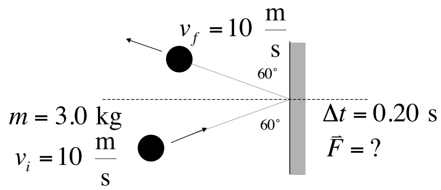
Example 1: (HO 16 #6)



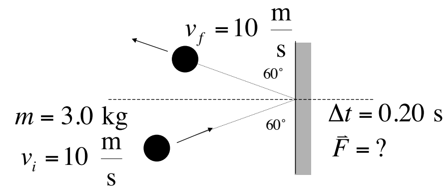
$$\vec{J} = \vec{F}\Delta t \text{ so } \vec{F} = \frac{\vec{J}}{\Delta t}$$

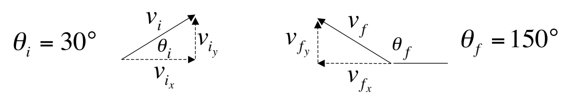
$$\vec{J} = \Delta\vec{p} = m\Delta\vec{v}$$

$$J_x = \Delta p_x = m\Delta v_x$$

$$J_y = \Delta p_y = m\Delta v_y$$

Example 1: (HO 16 #6)

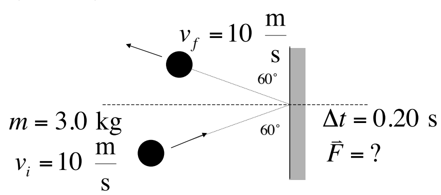




$$J_y = m\Delta v_y = 0$$

$$J_x = m\Delta v_x = m\left(v_{f_x} - v_{i_x}\right)$$

Example 1: (HO 16 #6)

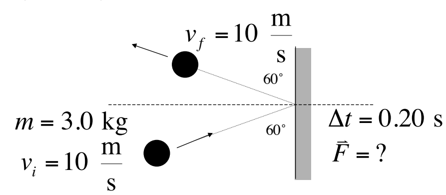


$$J_y = m\Delta v_y = 0 \qquad J_x = m\left(v_{f_x} - v_{i_x}\right)$$

$$F_y = \frac{J_y}{\Delta t} = 0 \qquad F_x = \frac{J_x}{\Delta t} = \frac{m\left(v_{f_x} - v_{i_x}\right)}{\Delta t}$$

$$F_x = \frac{m\left(v_f\cos\theta_f - v_i\cos\theta_i\right)}{\Delta t}$$

Example 1: (HO 16 #6)

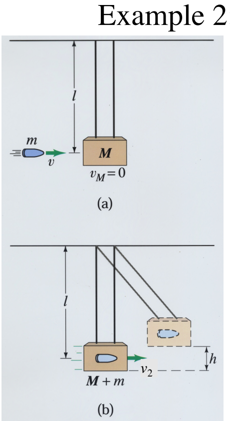


$$F_x = \frac{m\left(v_f\cos\theta_f - v_i\cos\theta_i\right)}{\Delta t}$$

$$F_x = \frac{3.0\text{ kg}\left(10\ \frac{\text{m}}{\text{s}}\cos(150°) - 10\ \frac{\text{m}}{\text{s}}\cos(30°)\right)}{0.2\text{ s}} = -260\text{ N}$$

$$\vec{F} = \left(-260\text{ N}, 0\right) = \boxed{260\text{ N}\angle 180°}$$

Example 2: Ballistic Pendulum



Find the height h assuming an inelastic collision between the bullet and the block.

Example 2: Ballistic Pendulum

Using Conservation of Momentum:

$$mv = (m + M)v_1$$

$$v_1 = \left(\frac{m}{m + M}\right)v$$

Using Energy Conservation:

$$K_1 + \cancel{U_1} = \cancel{K_2} + U_2$$

$$\frac{1}{2}(m + M)v_1^2 = (m + M)gh$$

$$h = \frac{1}{2g}v_1^2 = \boxed{\frac{1}{2g}\left(\frac{mv}{m + M}\right)^2}$$

Example 3:

A car with a mass of 5000 kg is moving at 20 m/s south is headed towards a second car with a mass of 2500 kg moving at 20 m/s east. The cars collide and lock together. What is the velocity of the coupled cars following the collision?

