{t_2} F\,dt = \int_0^{t_2} \left(A + Bt^2\right)dt = At + B\frac{t^3}{3}\bigg]_0^{t_2} = At_2 + \frac{1}{3}Bt_2^3 - 0 = \boxed{At_2 + \frac{1}{3}Bt_2^3}$$

b.) $v_i = 0$

Impulse-Momentum Theorem

$$J = \Delta p = m\Delta v = m(v_f - v_i) \text{ so } v_f = \frac{J}{m} + v_i = \frac{At_2 + \frac{1}{3}Bt_2^3}{m} + 0 = \boxed{\frac{At_2}{m} + \frac{1}{3m}Bt_2^3}$$

4.) $m = 0.046\text{ kg}$, $\theta = 45°$, $\Delta x = 200\text{ m}$, $\Delta t = 2 \text{ x } 10^{-3}\text{ s}$

ball is a projectile launched ground-to ground $v_x = v_o\cos\theta$ and $v_{y_o} = v_o\sin\theta$

$v_x = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta x}{t}$ or (1) $t = \dfrac{\Delta x}{v_x}$ and maximum height occurs at midpoint where $\Delta x = 100\text{ m}$

at maximum height $v_y = 0 = -gt + v_{y_o}$ so (2) $t = \dfrac{v_{y_o}}{g}$ and equating (1) and (2) $\dfrac{\Delta x}{v_x} = \dfrac{v_{y_o}}{g}$

therefore at max height $\dfrac{\Delta x}{v_o\cos\theta} = \dfrac{v_o\sin\theta}{g}$ and $g\Delta x = v_o^2\sin\theta\cos\theta$ or $v_o = \sqrt{\dfrac{g\Delta x}{\sin\theta\cos\theta}}$

4.) (continued)

so the speed right after the ball is hit is $v_o = \sqrt{\dfrac{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(100\ \text{m})}{\sin 45^\circ \cos 45^\circ}} = 44.3\ \dfrac{\text{m}}{\text{s}}$

before the ball is hit $v_x = v_y = 0$

after the ball is hit $v_x = v_o\cos\theta = \left(44.3\ \frac{\text{m}}{\text{s}}\right)\cos 45^\circ = 31.3\ \frac{\text{m}}{\text{s}}$ and $v_y = v_o\sin\theta = \left(44.3\ \frac{\text{m}}{\text{s}}\right)\sin 45^\circ = 31.3\ \frac{\text{m}}{\text{s}}$

using Impulse-Momentum Theorem

$$F_x\Delta t = \Delta p_x = m\Delta v_x = m(v_x - 0) \text{ so } F_x = \frac{mv_x}{\Delta t} = \frac{(0.046\ \text{kg})\left(31.3\ \frac{\text{m}}{\text{s}}\right)}{7.0 \text{ x } 10^{-3}\ \text{s}} = 205.7\ \text{N}$$

$$F_y\Delta t = \Delta p_y = m\Delta v_y = m(v_y - 0) \text{ so } F_y = \frac{mv_y}{\Delta t} = \frac{(0.046\ \text{kg})\left(31.3\ \frac{\text{m}}{\text{s}}\right)}{7.0 \text{ x } 10^{-3}\ \text{s}} = 205.7\ \text{N}$$

therefore $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(205.7\ \text{N})^2 + (205.7\ \text{N})^2} = \boxed{291\ \text{N}}$

since we are only interested in the magnitude of the force we could have just found F from Δv since they are in the same direction

$$F\Delta t = \Delta p = m\Delta v \text{ and } F = \frac{m\Delta v}{\Delta t} = \frac{(0.046\ \text{kg})\left(44.3\ \frac{\text{m}}{\text{s}} - 0\right)}{7.0 \text{ x } 10^{-3}\ \text{s}} = \boxed{291\ \text{N}}$$

In general, however, we should always treat impulse as a vector and look at components.

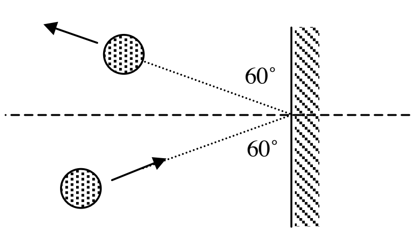
5.) $m_1 = 1800\ \text{kg}$, $v_{1_i} = 0$, $m_2 = 900\ \text{kg}$, $v_{2_i} = 20\ \frac{\text{m}}{\text{s}}$ inelastic collision so $v_{1_f} = v_{2_f} = v_f$

using Conservation of Momentum

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f} = (m_1 + m_2)v_f$$

$$v_f = \frac{m_1v_{1_i} + m_2v_{2_i}}{m_1 + m_2} = \frac{(1800\ \text{kg})0 + (900\ \text{kg})\left(20\ \frac{\text{m}}{\text{s}}\right)}{1800\ \text{kg} + 900\ \text{kg}} = \boxed{6.67\ \frac{\text{m}}{\text{s}}}$$

6.)



$m = 3.0\ \text{kg}$, $v_i = 10\ \frac{\text{m}}{\text{s}}$, $\theta_i = 30^\circ$ $v_f = 10\ \frac{\text{m}}{\text{s}}$, $\theta_f = 150^\circ$ $\Delta t = 0.20\ \text{s}$

Before hitting the wall $v_{x_i} = v_i\cos\theta_i = \left(10\ \frac{\text{m}}{\text{s}}\right)\cos 30^\circ = 8.66\ \frac{\text{m}}{\text{s}}$

$v_{y_i} = v_i\sin\theta_i = \left(10\ \frac{\text{m}}{\text{s}}\right)\sin 30^\circ = 5.0\ \frac{\text{m}}{\text{s}}$

After hitting the wall $v_{x_f} = v_f\cos\theta_f = \left(10\ \frac{\text{m}}{\text{s}}\right)\cos 150^\circ = -8.66\ \frac{\text{m}}{\text{s}}$

$v_{y_f} = v_f\sin\theta_f = \left(10\ \frac{\text{m}}{\text{s}}\right)\sin 150^\circ = 5.0\ \frac{\text{m}}{\text{s}}$

6.) (continued)

the only momentum change occurs in x-direction and $\Delta p_x = m\Delta v_x = m\left(v_{x_f} - v_{x_i}\right)$

