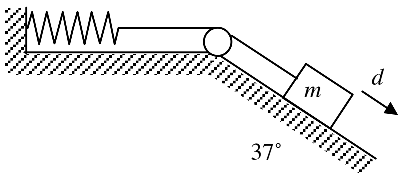
10.)



$m = 2.0 \text{ kg}$, $k = 100 \ \frac{\text{N}}{\text{m}}$, and frictionless surface, $v_o = 0$

$d = \Delta x = 0.20 \text{ m}$, $v = ?$

using Energy Conservation:

$$U_{g_1} = K_2 + U_{e_2}$$

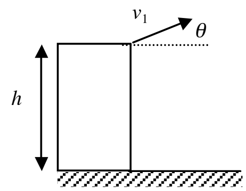
$$mgy_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}k\Delta x_2^2$$

$$mgd\sin\theta = \frac{1}{2}mv_2^2 + \frac{1}{2}k\Delta x_2^2$$

$$v_2 = \sqrt{\frac{2mgd\sin\theta - k\Delta x_2^2}{m}} = \sqrt{\frac{2\left((2.0 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(0.20 \text{ m})\sin 37° - \left(100 \ \frac{\text{N}}{\text{m}}\right)(0.20 \text{ m})^2\right)}{2.0 \text{ kg}}} = \boxed{0.60 \ \frac{\text{m}}{\text{s}}}$$

HO 14 Solutions

1.)



$h = 27.5\ \text{m}$, $\theta = 37.0°$, and $v_1 = 16.0\ \frac{\text{m}}{\text{s}}$

using the ground as a reference $y_1 = 27.5\ \text{m}$ and $y_2 = 0$

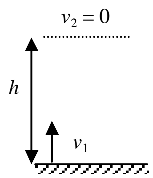
using Conservation of Energy

$$K_1 + U_{g_1} = K_2 + U_{g_2}$$

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \text{ or } \frac{1}{2}v_1^2 + gy_1 = \frac{1}{2}v_2^2 + gy_2$$

$$v_2 = \sqrt{v_1^2 + 2(gy_1 - gy_2)} = \sqrt{v_1^2 + 2g(y_1 - y_2)} = \sqrt{\left(16.0\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(27.5\ \text{m} - 0)} = \boxed{28.2\ \frac{\text{m}}{\text{s}}}$$

2.)



$v_1 = 20.0\ \frac{\text{m}}{\text{s}}$, $m = 0.145\ \text{kg}$, at maximum height $v_2 = 0$

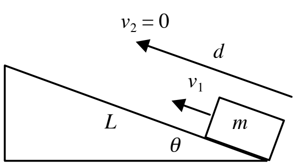
using the ground as a reference $y_1 = 0$

using Conservation of Energy

$$K_1 = U_{g_2}$$

$$\frac{1}{2}mv_1^2 = mgy_2 \text{ and } y_2 = \frac{v_1^2}{2g} = \frac{\left(20.0\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{20.4\ \text{m}}$$

3.)



$m = 12\ \text{kg}$, $\theta = 30°$, $L = 2.5\ \text{m}$, $v_1 = 5.0\ \frac{\text{m}}{\text{s}}$, $d = 1.6\ \text{m}$, and $v_2 = 0$

using the bottom of the ramp as a reference $y_1 = 0$

a.)

using Conservation of Energy

$$K_1 + W_f = U_{g_2}$$

$$\frac{1}{2}mv_1^2 + fd\cos\theta_f = mgd\sin\theta$$

$$f = \frac{mgd\sin\theta - \frac{1}{2}mv_1^2}{d\cos\theta_f} = \frac{m\left(2gd\sin\theta - v_1^2\right)}{2d\cos\theta_f} = \frac{(12\ \text{kg})\left(2\left(9.8\ \frac{\text{m}}{\text{s}}\right)(1.6\ \text{m})\sin30° - \left(5.0\ \frac{\text{m}}{\text{s}}\right)^2\right)}{2(1.6\ \text{m})\cos180°} = \boxed{35\ \text{N}}$$

b.) $v_1 = 0$ and $y_2 = 0$

using Conservation of Energy

$$U_{g_1} + W_f = K_2$$

3.)