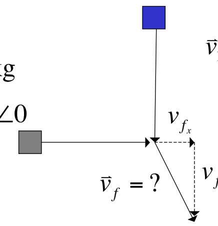
Example 3:

$m_1 = 5000 \text{ kg}$

$\vec{v}_{1_i} = 20 \ \frac{\text{m}}{\text{s}} \angle -90°$

$m_2 = 2500 \text{ kg}$

$\vec{v}_{2_i} = 20 \ \frac{\text{m}}{\text{s}} \angle 0$



$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$m_1 v_{1_{xi}} + m_2 v_{2_{xi}} = (m_1 + m_2) v_{f_x}$$

$$m_1 v_{1_i} \cos\theta_{1_i} + m_2 v_{2_i} \cos\theta_{2_i} = (m_1 + m_2) v_{f_x}$$

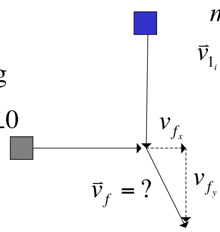
Example 3:

$m_1 = 5000 \text{ kg}$

$\vec{v}_{1_i} = 20 \ \frac{\text{m}}{\text{s}} \angle -90°$

$m_2 = 2500 \text{ kg}$

$\vec{v}_{2_i} = 20 \ \frac{\text{m}}{\text{s}} \angle 0$



$$m_1 v_{1_i} \cos\theta_{1_i} + m_2 v_{2_i} \cos\theta_{2_i} = (m_1 + m_2) v_{f_x}$$

$$v_{f_x} = \frac{m_1 v_{1_i} \cos\theta_{1_i} + m_2 v_{2_i} \cos\theta_{2_i}}{m_1 + m_2}$$

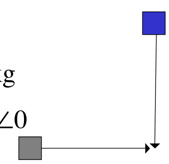
Example 3:

$m_1 = 5000 \text{ kg}$

$\vec{v}_{1_i} = 20 \ \frac{\text{m}}{\text{s}} \angle -90°$

$m_2 = 2500 \text{ kg}$

$\vec{v}_{2_i} = 20 \ \frac{\text{m}}{\text{s}} \angle 0$



$$v_{f_x} = \frac{m_1 v_{1_i} \cos\theta_{1_i} + m_2 v_{2_i} \cos\theta_{2_i}}{m_1 + m_2}$$

$$v_{f_x} = \frac{(5000 \text{ kg})\left(20 \ \frac{\text{m}}{\text{s}}\right)\cos(-90°) + (2500 \text{ kg})\left(20 \ \frac{\text{m}}{\text{s}}\right)\cos(0)}{(5000 \text{ kg}) + (2500 \text{ kg})}$$

$$v_{f_x} = 6.7 \ \frac{\text{m}}{\text{s}}$$

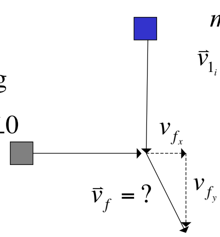
Example 3:

$m_1 = 5000 \text{ kg}$

$\vec{v}_{1_i} = 20 \ \frac{\text{m}}{\text{s}} \angle -90°$

$m_2 = 2500 \text{ kg}$

$\vec{v}_{2_i} = 20 \ \frac{\text{m}}{\text{s}} \angle 0$



$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$m_1 v_{1_{yi}} + m_2 v_{2_{yi}} = (m_1 + m_2) v_{f_y}$$

$$m_1 v_{1_i} \sin\theta_{1_i} + m_2 v_{2_i} \sin\theta_{2_i} = (m_1 + m_2) v_{f_y}$$