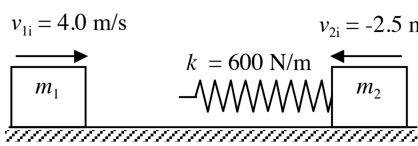
using Impulse-Momentum Theorem

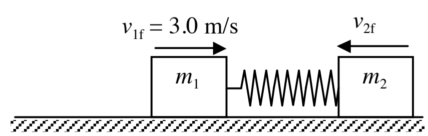
$$\vec{J} = \Delta\vec{p} \text{ or } J_x = \Delta p_x = m\left(v_{x_f} - v_{x_i}\right) \text{ so } J_x = F_x\Delta t \text{ or } F_x = \frac{J_x}{\Delta t} \text{ and } J_y = F_y\Delta t = 0 \text{ or } F_y = 0$$

$$\text{therefore } F_x = \frac{J_x}{\Delta t} = \frac{m\left(v_{x_f} - v_{x_i}\right)}{\Delta t} = \frac{(3.0\text{ kg})\left(-8.66\ \frac{\text{m}}{\text{s}} - 8.66\ \frac{\text{m}}{\text{s}}\right)}{0.20\text{ s}} = -260\text{ N}$$

so the force from the wall is $\vec{F} = \boxed{260\text{ N}\angle -180°}$

7.)





$m_1 = 1.60$ kg and $m_2 = 2.10$ kg

a.)

using Conservation of Momentum

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f}$$

$$v_{2_f} = \frac{m_1v_{1_i} + m_2v_{2_i} - m_1v_{1_f}}{m_2} = \frac{(1.60\text{ kg})\left(4.0\ \frac{\text{m}}{\text{s}}\right) + (2.10\text{ kg})\left(-2.5\ \frac{\text{m}}{\text{s}}\right) - (1.60\text{ kg})\left(3.0\ \frac{\text{m}}{\text{s}}\right)}{2.10\text{ kg}} = \boxed{-1.74\ \frac{\text{m}}{\text{s}}}$$

b.)

using Conservation of Energy

$$K_1 + U_{g1} + U_{e1} + W_{other} = K_2 + U_{g2} + U_{e2} \text{ so } K_1 + 0 + 0 + 0 = K_2 + 0 + U_{e2} \text{ or } K_1 = K_2 + U_{e2}$$

$$\frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2 = \frac{1}{2}m_1v_{1_f}^2 + \frac{1}{2}m_2v_{2_f}^2 + \frac{1}{2}k\Delta x_2^2$$

$$\Delta x_2 = \sqrt{\frac{m_1v_{1_i}^2 + m_2v_{2_i}^2 - m_1v_{1_f}^2 - m_2v_{2_f}^2}{k}}$$

$$\Delta x_2 = \sqrt{\frac{(1.60\text{ kg})\left(4.0\ \frac{\text{m}}{\text{s}}\right)^2 + (2.10\text{ kg})\left(-2.5\ \frac{\text{m}}{\text{s}}\right)^2 - (1.60\text{ kg})\left(3.0\ \frac{\text{m}}{\text{s}}\right)^2 - (2.10\text{ kg})\left(-1.74\ \frac{\text{m}}{\text{s}}\right)^2}{600\ \frac{\text{N}}{\text{m}}}} = \boxed{0.173\text{ m}}$$

c.) $v_{2_f} = 0$

using Conservation of Momentum

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1v_{1_i} + m_2v_{2_i} = m_1v_{1f} + m_2v_{2_f}$$

$$v_{1_f} = \frac{m_1v_{1_i} + m_2v_{2_i} - m_2v_{2_f}}{m_1} = \frac{(1.60\text{ kg})\left(4.0\ \frac{\text{m}}{\text{s}}\right) + (2.10\text{ kg})\left(-2.5\ \frac{\text{m}}{\text{s}}\right) - 0}{1.60\text{ kg}} = \boxed{0.72\ \frac{\text{m}}{\text{s}}}$$

7.) c.) (continued)

using Conservation of Energy

$$K_1 + U_{g_1} + U_{e_1} + W_{other} = K_2 + U_{g_2} + U_{e_2} \text{ so } K_1 + 0 + 0 + 0 = K_2 + 0 + U_{e_2} \text{ or } K_1 = K_2 + U_{e_2}$$

$$\frac{1}{2} m_1 v_{1_i}^2 + \frac{1}{2} m_2 v_{2_i}^2 = \frac{1}{2} m_1 v_{1_f}^2 + \frac{1}{2} m_2 v_{2_f}^2 + \frac{1}{2} k \Delta x_2^2$$

$$\Delta x_2 = \sqrt{\frac{m_1 v_{1_i}^2 + m_2 v_{2_i}^2 - m_1 v_{1_f}^2 - m_2 v_{2_f}^2}{k}}$$

$$\Delta x_2 = \sqrt{\frac{(1.60\text{ kg})\left(4.0\ \frac{\text{m}}{\text{s}}\right)^2 + (2.10\text{ kg})\left(-2.5\ \frac{\text{m}}{\text{s}}\right)^2 - (1.60\text{ kg})\left(0.72\ \frac{\text{m}}{\text{s}}\right)^2 - 0}{600\ \frac{\text{N}}{\text{m}}}} = \boxed{0.251\text{ m}}$$

8.) $m_1 = 1500\text{ kg}$, $v_{1_i} = 25.0\ \frac{\text{m}}{\text{s}}$, $\theta_{1_i} = 0$, $m_2 = 2500\text{ kg}$, $v_{2_i} = 20.0\ \frac{\text{m}}{\text{s}}$, and $\theta_{2_i} = 90°$

inelastic collision so $v_{1_{fx}} = v_{2_{fx}} = v_{fx}$ and $v_{1_{fy}} = v_{2_{fy}} = v_{fy}$

using Conservation of Momentum in the *x*-direction

$$p_{1_{ix}} + p_{2_{ix}} = p_{1_{fx}} + p_{2_{fx}} \text{ so } m_1 v_{1_{ix}} + m_2 v_{2_{ix}} = m_1 v_{fx} + m_2 v_{fx} = (m_1 + m_2) v_{fx}$$

$$v_{fx} = \frac{m_1 v_{1_{ix}} + m_2 v_{2_{ix}}}{m_1 + m_2} = \frac{m_1 v_{1_i} \cos\theta_{1_i} + m_2 v_{2_i} \cos\theta_{2_i}}{m_1 + m_2}$$

$$v_{fx} = \frac{(1500\text{ kg})\left(25.0\ \frac{\text{m}}{\text{s}}\right)\cos 0 + (2500\text{ kg})\left(20.0\ \frac{\text{m}}{\text{s}}\right)\cos 90°}{1500\text{ kg} + 2500\text{ kg}} = 9.375\ \frac{\text{m}}{\text{s}}$$

using Conservation of Momentum in the *y*-direction

$$p_{1_{iy}} + p_{2_{iy}} = p_{1_{fy}} + p_{2_{fy}} \text{ so } m_1 v_{1_{iy}} + m_2 v_{2_{iy}} = m_1 v_{fy} + m_2 v_{fy} = (m_1 + m_2) v_{fy}$$