b.) continued

$$mgy_1 + fd\cos\theta_f = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{\frac{2d\left(mg\sin\theta + f\cos\theta_f\right)}{m}} = \sqrt{\frac{2(1.6\text{ m})\left((12\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}}\right)\sin 30° + (35\text{ N})\cos 180°\right)}{12\text{ kg}}} = \boxed{2.52\ \frac{\text{m}}{\text{s}}}$$

c.) $v_2 = 0$ and $d = 2.5$ m

using Conservation of Energy

$$K_1 + W_f = U_{g_2}$$

$$\frac{1}{2}mv_1^2 + fd\cos\theta_f = mgd\sin\theta$$

$$v_1 = \sqrt{\frac{2(2.5\text{ m})\left((12\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}}\right)\sin 30° - (35\text{ N})\cos 180°\right)}{12\text{ kg}}} = \boxed{6.25\ \frac{\text{m}}{\text{s}}}$$

d.) $v_1 = 10.0\ \frac{\text{m}}{\text{s}}$ and $d = 2.5$ m

using Conservation of Energy

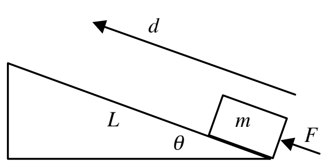
$$K_1 + W_f = K_2 + U_{g_2}$$

$$\frac{1}{2}mv_1^2 + mgy_1 + W_f = \frac{1}{2}mv_2^2 + mgy_2 \text{ and } \frac{1}{2}mv_1^2 + 0 + fd\cos\theta_f = \frac{1}{2}mv_2^2 + mgd\sin\theta$$

$$v_2 = \sqrt{\frac{mv_1^2 + 2d\left(f\cos\theta_f - mg\sin\theta\right)}{m}}$$

$$v_2 = \sqrt{\frac{(12\text{ kg})\left(10.0\ \frac{\text{m}}{\text{s}}\right)^2 + 2(2.5\text{ m})\left((35\text{ N})\cos 180° - (12\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}}\right)\sin 30°\right)}{12\text{ kg}}} = \boxed{7.8\ \frac{\text{m}}{\text{s}}}$$

4.)



$m = 12.0$ kg, $d = 14.0$ m, $\theta = 37°$, $F = 120$ N, and $\mu_k - 0.25$

a.) $W_F = Fd\cos\theta_F = (120\text{ N})(14.0\text{ m})\cos 0 = \boxed{1680\text{ J}}$

b.) $W_f = fd\cos\theta_f = \mu_k F_N d\cos\theta_f = \mu_k mg\cos\theta d\cos\theta_f = \mu_k mgd\cos\theta\cos\theta_f$

$$W_f = 0.25(12.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(14.0\text{ m})\cos 37°\cos 180° = \boxed{-329\text{ J}}$$

4.) continued

c.) assuming $y_1 = 0$

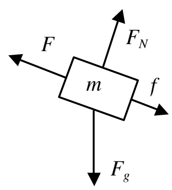
$$\Delta U_g = U_{g2} - U_{g1} = mgy_2 - mgy_1 = mg(y_2 - y_1) = mgy_2 = mgd\sin\theta = (12.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(14.0\text{ m})\sin 37° = \boxed{991\text{ J}}$$

d.) using Conservation of Energy

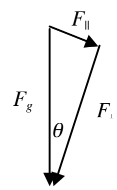
$$K_1 + U_{g_1} + W_F + W_f = K_2 + U_{g_2} \text{ or } U_{g_1} - U_{g_2} + W_F + W_f = K_2 - K_1$$

$$\Delta K = U_{g_1} - U_{g_2} + W_F + W_f = -\left(U_{g_2} - U_{g_1}\right) + W_F + W_f = -\Delta U_g + W_F + W_f = -991\text{ J} + 1680\text{ J} + (-329\text{ J}) = \boxed{360\text{ J}}$$

e.) the force-diagram



the components of the weight



$$F_{\parallel} = F_g\sin\theta$$
$$F_{\perp} = F_g\cos\theta = F_N$$

Newton's 2nd Law (up the plane)

$$F_{net} = \sum F = ma$$

$$F - f - F_{\parallel} = ma \text{ so } a = \frac{F - f - F_{\parallel}}{m} = \frac{F - \mu_k F_N - F\sin\theta}{m} = \frac{F - \mu_k F_g\cos\theta - F_g\sin\theta}{m}$$

$$a = \frac{F - \mu_k mg\cos\theta - mg\sin\theta}{m} = \frac{F - mg\left(\mu_k\cos\theta + \sin\theta\right)}{m}$$

$$a = \frac{120\text{ N} - (12.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.25\cos 37° + \sin 37°)}{12.0\text{ kg}} = \boxed{2.15\ \frac{\text{m}}{\text{s}^2}}$$

f.) assuming $v_1 = 0$ and uniform acceleration

$$v^2 = v_o^2 + 2ad \text{ so } v_2 = \sqrt{v_1^2 + 2ad} = \sqrt{2ad} = \sqrt{2\left(2.15\ \frac{\text{m}}{\text{s}^2}\right)(14.0\text{ m})} = \boxed{7.75\ \frac{\text{m}}{\text{s}}}$$