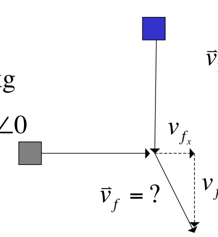
Example 3:

$m_1 = 5000 \text{ kg}$

$\vec{v}_{1_i} = 20 \ \frac{\text{m}}{\text{s}} \angle -90°$

$m_2 = 2500 \text{ kg}$

$\vec{v}_{2_i} = 20 \ \frac{\text{m}}{\text{s}} \angle 0$



$$m_1 v_{1_i} \sin\theta_{1_i} + m_2 v_{2_i} \sin\theta_{2_i} = (m_1 + m_2) v_{f_y}$$

$$v_{f_y} = \frac{m_1 v_{1_i} \sin\theta_{1_i} + m_2 v_{2_i} \sin\theta_{2_i}}{m_1 + m_2}$$

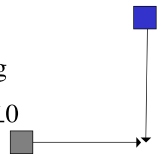
Example 3:

$m_1 = 5000 \text{ kg}$

$\vec{v}_{1_i} = 20 \ \frac{\text{m}}{\text{s}} \angle -90°$

$m_2 = 2500 \text{ kg}$

$\vec{v}_{2_i} = 20 \ \frac{\text{m}}{\text{s}} \angle 0$



$$v_{f_y} = \frac{m_1 v_{1_i} \sin\theta_{1_i} + m_2 v_{2_i} \sin\theta_{2_i}}{m_1 + m_2}$$

$$v_{f_y} = \frac{(5000 \text{ kg})\left(20 \ \frac{\text{m}}{\text{s}}\right)\sin(-90°) + (2500 \text{ kg})\left(20 \ \frac{\text{m}}{\text{s}}\right)\sin(0)}{(5000 \text{ kg}) + (2500 \text{ kg})}$$

$$v_{f_y} = -13.3 \ \frac{\text{m}}{\text{s}}$$

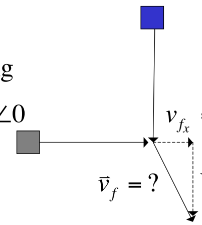
Example 3:

$m_1 = 5000 \text{ kg}$

$\vec{v}_{1_i} = 20 \ \frac{\text{m}}{\text{s}} \angle -90°$

$m_2 = 2500 \text{ kg}$

$\vec{v}_{2_i} = 20 \ \frac{\text{m}}{\text{s}} \angle 0$



$v_{f_x} = 6.7 \ \frac{\text{m}}{\text{s}}$

$\vec{v}_f = ?$

$v_{f_y} = -13.3 \ \frac{\text{m}}{\text{s}}$

$$v_f = \sqrt{{v_{f_x}}^2 + {v_{f_y}}^2} = \boxed{14.9 \ \frac{\text{m}}{\text{s}}}$$

$$\theta_f = \tan^{-1}\left(\frac{v_{f_y}}{v_{f_x}}\right) = \boxed{-63.3°}$$

Example 4:

$m_1 = 6.0 \text{ kg}$

$v_{1_i} = 1.0 \text{ m/s}$

$m_2 = 3.0 \text{ kg}$

$v_{2_i} = -5.0 \text{ m/s}$

Two balls are approaching each other as shown in the figure above. What are the final velocities of each ball if the collision is perfectly elastic?

Example 4:

$m_1 = 6.0 \text{ kg}$

$v_{1_i} = 1.0 \text{ m/s}$

$m_2 = 3.0 \text{ kg}$

$v_{2_i} = -5.0 \text{ m/s}$

$$(1) \quad m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f}$$

$$(1^*) \quad v_{1_i} + \frac{m_2}{m_1} v_{2_i} = v_{1_f} + \frac{m_2}{m_1} v_{2_f}$$

$$(6) \quad (v_{1_i} - v_{2_i}) = -(v_{1_f} - v_{2_f})$$

$$(6^*) \quad v_{1_i} - v_{2_i} = -v_{1_f} + v_{2_f}$$

$$(1^* + 6^*) \quad 2v_{1_i} + \frac{m_2}{m_1} v_{2_i} - v_{2_i} = \frac{m_2}{m_1} v_{2_f} + v_{2_f}$$