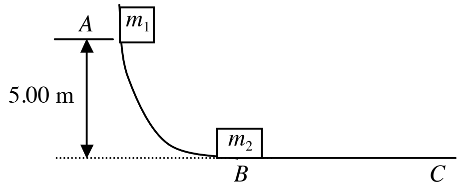
$$v_{fy} = \frac{m_1 v_{1_{iy}} + m_2 v_{2_{iy}}}{m_1 + m_2} = \frac{m_1 v_{1_i} \sin\theta_{1_i} + m_2 v_{2_i} \sin\theta_{2_i}}{m_1 + m_2}$$

$$v_{fy} = \frac{(1500\text{ kg})\left(25.0\ \frac{\text{m}}{\text{s}}\right)\sin 0 + (2500\text{ kg})\left(20.0\ \frac{\text{m}}{\text{s}}\right)\sin 90°}{1500\text{ kg} + 2500\text{ kg}} = 12.50\ \frac{\text{m}}{\text{s}}$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(9.375\ \frac{\text{m}}{\text{s}}\right)^2 + \left(12.50\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{15.625\ \frac{\text{m}}{\text{s}}}$$

$$\theta_f = \tan^{-1} \frac{v_{yf}}{v_{xf}} = \tan^{-1}\left(\frac{12.5\ \frac{\text{m}}{\text{s}}}{9.375\ \frac{\text{m}}{\text{s}}}\right) = \boxed{53.13°}$$

9.)



$m_1 = 5.00$ kg, $v_{1_i} = 0$ and $m_2 = 10.0$ kg, $v_{2_i} = 0$

(1) determine the speed of m_1 just before its collision with m_2 using Conservation of Energy

using the lowest point as a reference for m_1, $y_1 = 5.00$ m and $y_2 = 0$

$$K_1 + U_{g_1} + U_{e_1} + W_{other} = K_2 + U_{g2} + U_{e_2} \text{ so } 0 + U_{g_1} + 0 + 0 = K_2 + 0 + 0 \text{ or } U_{g_1} = K_2$$

$$m_1 g y_1 = \frac{1}{2} m_1 v^2 \text{ and } v = \sqrt{2gy_1} = v_{1_i} \text{ (before the collision)}$$

(2) determine the speed of m_1 just after its collision with m_2 using Conservation of Momentum

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f} \text{ and } m_1 v_{1_i} + 0 = m_1 v_{1_f} + m_2 v_{2_f}$$

therefore (A) $m_1 v_{1_i} = m_1 v_{1_f} + m_2 v_{2_f}$

since the collision is elastic kinetic energy is also conserved the relative speed of the two blocks before the collision equals the negative of their relative velocities after the collision

$$v_{1_i} - v_{2_i} = -\left(v_{1_f} - v_{2_f}\right) \text{ therefore (B) } v_{1_i} = -v_{1_f} + v_{2_f}$$

from (A) $v_{1_i} = v_{1_f} + \dfrac{m_2}{m_1} v_{2_f}$ and combining this with (B) $2v_{1_i} = \left(1 + \dfrac{m_2}{m_1}\right) v_{2_f} = \left(\dfrac{m_1 + m_2}{m_1}\right) v_{2_f}$

and $v_{2_f} = \dfrac{2m_1}{m_1 + m_2} v_{1_i}$ and from (B) $v_{1_f} = v_{2_f} - v_{1_i} = \dfrac{2m_1}{m_1 + m_2} v_{1_i} - v_{1_i}$

$v_{1_f} = v_{1_i}\left(\dfrac{2m_1}{m_1 + m_2} - 1\right)$ and using result from (1) $v_{1_f} = \sqrt{2gy_1}\left(\dfrac{2m_1}{m_2 + m_1} - 1\right)$

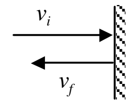
(3) determine the height of m_1 after the collision using conservation of energy $(K = U)$

$$\frac{1}{2} m_1 v_{1_f}^2 = m_1 g y_2 \text{ or } y_2 = \frac{v_{1_f}^2}{2g} = \frac{1}{2g}\left(\sqrt{2gy_1}\left(\frac{2m_1}{m_2 + m_1} - 1\right)\right)^2 = \frac{1}{2g}\left(2gy_1\left(\frac{2m_1}{m_2 + m_1} - 1\right)^2\right) = y_1\left(\frac{2m_1}{m_2 + m_1} - 1\right)^2$$

$$y_2 = (5.00 \text{ m})\left(\frac{2(5.0 \text{ kg})}{5.00 \text{ kg} + 10.0 \text{ kg}} - 1\right)^2 = \boxed{0.56 \text{ m}}$$

HO 17 Solutions

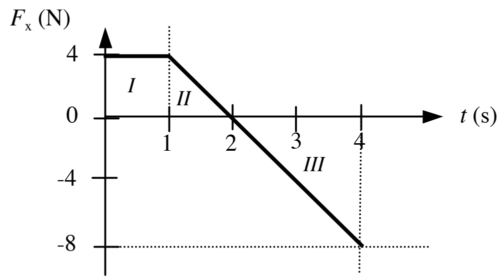
1.) $m = 1.2 \text{ kg}$, $v_i = 8.0 \ \frac{\text{m}}{\text{s}}$, $v_f = -6.0 \ \frac{\text{m}}{\text{s}}$, $\Delta t = 2.0 \text{ x } 10^{-3} \text{ s}$



using Impulse-Momentum Theorem

$$J = \Delta p = mv_f - mv_i = m(v_f - v_i) \text{ and } J = F\Delta t \text{ so } F = \frac{J}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t} = \frac{(1.2 \text{ kg})\left(-6.0 \ \frac{\text{m}}{\text{s}} - 8.0 \ \frac{\text{m}}{\text{s}}\right)}{2.0 \text{ x } 10^{-3} \text{ s}} = \boxed{-8400 \text{ N}}$$

2.)