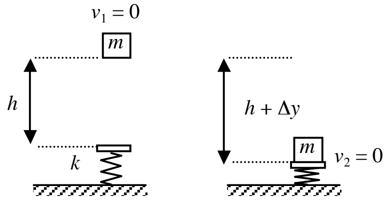
g.)

$$\Delta K = K_2 - K_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\Delta K = \frac{1}{2}(12.0\text{ kg})\left(7.75\ \frac{\text{m}}{\text{s}}\right)^2 - 0 = \boxed{360\text{ J}}$$

This is the same as that obtained in part (d.) using energy methods.

5.)



$h = 0.80\ \text{m}$, $m = 1.20\ \text{kg}$, $v_1 = v_2 = 0$, and $k = 1960\ \dfrac{\text{N}}{\text{m}}$

using the lowest point as a reference $y_1 = h + \Delta y$ and $y_2 = 0$

using Conservation of Energy

$$U_{g_1} = U_{e_2}$$

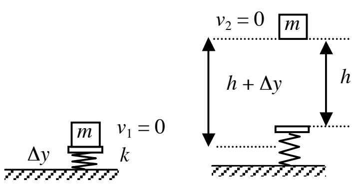
$$mgy_1 = \frac{1}{2}k\Delta y^2 \text{ so } mg(h + \Delta y) = \frac{1}{2}k\Delta y^2 \text{ or } mgh + mg\Delta y = \frac{1}{2}k\Delta y^2$$

$$0 = \frac{1}{2}k\Delta y^2 - mg\Delta y - mgh = \frac{1}{2}\left(1960\ \frac{\text{N}}{\text{m}}\right)\Delta y^2 - (1.20\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\Delta y - (1.20\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.80\ \text{m})$$

$$0 = \left(980\ \frac{\text{N}}{\text{m}}\right)\Delta y^2 - (11.76\ \text{N})\Delta y - (9.408\ \text{J})$$

using Quadratic Formula $\Delta y = 0.1042\ \text{m}$ or $\Delta y = -0.0922\ \text{m}$ so $\Delta y = \boxed{0.1042\ \text{m}}$

6.)



$m = 1.60\ \text{kg}$, $k = 1500\ \dfrac{\text{N}}{\text{m}}$, $\Delta y = 0.20\ \text{m}$, and $v_1 = v_2 = 0$

using lowest point as a reference $y_1 = 0$ and $y_2 = h + \Delta y$

using Conservation of Energy

$$U_{e_1} = U_{g_2}$$

$$\frac{1}{2}k\Delta y^2 = mgy_2 = mg(h + \Delta y) = mgh + mg\Delta y \text{ or } h = \frac{\frac{1}{2}k\Delta y^2 - mg\Delta y}{mg} = \frac{\Delta y(k\Delta y - 2mg)}{2mg}$$

$$h = \frac{(0.20\ \text{m})\left(\left(1500\ \frac{\text{N}}{\text{m}}\right)(0.20\ \text{m}) - 2(1.60\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\right)}{2(1.60\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)} = \boxed{1.71\ \text{m}}$$

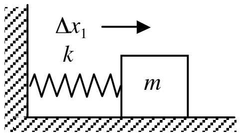
alternatively one could solve for y_2 and subtract out the amount the spring was compressed

$$U_{e_1} = U_{g_2} \text{ so } \frac{1}{2}k\Delta y^2 = mgy_2$$

$$y_2 = \frac{k\Delta y^2}{2mg} = \frac{\left(1500\ \frac{\text{N}}{\text{m}}\right)(0.20\ \text{m})^2}{2(1.60\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)} = 0.191\ \text{m}$$

so the height above the spring is $y_2 - \Delta y = 1.91\ \text{m} - 0.20\ \text{m} = \boxed{1.71\ \text{m}}$

7.)