Example 4:

$m_1 = 6.0 \text{ kg}$

$v_{1_i} = 1.0 \text{ m/s}$

$m_2 = 3.0 \text{ kg}$

$v_{2_i} = -5.0 \text{ m/s}$

$$(1^* + 6^*) \quad 2v_{1_i} + \frac{m_2}{m_1} v_{2_i} - v_{2_i} = \frac{m_2}{m_1} v_{2_f} + v_{2_f}$$

$$2v_{1_i} + \left(\frac{m_2}{m_1} - 1\right) v_{2_i} = \left(\frac{m_2}{m_1} + 1\right) v_{2_f}$$

$$v_{2_f} = \frac{2v_{1_i} + \left(\frac{m_2}{m_1} - 1\right) v_{2_i}}{\left(\frac{m_2}{m_1} + 1\right)} = \frac{2\left(1.0 \ \frac{\text{m}}{\text{s}}\right) + \left(\frac{3.0 \text{ kg}}{6.0 \text{ kg}} - 1\right)\left(-5.0 \ \frac{\text{m}}{\text{s}}\right)}{\left(\frac{3.0 \text{ kg}}{6.0 \text{ kg}} + 1\right)}$$

$$\boxed{v_{2_f} = 3.0 \ \frac{\text{m}}{\text{s}}}$$

Example 4:

$m_1 = 6.0 \text{ kg}$

$v_{1_i} = 1.0 \text{ m/s}$

$m_2 = 3.0 \text{ kg}$

$v_{2_i} = -5.0 \text{ m/s}$

$$(6) \quad (v_{1_i} - v_{2_i}) = -(v_{1_f} - v_{2_f})$$

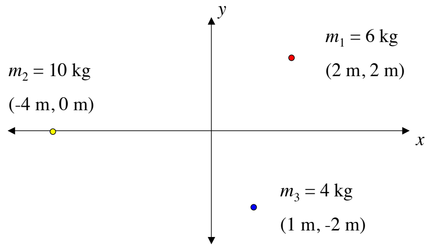
$$v_{1_f} = v_{2_f} - v_{1_i} + v_{2_i}$$

$$v_{1_f} = 3.0 \ \frac{\text{m}}{\text{s}} - 1.0 \ \frac{\text{m}}{\text{s}} + \left(-5.0 \ \frac{\text{m}}{\text{s}}\right)$$

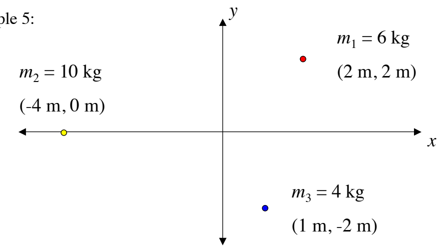
$$\boxed{v_{1_f} = -3.0 \ \frac{\text{m}}{\text{s}}}$$

Example 5:

Find the center of mass for the following arrangement of particles.



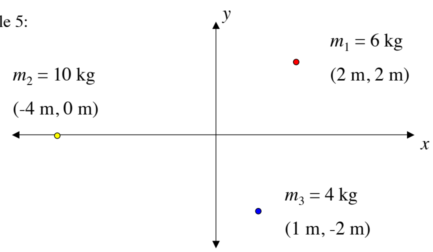
Example 5:



$$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$x_{cm} = \frac{(6\text{ kg})(2\text{ m}) + (10\text{ kg})(-4\text{ m}) + (4\text{ kg})(1\text{ m})}{6\text{ kg} + 10\text{ kg} + 4\text{ kg}} = -1.20\text{ m}$$

Example 5:



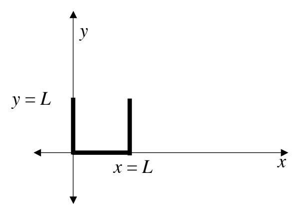
$$y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$y_{cm} = \frac{(6\text{ kg})(2\text{ m}) + (10\text{ kg})(0) + (4\text{ kg})(-2\text{ m})}{10\text{ kg} + 6\text{ kg} + 4\text{ kg}} = 0.2\text{ m}$$

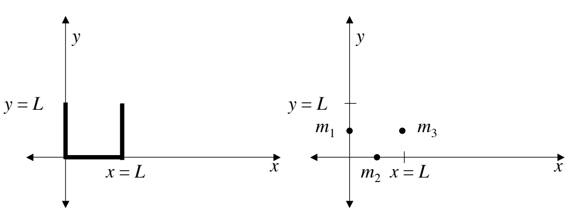
$$\boxed{(x_{cm}, y_{cm}) = (-1.2\text{ m},\ 0.2\text{ m})}$$

Example 6:

A thin uniform rod with a mass of M is bent in the shape of a U. Find the center of mass.



Example 6:

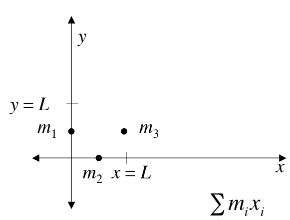


$$m_1 = \frac{M}{3} \quad x_1 = 0 \quad y_1 = \frac{L}{2}$$

$$m_2 = \frac{M}{3} \quad x_2 = \frac{L}{2} \quad y_2 = 0$$

$$m_3 = \frac{M}{3} \quad x_3 = L \quad y_3 = \frac{L}{2}$$

Example 6:



$$m_1 = \frac{M}{3} \quad x_1 = 0 \quad y_1 = \frac{L}{2}$$

$$m_2 = \frac{M}{3} \quad x_2 = \frac{L}{2} \quad y_2 = 0$$

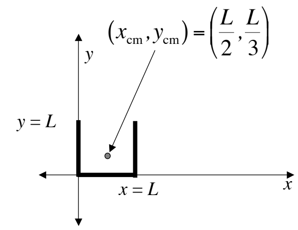
$$m_3 = \frac{M}{3} \quad x_3 = L \quad y_3 = \frac{L}{2}$$

$$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_2}$$

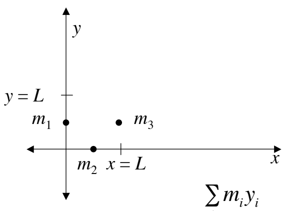
$$x_{cm} = \frac{\left(\frac{M}{3}\right)(0) + \left(\frac{M}{3}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{3}\right)(L)}{\frac{M}{3} + \frac{M}{3} + \frac{M}{3}} = \frac{\frac{3ML}{6}}{M} \boxed{= \frac{1}{2}L}$$

Example 6:

A thin uniform rod with a mass of M is bent in the shape of a U. Find the center of mass.



Example 6:



$$m_1 = \frac{M}{3} \quad x_1 = 0 \quad y_1 = \frac{L}{2}$$

$$m_2 = \frac{M}{3} \quad x_2 = \frac{L}{2} \quad y_2 = 0$$

$$m_3 = \frac{M}{3} \quad x_3 = L \quad y_3 = \frac{L}{2}$$

$$y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_2}$$

$$y_{cm} = \frac{\left(\frac{M}{3}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{3}\right)(0) + \left(\frac{M}{3}\right)\left(\frac{L}{2}\right)}{\frac{M}{3} + \frac{M}{3} + \frac{M}{3}} = \frac{\frac{2ML}{6}}{M} \boxed{= \frac{1}{3}L}$$

Example 7:

A 1200 kg station wagon is moving along a straight highway at 12 m/s. Another car, with a mass 1800 kg and speed 20 m/s, has its center of mass 40 m ahead of the center of mass of the station wagon.