$m = 2.0 \text{ kg}$, $v_x(0) = -2.0 \ \frac{\text{m}}{\text{s}}$

$$\vec{J} = \int \vec{F}dt = Area = A_I + A_{II} + A_{III}$$

$$J = (4 \text{ N})(1 \text{ s}) + \frac{1}{2}(4 \text{ N})(1 \text{ s}) + \frac{1}{2}(-8 \text{ N})(2 \text{ s}) = -2 \text{ N}\cdot\text{s}$$

using Impulse-Momentum Theorem

$$J = \Delta p = mv_f - mv_i = m(v_f - v_i) \text{ so } v_f = \frac{J}{m} + v_i = \frac{-2 \text{ N}\cdot\text{s}}{2.0 \text{ kg}} + \left(-2.0 \ \frac{\text{m}}{\text{s}}\right) = \boxed{-3.0 \ \frac{\text{m}}{\text{s}}}$$

3.) $m = 2.0 \text{ kg}$, $F_x = (4.0t) \text{ N} = \left(4.0 \ \frac{\text{N}}{\text{s}}\right)t$, $v_x(0) = 3.0 \ \frac{\text{m}}{\text{s}}$, $v_x(t) = 8.0 \ \frac{\text{m}}{\text{s}}$ find t

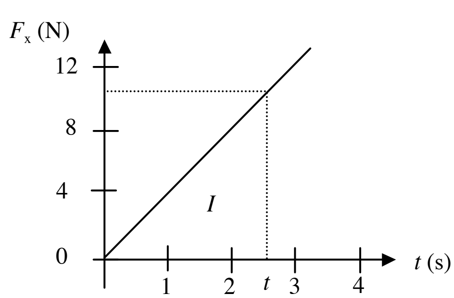
$$\vec{J} = \int \vec{F}dt \Rightarrow J_x = \int F_x dt = \int_0^t \left(4.0 \ \frac{\text{N}}{\text{s}}\right)tdt = \left.\left(4.0 \ \frac{\text{N}}{\text{s}}\right)\frac{t^2}{2}\right]_0^t = \left(2.0 \ \frac{\text{N}}{\text{s}}\right)t^2$$

using Impulse-Momentum Theorem

$$J = \Delta p = mv_f - mv_i = m(v_f - v_i) = (2.0 \text{ kg})\left(8.0 \ \frac{\text{m}}{\text{s}} - 3.0 \ \frac{\text{m}}{\text{s}}\right) = 10.0 \text{ N}\cdot\text{s}$$

$$J(t) = (2.0t^2)\text{N} = 10.0 \text{ N}\cdot\text{s} \text{ and } t = \sqrt{\frac{10.0 \text{ N}\cdot\text{s}}{2.0 \ \frac{\text{N}}{\text{s}}}} = \boxed{2.24 \text{ s}}$$

Alternatively graph F versus t and find the area

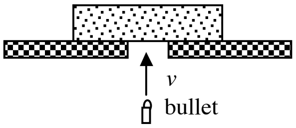


$$\vec{J} = \int \vec{F}dt = Area = A_I = \frac{1}{2}t\left(\left(4.0 \ \frac{\text{N}}{\text{s}}\right)t\right) = \left(2.0 \ \frac{\text{N}}{\text{s}}\right)t^2$$

using Impulse-Momentum Theorem

$$J = \Delta p = mv_f - mv_i = m(v_f - v_i) = (2.0 \text{ kg})\left(8.0 \ \frac{\text{m}}{\text{s}} - 3.0 \ \frac{\text{m}}{\text{s}}\right) = 10.0 \text{ N}\cdot\text{s}$$

$$\left(2.0 \ \frac{\text{N}}{\text{s}}\right)t^2 = 10.0 \text{ N}\cdot\text{s} \text{ and } t = \sqrt{\frac{10.0 \text{ N}\cdot\text{s}}{2.0 \ \frac{\text{N}}{\text{s}}}} = \boxed{2.24 \text{ s}}$$

4.) 

$m_1 = 0.010\text{ kg}$, $v_{1_i} = 1000\ \frac{\text{m}}{\text{s}}$, $v_{1f} = 400\ \frac{\text{m}}{\text{s}}$, $m_2 = 2.0\text{ kg}$, and $v_{2_i} = 0$

using Conservation of Momentum

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f}$$

$$v_{2_f} = \frac{m_1v_{1_i} + m_2v_{2_i} - m_2v_{2_f}}{m_2} = \frac{(0.010\text{ kg})\left(1000\ \frac{\text{m}}{\text{s}}\right) + 0 - (0.010\text{ kg})\left(400\ \frac{\text{m}}{\text{s}}\right)}{2.0\text{ kg}} = 3.0\ \frac{\text{m}}{\text{s}}$$

using Conservation of Energy for the block m_2 with $y_1 = 0$ and $v_2 = 0$

$$K_1 + U_{g1} + U_{e1} + W_{other} = K_2 + U_{g2} + U_{e2} \text{ so } K_1 + 0 + 0 + 0 = 0 + U_{g2} + 0 \text{ or } K_1 = U_{g2}$$

$$\frac{1}{2}m_2v_1^2 = m_2gy_2 \text{ and } y_2 = \frac{v_1^2}{2g} = \frac{\left(3.0\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{0.46\text{ m}}$$

5.) $m_1 = 6.0\text{ kg}$, $v_{1_i} = 5.0\ \frac{\text{m}}{\text{s}}$, $m_2 = 2.0\text{ kg}$, $v_{1_f} = v_{2_f} = v_f = -2.0\ \frac{\text{m}}{\text{s}}$

using Conservation of Momentum

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f} = (m_1 + m_2)v_f$$

$$v_{2_i} = \frac{(m_1 + m_2)v_f - m_1v_{1_i}}{m_2} = \frac{(6.0\text{ kg} + 2.0\text{ kg})\left(-2.0\ \frac{\text{m}}{\text{s}}\right) - (6.0\text{ kg})\left(5.0\ \frac{\text{m}}{\text{s}}\right)}{2.0\text{ kg}} = \boxed{-23\ \frac{\text{m}}{\text{s}}}$$