$m = 0.200\text{ kg}$, $k = 5.00\ \frac{\text{N}}{\text{m}}$, stretched $\Delta x_1 = -0.100\text{ m}$, and $v_1 = 0$

surface is frictionless and the block is connected to the spring

a.) when $\Delta x_2 = 0.080\text{ m}$

using Conservation of Energy

$$U_{e_1} = K_2 + U_{e_2}$$

$$\frac{1}{2}k\Delta x_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}k\Delta x_2^2 \text{ so } v_2 = \pm\sqrt{\frac{k\Delta x_1^2 - k\Delta x_2^2}{m}} = \pm\sqrt{\frac{k\left(\Delta x_1^2 - \Delta x_2^2\right)}{m}}$$

(could moving towards or away from the wall) $$v_2 = \pm\sqrt{\frac{\left(5.00\ \frac{\text{N}}{\text{m}}\right)\left((-0.100\text{ m})^2 - (0.080\text{ m})^2\right)}{0.200\text{ kg}}} = \boxed{\pm 0.30\ \frac{\text{m}}{\text{s}}}$$

b.) when $\Delta x_2 = 0$

using Conservation of Energy

$$U_{e_1} = K_2$$

$$\frac{1}{2}k\Delta x_1^2 = \frac{1}{2}mv_2^2 \text{ so } v_2 = -\sqrt{\frac{k\Delta x_1^2}{m}} = -\Delta x_1\sqrt{\frac{k}{m}}$$

$$v_2 = -(0.100\text{ m})\sqrt{\frac{\left(5.00\ \frac{\text{N}}{\text{m}}\right)}{0.200\text{ kg}}} = \boxed{-0.50\ \frac{\text{m}}{\text{s}}}$$

c.) $v_2 = 2.00\ \frac{\text{m}}{\text{s}}$ when $\Delta x_2 = 0$ (maximum speed occurs at equilibrium point)

using Conservation of Energy

$$U_{e_1} = K_2$$

$$\frac{1}{2}k\Delta x_1^2 = \frac{1}{2}mv_2^2 \text{ so } \Delta x_1 = \sqrt{\frac{mv_2^2}{k}} = \sqrt{\frac{(0.200\text{ kg})\left(2.00\ \frac{\text{m}}{\text{s}}\right)^2}{\left(5.00\ \frac{\text{N}}{\text{m}}\right)}} = \boxed{0.40\text{ m}}$$

d.) when $v_2 = 0.40\ \frac{\text{m}}{\text{s}}$

using Conservation of Energy

$$U_{e_1} = K_2 + U_{e_2}$$

$$\frac{1}{2}k\Delta x_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}k\Delta x_2^2 \text{ so } \Delta x_2 = \pm\sqrt{\frac{k\Delta x_1^2 - mv_2^2}{k}} = \pm\sqrt{\frac{\left(5.00\ \frac{\text{N}}{\text{m}}\right)(0.100\text{ m})^2 - (0.200\text{ kg})\left(0.40\ \frac{\text{m}}{\text{s}}\right)^2}{\left(5.00\ \frac{\text{N}}{\text{m}}\right)}} = \boxed{\pm 0.060\text{ m}}$$

7.) continued

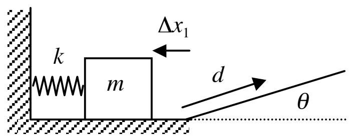
e.) if there is friction and $v_2 = 0$ when $\Delta x_2 = 0$

using Conservation of Energy

$$U_{e_1} + W_f = 0$$

$$\frac{1}{2}k\Delta x_1^2 + fd\cos\theta_f = \frac{1}{2}k\Delta x_1^2 + \mu_k F_N d\cos\theta_f = \frac{1}{2}k\Delta x_1^2 + \mu_k F_g d\cos\theta_f = \frac{1}{2}k\Delta x_1^2 + \mu_k mgd\cos\theta_f = 0$$

$$\mu_k = \frac{-k\Delta x_1^2}{2mgd\cos\theta_f} = \frac{-\left(5.00\ \frac{\text{N}}{\text{m}}\right)(0.100\ \text{m})^2}{2(0.200\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.100\ \text{m})\cos 180°} = \boxed{0.128}$$

8.) 

$m = 2.00$ kg, $k = 400\ \frac{\text{N}}{\text{m}}$, compressed $\Delta x_1 = 0.220$ m, and $\theta = 37°$

all surfaces are frictionless and block is not attached $v_1 = 0$

a.) when block leaves the spring $\Delta x_2 = 0$

using Conservation of Energy

$$U_{e_1} = K_2$$

$$\frac{1}{2}k\Delta x_1^2 = \frac{1}{2}mv_2^2 \text{ and } v_2 = \sqrt{\frac{k\Delta x_1^2}{m}} = \sqrt{\frac{\left(400\ \frac{\text{N}}{\text{m}}\right)(0.220\ \text{m})^2}{2.00\ \text{kg}}} = \boxed{3.11\ \frac{\text{m}}{\text{s}}}$$