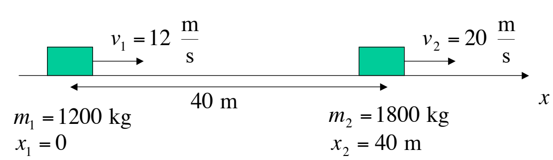
a.) Find the position of the center of mass of the system.

b.) Find the magnitude of the total momentum of the system.

c.) Find the speed of the center of mass of the system.

d.) Find the total momentum of the system, using the speed of the center of mass. Compare with the result in part (b).

Example 7:

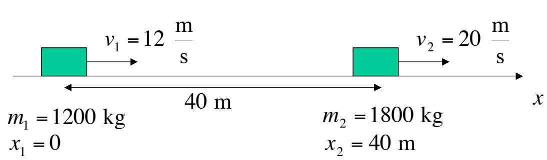


a.) Find the position of the center of mass of the system.

$$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_{cm} = \frac{(1200 \text{ kg})(0) + (1800 \text{ kg})(40 \text{ m})}{1200 \text{ kg} + 1800 \text{ kg}} \boxed{= 24 \text{ m}}$$

Example 7:



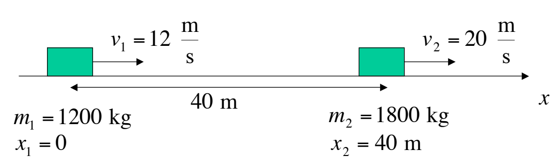
b.) Find the magnitude of the total momentum of the system.

$$\bar{p} = \bar{p}_1 + \bar{p}_2 = m_1 v_1 + m_2 v_2$$

$$p = (1200 \text{ kg})\left(12 \ \frac{\text{m}}{\text{s}}\right) + (1800 \text{ kg})\left(20 \ \frac{\text{m}}{\text{s}}\right)$$

$$\boxed{p = 50{,}400 \ \frac{\text{kg} \cdot \text{m}}{\text{s}}}$$

Example 7:

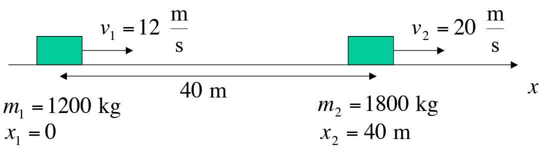


c.) Find the speed of the center of mass of the system.

$$v_{cm_x} = \frac{\sum_i m_i v_{i_x}}{\sum_i m_i} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$v_{cm_x} = \frac{(1200 \text{ kg})\left(12 \ \frac{\text{m}}{\text{s}}\right) + (1800 \text{ kg})\left(20 \ \frac{\text{m}}{\text{s}}\right)}{1200 \text{ kg} + 1800 \text{ kg}} \boxed{= 16.8 \ \frac{\text{m}}{\text{s}}}$$

Example 7:



d.) Find the total momentum of the system, using the speed of the center of mass.

$$p = Mv_{\text{cm}_x}$$

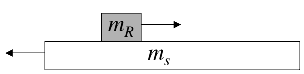
$$p = (1200\ \text{kg} + 1800\ \text{kg})\left(16.8\ \frac{\text{m}}{\text{s}}\right) \boxed{= 50,400\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

Example 8:

Rat is standing on a slab of concrete, which is in turn is resting on a frozen lake. Assume that there is no friction between the slab and the ice. The slab has a weight four times that of Rat. If she begins walking forward at 3 m/s, relative to the ice, with what speed relative to the ice does the slab move?

Example 8:

$$v_R = 3\ \frac{\text{m}}{\text{s}}$$



$$m_s = 4m_R$$

$$\bar{p}_i = \bar{p}_f$$

$$0 = m_R v_R + m_s v_s$$

$$v_s = -\frac{m_R v_R}{m_s} = -\frac{m_R v_R}{4m_R} = -\frac{v_R}{4}$$

$$v_s = -\frac{\left(3\ \frac{\text{m}}{\text{s}}\right)}{4} \boxed{= -0.75\ \frac{\text{m}}{\text{s}}}$$