6.) $m_1 = 2.0\text{ kg}$, $v_{1_i} = 5.0\ \frac{\text{m}}{\text{s}}$, $m_2 = 8.0\text{ kg}$, $v_{2_i} = 0$, and $v_{1_f} = v_{2_f} = v_f$

using Conservation of Momentum

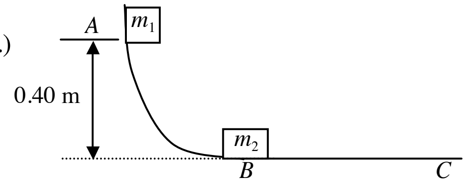
$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f} = (m_1 + m_2)v_f$$

$$v_f = \frac{m_1v_{1_i} + m_2v_{2_i}}{m_1 + m_2} = \frac{(2.0\text{ kg})\left(5.0\ \frac{\text{m}}{\text{s}}\right) + 0}{2.0\text{ kg} + 8.0\text{ kg}} = 1.0\ \frac{\text{m}}{\text{s}}$$

$$\Delta K = \left(K_{1_f} + K_{2_f}\right) - \left(K_{1_i} + K_{2_i}\right) = \left(\frac{1}{2}m_1v_{1_f}^2 + \frac{1}{2}m_2v_{2_f}^2\right) - \left(\frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2\right)$$

$$\Delta K = \left(\frac{1}{2}m_1 + \frac{1}{2}m_2\right)v_f^2 - \left(\frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2\right)$$

$$\Delta K = \left(\frac{1}{2}(2.0\text{ kg}) + \frac{1}{2}(8.0\text{ kg})\right)\left(1.0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(\frac{1}{2}(2.0\text{ kg})\left(5.0\ \frac{\text{m}}{\text{s}}\right)^2 + 0\right) = \boxed{-20\text{ J}}$$

7.) 

$m_1 = 3.0\text{ kg}$, $m_2 = 1.4\text{ kg}$, and $v_1 = v_2 = 0$

(1) determine the speed of m_1 just before its collision with m_2 using Conservation of Energy

using the lowest point as a reference for m_1, $y_1 = 0.40\text{ m}$ and $y_2 = 0$

$$K_1 + U_{g1} + U_{e1} + W_{other} = K_2 + U_{g2} + U_{e2} \text{ so } 0 + U_{g1} + 0 + 0 = K_2 + 0 + 0 \text{ or } U_{g1} = K_2$$

$$m_1 g y_1 = \frac{1}{2} m_1 v^2 \text{ and } v = \sqrt{2gy_1} = v_{1_i} \text{ (before the collision)}$$

(2) determine the speed of m_1 and m_2 just after the inelastic collision using Conservation of Momentum

$v_{2_i} = 0$ and $v_{1_f} = v_{2_f} = v_f$

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1_i} + m_2 v_{2_i}}{m_1 + m_2} = \frac{m_1\sqrt{2gy_1}}{m_1 + m_2} = \frac{(3.0\text{ kg})\sqrt{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.40\text{ m})}}{3.0\text{ kg} + 1.4\text{ kg}} = \boxed{1.91\ \frac{\text{m}}{\text{s}}}$$

8.) $m_1 = 0.010\text{ kg}$, $v_{1_i} = 2000\ \frac{\text{m}}{\text{s}}$, $m_2 = 4.0\text{ kg}$, $v_{2_i} = -4.2\ \frac{\text{m}}{\text{s}}$, and $v_{2_f} = 0$

using Conservation of Momentum

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f} \text{ so } m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f} \text{ and } v_{1_f} = \frac{m_1 v_{1_i} + m_2 v_{2_i} - m_2 v_{2_f}}{m_1}$$

$$v_{1_f} = \frac{(0.010\text{ kg})\left(2000\ \frac{\text{m}}{\text{s}}\right) + (4.0\text{ kg})\left(-4.2\ \frac{\text{m}}{\text{s}}\right) - 0}{0.010\text{ kg}} = 320\ \frac{\text{m}}{\text{s}}$$

$$K_{1_f} = \frac{1}{2} m_1 v_{1_f}^2 = \frac{1}{2}(0.010\text{ kg})\left(320\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{512\text{ J}}$$

9.) $m_1 = 3.0\text{ kg}$, $v_{1_i} = 10\ \frac{\text{m}}{\text{s}}$, $\theta_{1_i} = 0$, $v_{1_f} = 8.0\ \frac{\text{m}}{\text{s}}$, $\theta_{1_f} = 35°$, $m_2 = 6.0\text{ kg}$, $v_{2_i} = 0$

using Conservation of Momentum in the *x*-direction

$$p_{1_{ix}} + p_{2_{ix}} = p_{1_{fx}} + p_{2_{fx}} \text{ so } m_1 v_{1_{ix}} + m_2 v_{2_{ix}} = m_1 v_{1_{fx}} + m_2 v_{2_{fx}}$$

$$v_{2_{fx}} = \frac{m_1 v_{1_{ix}} + m_2 v_{2_{ix}} - m_1 v_{1_{fx}}}{m_2} = \frac{m_1 v_{1_i}\cos\theta_{1_i} + m_2 v_{2_i}\cos\theta_{2_i} - m_1 v_{1_f}\cos\theta_{1f}}{m_2}$$

$$v_{2_{fx}} = \frac{(3.0\text{ kg})\left(10\ \frac{\text{m}}{\text{s}}\right)\cos 0 + 0 - (3.0\text{ kg})\left(8.0\ \frac{\text{m}}{\text{s}}\right)\cos 35°}{6.0\text{ kg}} = 1.72\ \frac{\text{m}}{\text{s}}$$

9.) (continued)

using Conservation of Momentum in the y-direction

$$p_{1_{iy}} + p_{2_{iy}} = p_{1_{fy}} + p_{2_{fy}} \text{ so } m_1 v_{1_{iy}} + m_2 v_{2_{iy}} = m_1 v_{1_{fy}} + m_2 v_{2_{fy}}$$

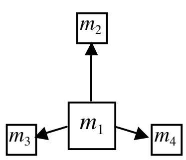
$$v_{2_{fy}} = \frac{m_1 v_{1_{iy}} + m_2 v_{2_{iy}} - m_1 v_{1_{fy}}}{m_2} = \frac{m_1 v_{1_i} \sin\theta_{1_i} + m_2 v_{2_i} \sin\theta_{2_i} - m_1 v_{1_f} \sin\theta_{1f}}{m_2}$$

$$v_{2_{fy}} = \frac{(3.0\text{ kg})\left(10\ \frac{\text{m}}{\text{s}}\right)\sin 0 + 0 - (3.0\text{ kg})\left(8.0\ \frac{\text{m}}{\text{s}}\right)\sin 35°}{6.0\text{ kg}} = -2.29\ \frac{\text{m}}{\text{s}}$$

$$v_{2_f} = \sqrt{v_{2_{fx}}^2 + v_{2_{fy}}^2} = \sqrt{\left(1.72\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-2.29\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{2.86\ \frac{\text{m}}{\text{s}}}$$

$$\theta_{2_f} = \tan^{-1}\frac{v_{2_{fy}}}{v_{2_{fx}}} = \tan^{-1}\left(\frac{-2.29\ \frac{\text{m}}{\text{s}}}{1.72\ \frac{\text{m}}{\text{s}}}\right) = \boxed{-53.1°}$$

10.) $m_1 = 3.0\text{ kg}$, $m_2 = 1.0\text{ kg}$, $v_{2_f} = 9.0\ \frac{\text{m}}{\text{s}}$, $\theta_{2_f} = 90°$, $m_3 = 1.0\text{ kg}$, $v_{3_f} = 4.0\ \frac{\text{m}}{\text{s}}$, $\theta_{3_f} = 210°$, $m_4 = 1.0\text{ kg}$, $v_{4_f} = 4.0\ \frac{\text{m}}{\text{s}}$, $\theta_{4_f} = -30°$



The x-components of m_3 and m_4 cancel each other out and m_2 only has a y-component. Therefore, the final total momentum has only a y-component which means the initial momentum of m_1 only has a y-component.

using Conservation of Momentum in the y-direction

$$p_{1_{iy}} = p_{2_{fy}} + p_{3_{fy}} + p_{4_{fy}} \text{ so } m_1 v_{1_{iy}} = m_2 v_{2_{fy}} + m_3 v_{3_{fy}} + m_4 v_{4_{fy}}$$

$$v_{1_{iy}} = \frac{m_2 v_{2_{fy}} + m_3 v_{3_{fy}} + m_4 v_{4_{fy}}}{m_1} = \frac{m_2 v_{2_f} \sin\theta_{2_f} + m_3 v_{3_f} \sin\theta_{3_f} + m_4 v_{4_f} \sin\theta_{4_f}}{m_1}$$

$$v_{1_{iy}} = \frac{(1.0\text{ kg})\left(9.0\ \frac{\text{m}}{\text{s}}\right)\sin 90° + (1.0\text{ kg})\left(4.0\ \frac{\text{m}}{\text{s}}\right)\sin 210° + (1.0\text{ kg})\left(4.0\ \frac{\text{m}}{\text{s}}\right)\sin(-30°)}{3.0\text{ kg}} = 1.67\ \frac{\text{m}}{\text{s}}$$

so the magnitude of the initial velocity of m_1 was $v_{1_i} = \boxed{1.67\ \frac{\text{m}}{\text{s}}}$

HO 18 Solutions

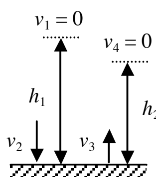
1.) $m_1 = 2.00\text{ kg}$, $(x_1, y_1) = (2.00\text{ m},\ 3.00\text{ m})$, $m_2 = 3.00\text{ kg}$, $(x_2, y_2) = (2.00\text{ m},\ -2.00\text{ m})$,
$m_3 = 4.00\text{ kg}$, $(x_3, y_3) = (-3.00\text{ m},\ 6.00\text{ m})$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(2.00\text{ m}) + (3.00\text{ kg})(2.00\text{ m}) + (4.00\text{ kg})(-3.00\text{ m})}{2.00\text{ kg} + 3.00\text{ kg} + 4.00\text{ kg}} = -0.22\text{ m}$$

$$y_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(3.00\text{ m}) + (3.00\text{ kg})(-2.00\text{ m}) + (4.00\text{ kg})(6.00\text{ m})}{2.00\text{ kg} + 3.00\text{ kg} + 4.00\text{ kg}} = 2.67\text{ m}$$

the center-of-mass is therefore $(x_{cm}, y_{cm}) = \boxed{(-0.22\text{ m},\ 2.67\text{ m})}$

2.)



$m = 0.200\text{ kg}$, $h_1 = 4.00\text{ m}$, $h_2 = 3.80\text{ m}$

a.) use Conservation of Energy find the speed v_2 that the ball hits the ground

using the ground as a reference $y_1 = 4.00\text{ m}$ and $y_2 = 0$

$K_1 + U_{g1} + W_{other} = K_2 + U_{g2}$ so $0 + U_{g1} + 0 = K_2 + 0$ or $U_{g1} = K_2$

$$mgy_1 = \frac{1}{2}mv_2^2 \text{ and } v_2 = \sqrt{2gy_1} = \sqrt{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(4.00\text{ m})} = 8.85\ \frac{\text{m}}{\text{s}}$$

use Conservation of Energy find the speed v_3 that the ball leaves the ground

using the ground as a reference $y_3 = 0$ and $y_4 = 3.80\text{ m}$

$K_3 + U_{g3} + W_{other} = K_4 + U_{g4}$ so $K_3 + 0 + 0 = 0 + U_{g4}$ or $K_3 = U_{g4}$

$$\frac{1}{2}mv_3^2 = mgy_4 \text{ and } v_3 = \sqrt{2gy_4} = \sqrt{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(3.80\text{ m})} = 8.63\ \frac{\text{m}}{\text{s}}$$

using the Impulse-Momentum theorem

$$J = \Delta p = m\Delta v = m(v_3 - v_2) = m(v_3 - v_2) = (0.200\text{ kg})\left(8.63\ \frac{\text{m}}{\text{s}} - \left(-8.85\ \frac{\text{m}}{\text{s}}\right)\right) = \boxed{3.5\text{ N}\cdot\text{s}}$$

b.) $\Delta t = 2.00 \text{ x } 10^{-3}\text{ s}$

$$J = F\Delta t \text{ and } F = \frac{J}{\Delta t} = \frac{3.5\text{ N}\cdot\text{s}}{2.00 \text{ x } 10^{-3}\text{ s}} = \boxed{1750\text{ N}}$$

3.) $m_1 = 1500\text{ kg}$, $\theta_1 = 270°$, $m_2 = 2000\text{ kg}$, $\theta_2 = 180°$, $\vec{p}_{total} = 8000\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\angle 240°$

total momentum *x*-component

$$p_{total_x} = p_{1_x} + p_{2_x} \text{ so } p_{total_x} = m_1 v_{1_x} + m_2 v_{2_x} \text{ and } p_{total}\cos\theta_{total} = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

$$\left(8000\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)\cos 240° = (1500\text{ kg})v_1\cos 270° + (2000\text{ kg})v_2\cos 180° = 0 - (2000\text{ kg})v_2$$

$$v_2 = \frac{-\left(8000\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)\cos 240°}{2000\text{ kg}} = \boxed{2.00\ \frac{\text{m}}{\text{s}}}$$

3.) (continued)

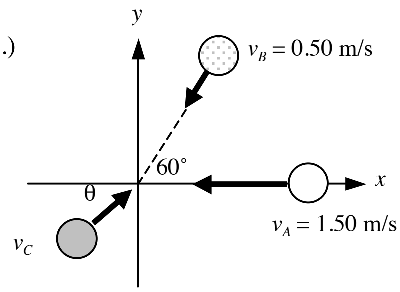
total momentum y-component

$$p_{total_y} = p_{1_y} + p_{2_y} \text{ so } p_{total_y} = m_1 v_{1_y} + m_2 v_{2_y} \text{ and } p_{total}\sin\theta_{total} = m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2$$

$$\left(8000\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)\sin240° = (1500\text{ kg})v_1\sin270° + (2000\text{ kg})v_2\sin180° = -(1500\text{ kg})v_1 + 0$$

$$v_1 = \frac{-\left(8000\ \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)\sin240°}{1500\text{ kg}} = \boxed{4.62\ \frac{\text{m}}{\text{s}}}$$

4.)



$$m_A = 0.020\text{ kg},\ m_B = 0.030\text{ kg},\ m_C = 0.050\text{ kg}$$

$$v_A = 1.50\ \frac{\text{m}}{\text{s}},\ \theta_A = 180°$$

$$v_B = 0.50\ \frac{\text{m}}{\text{s}},\ \theta_B = 240°$$

a.)

using Conservation of Momentum in the x-direction

$$p_{A_x} + p_{B_x} + p_{C\,x} = 0$$

$$m_A v_{A_x} + m_B v_{B_x} + m_C v_{C_x} = 0$$

$$m_A v_A \cos\theta_A + m_B v_B \cos\theta_B + m_C v_{C_x} = 0$$

$$v_{C_x} = \frac{-m_A v_A \cos\theta_A - m_B v_B \cos\theta_B}{m_C} = \frac{-(0.020\text{ kg})\left(1.50\ \frac{\text{m}}{\text{s}}\right)\cos180° - (0.030\text{ kg})\left(0.50\ \frac{\text{m}}{\text{s}}\right)\cos240°}{0.050\text{ kg}} = \boxed{0.75\ \frac{\text{m}}{\text{s}}}$$

using Conservation of Momentum in the y-direction

$$p_{Ay} + p_{By} + p_{C\,y} = 0$$

$$m_A v_{A_y} + m_B v_{B_y} + m_C v_{C_y} = 0$$

$$m_A v_A \sin\theta_A + m_B v_B \sin\theta_B + m_C v_{C_y} = 0$$

$$v_{C_y} = \frac{-m_A v_A \sin\theta_A - m_B v_B \sin\theta_B}{m_C} = \frac{-(0.020\text{ kg})\left(1.50\ \frac{\text{m}}{\text{s}}\right)\sin180° - (0.030\text{ kg})\left(0.50\ \frac{\text{m}}{\text{s}}\right)\sin240°}{0.050\text{ kg}} = \boxed{0.26\ \frac{\text{m}}{\text{s}}}$$

b.)

$$\theta_C = \tan^{-1}\frac{v_{C_y}}{v_{C_x}} = \tan^{-1}\left(\frac{0.26\ \frac{\text{m}}{\text{s}}}{0.75\ \frac{\text{m}}{\text{s}}}\right) = \boxed{19.1°}$$

5.) $m_1 = 0.040\text{ kg}$, $(x_1, y_1) = (4.0\text{ m, }3.0\text{ m})$, $m_2 = 0.050\text{ kg}$, $(x_2, y_2) = (-2.0\text{ m, }-2.0\text{ m})$, $m_3 = 0.020\text{ kg}$, and $(x_{cm}, y_{cm}) = (0,\ 0)$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = 0$$

$$x_3 = \frac{-(m_1 x_1 + m_2 x_2)}{m_3} = \frac{-((0.040\text{ kg})(4.0\text{ m}) + (0.050\text{ kg})(-2.00\text{ m}))}{0.020\text{ kg}} = \boxed{-3.0\text{ m}}$$

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = 0$$

$$y_3 = \frac{-(m_1 x_1 + m_2 x_2)}{m_3} = \frac{-((0.040\text{ kg})(3.0\text{ m}) + (0.050\text{ kg})(-2.00\text{ m}))}{0.020\text{ kg}} = \boxed{-1.0\text{ m}}$$

6.) $m_1 = 0.030\text{ kg}$, $(x_1, y_1) = (3.0\text{ m, }4.0\text{ m})$, $m_2 = 0.040\text{ kg}$, $(x_2, y_2) = (-2.0\text{ m, }-2.0\text{ m})$, $m_3 = 0.020\text{ kg}$, and $(x_{cm}, y_{cm}) = (0,\ 0)$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = 0$$

$$x_3 = \frac{-(m_1 x_1 + m_2 x_2)}{m_3} = \frac{-((0.030\text{ kg})(3.0\text{ m}) + (0.040\text{ kg})(-2.00\text{ m}))}{0.020\text{ kg}} = \boxed{-0.5\text{ m}}$$

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = 0$$

$$y_3 = \frac{-(m_1 x_1 + m_2 x_2)}{m_3} = \frac{-((0.030\text{ kg})(4.0\text{ m}) + (0.040\text{ kg})(-2.00\text{ m}))}{0.020\text{ kg}} = \boxed{-2.0\text{ m}}$$

7.) $m_1 = 2.0\text{ kg}$, $\vec{v}_1 = 4.0\ \frac{\text{m}}{\text{s}}\angle 0$, $m_2 = 3.0\text{ kg}$, $\vec{v}_2 = 5.0\ \frac{\text{m}}{\text{s}}\angle 90°$

$$v_{cm_x} = \frac{\sum p_{x_i}}{\sum m_i} = \frac{\sum m_i v_{x_i}}{\sum m_i} = \frac{m_1 v_{1_x} + m_2 v_{2_x}}{m_1 + m_2} = \frac{m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2}{m_1 + m_2}$$

$$v_{cm_x} = \frac{(2.0\text{ kg})\left(4.0\ \frac{\text{m}}{\text{s}}\right)\cos 0 + (3.0\text{ kg})\left(5.0\ \frac{\text{m}}{\text{s}}\right)\cos 90°}{2.0\text{ kg} + 3.0\text{ kg}} = 1.6\ \frac{\text{m}}{\text{s}}$$

$$v_{cm_y} = \frac{\sum p_{y_i}}{\sum m_i} = \frac{\sum m_i v_{y_i}}{\sum m_i} = \frac{m_1 v_{1_y} + m_2 v_{2_y}}{m_1 + m_2} = \frac{m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2}{m_1 + m_2}$$

$$v_{cm_y} = \frac{(2.0\text{ kg})\left(4.0\ \frac{\text{m}}{\text{s}}\right)\sin 0 + (3.0\text{ kg})\left(5.0\ \frac{\text{m}}{\text{s}}\right)\sin 90°}{2.0\text{ kg} + 3.0\text{ kg}} = 3.0\ \frac{\text{m}}{\text{s}}$$

$$v_{cm} = \sqrt{v_{cm_x}^2 + v_{cm_y}^2} = \sqrt{\left(1.6\ \frac{\text{m}}{\text{s}}\right)^2 + \left(3.0\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{3.4\ \frac{\text{m}}{\text{s}}}$$

8.) $m_1 = 3.0\text{ kg}$, $\vec{v}_1 = 6.0\ \frac{\text{m}}{\text{s}}\angle 270°$, $m_2 = 4.0\text{ kg}$, $\vec{v}_2 = 7.0\ \frac{\text{m}}{\text{s}}\angle 0$

$$v_{cm_x} = \frac{\sum p_{x_i}}{\sum m_i} = \frac{\sum m_i v_{x_i}}{\sum m_i} = \frac{m_1 v_{1_x} + m_2 v_{2_x}}{m_1 + m_2} = \frac{m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2}{m_1 + m_2}$$

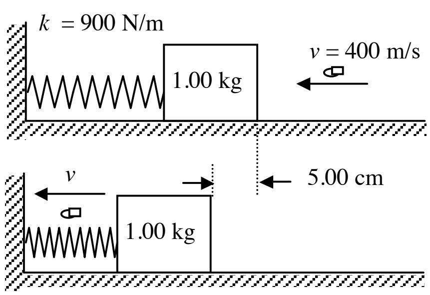
$$v_{cm_x} = \frac{(3.0\text{ kg})\left(6.0\ \frac{\text{m}}{\text{s}}\right)\cos 270° + (4.0\text{ kg})\left(7.0\ \frac{\text{m}}{\text{s}}\right)\cos 0}{3.0\text{ kg} + 4.0\text{ kg}} = 4.0\ \frac{\text{m}}{\text{s}}$$

$$v_{cm_y} = \frac{\sum p_{y_i}}{\sum m_i} = \frac{\sum m_i v_{y_i}}{\sum m_i} = \frac{m_1 v_{1_y} + m_2 v_{2_y}}{m_1 + m_2} = \frac{m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2}{m_1 + m_2}$$

$$v_{cm_y} = \frac{(3.0\text{ kg})\left(6.0\ \frac{\text{m}}{\text{s}}\right)\sin 270° + (4.0\text{ kg})\left(7.0\ \frac{\text{m}}{\text{s}}\right)\sin 0}{3.0\text{ kg} + 4.0\text{ kg}} = -2.57\ \frac{\text{m}}{\text{s}}$$

$$v_{cm} = \sqrt{v_{cm_x}^2 + v_{cm_y}^2} = \sqrt{\left(4.0\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-2.57\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{4.75\ \frac{\text{m}}{\text{s}}}$$

9.)



$m_b = 0.0050\text{ kg}$, $v_{b_i} = 400\ \frac{\text{m}}{\text{s}}$, $m_B = 1.00\text{ kg}$, $v_{B_i} = 0$

after impact $\Delta x = 0.050\text{ m}$

a.) use Conservation of Energy to get the speed of the block after the collision with the bullet (assuming that only the motion of the block is responsible for the compression of the spring)

$K_B = U_e$ so $\frac{1}{2} m_B v_{B_f}^2 = \frac{1}{2} k \Delta x^2$ and $v_{B_f} = \sqrt{\frac{k\Delta x^2}{m_B}} = \Delta x \sqrt{\frac{k}{m_B}}$

$$v_{B_f} = (0.050\text{ m})\sqrt{\frac{900\ \frac{\text{N}}{\text{m}}}{1.00\text{ kg}}} = 1.5\ \frac{\text{m}}{\text{s}}$$

use Conservation of Momentum to get the speed at which the bullet emerges from the block

$p_{b_i} + p_{B_i} = p_{b_f} + p_{B_f}$ so $m_b v_{b_i} + m_B v_{B_i} = m_b v_{b_f} + m_B v_{B_f}$

$$v_{b_f} = \frac{m_b v_{b_i} + m_B v_{B_i} - m_B v_{B_f}}{m_b} = \frac{(0.0050\text{ kg})\left(400\ \frac{\text{m}}{\text{s}}\right) + 0 - (1.00\text{ kg})\left(1.5\ \frac{\text{m}}{\text{s}}\right)}{0.0050\text{ kg}} = \boxed{100\ \frac{\text{m}}{\text{s}}}$$

b.) $\Delta K = K_f - K_i = \left(K_{B_f} + K_{b_f}\right) - \left(K_{B_i} + K_{b_i}\right) = \left(\frac{1}{2} m_B v_{B_f}^2 + \frac{1}{2} m_b v_{b_f}^2\right) - \left(\frac{1}{2} m_B v_{B_i}^2 + \frac{1}{2} m_b v_{b_i}^2\right)$

$$\Delta K = \left(\frac{1}{2}(1.00\text{ kg})\left(1.5\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(0.0050\text{ kg})\left(100\ \frac{\text{m}}{\text{s}}\right)^2\right) - \left(0 + \frac{1}{2}(0.0050\text{ kg})\left(400\ \frac{\text{m}}{\text{s}}\right)^2\right) = -374\text{ J}$$

so there are $\boxed{374\text{ J}}$ of energy lost in the collision

10.) $m_1 = 4.00\ \text{kg}$, $(x_1, y_1) = (2.00\ \text{m},\ -3.00\ \text{m})$, $m_2 = 3.00\ \text{kg}$, $(x_2, y_2) = (-2.00\ \text{m},\ 2.00\ \text{m})$,
$m_3 = 6.00\ \text{kg}$, $(x_3, y_3) = (3.00\ \text{m},\ -2.00\ \text{m})$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(4.00\ \text{kg})(2.00\ \text{m}) + (2.00\ \text{kg})(\text{-}2.00\ \text{m}) + (6.00\ \text{kg})(3.00\ \text{m})}{4.00\ \text{kg} + 2.00\ \text{kg} + 6.00\ \text{kg}} = 1.83\ \text{m}$$

$$y_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(4.00\ \text{kg})(-3.00\ \text{m}) + (2.00\ \text{kg})(2.00\ \text{m}) + (6.00\ \text{kg})(-2.00\ \text{m})}{4.00\ \text{kg} + 2.00\ \text{kg} + 6.00\ \text{kg}} = -1.67\ \text{m}$$

the center-of-mass is therefore $(x_{cm}, y_{cm}) = \boxed{(1.83\ \text{m},\ -1.67\ \text{m})}$