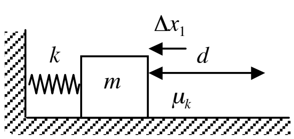
b.) started at the point where block has left the spring $\Delta x_1 = \Delta x_2 = 0$, $v_1 = 3.11\ \frac{\text{m}}{\text{s}}$, and $y_1 = 0$

block will travel up the incline until $v_2 = 0$

using Conservation of Energy

$$K_1 = U_{g_2}$$

$$\frac{1}{2}mv_1^2 = mgy_2 = mgd\sin\theta \text{ and } d = \frac{v_1^2}{2g\sin\theta} = \frac{\left(3.11\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\sin 37°} = \boxed{0.82\ \text{m}}$$

9.) 

$m = 0.50$ kg, $\Delta x_1 = 0.20$ m, $d = 1.00$ m, $k = 100\ \frac{\text{N}}{\text{m}}$, and $v_1 = v_2 = 0$

block is not attached

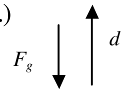
$$U_{e_1} + W_f = 0$$

$$\frac{1}{2}k\Delta x_1^2 + fd\cos\theta_f = 0 \text{ so } \frac{1}{2}k\Delta x_1^2 + \mu_k F_N d\cos\theta_f = \frac{1}{2}k\Delta x_1^2 + \mu_k mgd\cos\theta_f = 0$$

$$\mu_k = \frac{-k\Delta x_1^2}{2mgd\cos\theta_f} = \frac{-\left(100\ \frac{\text{N}}{\text{m}}\right)(0.20\ \text{m})^2}{2(0.50\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(1.00\ \text{m})\cos 180°} = \boxed{0.41}$$

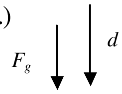
1.) $m = 0.50 \text{ kg}$,

a.)



$$W_{F_g} = \vec{F}_g \cdot \vec{d} = F_g d\cos\theta_g = mgd\cos\theta_g = (0.50 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(12 \text{ m})\cos 180° = \boxed{-58.8 \text{ J}}$$

b.)



$$W_{F_g} = \vec{F}_g \cdot \vec{d} = F_g d\cos\theta_g = mgd\cos\theta_g = (0.50 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(12 \text{ m})\cos 0 = \boxed{58.8 \text{ J}}$$

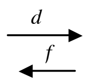
c.)

$$W_{total} = W_{up} + W_{down} = -58.8 \text{ J} + 58.8 \text{ J} = \boxed{0}$$

d.) The force of gravity is **conservative** since the total work is zero when the starting and end points are the same.

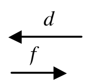
2.) $m = 0.50 \text{ kg}$, $f = 1.2 \text{ N}$, $d = 4.0 \text{ m}$

a.)



$$W_f = \vec{F} \cdot \vec{d} = fd\cos\theta_f = (1.2 \text{ N})(4.0 \text{ m})\cos 180° = \boxed{-4.8 \text{ J}}$$

b.)



$$W_f = \vec{F} \cdot \vec{d} = fd\cos\theta_f = (1.2 \text{ N})(4.0 \text{ m})\cos 180° = \boxed{-4.8 \text{ J}}$$

c.)

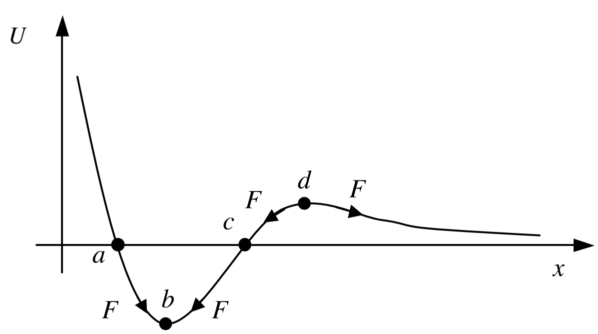
$$W_{total} = W_{right} + W_{left} = -4.8 \text{ J} + (-4.8 \text{ J}) = \boxed{-9.6 \text{ J}}$$

d.) The force of friction is **nonconservative** since the total work is not zero when the starting and end points are the same.

3.) $U(x) = \alpha x^3$ where $\alpha = 2.5 \ \frac{\text{J}}{\text{m}^3}$, $x = 1.60 \text{ m}$

$$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\alpha x^3\right) = -3\alpha x^2 = -3\left(2.5 \ \frac{\text{J}}{\text{m}^3}\right)(1.60 \text{ m})^2 = \boxed{-19.2 \text{ N}}$$

4.)



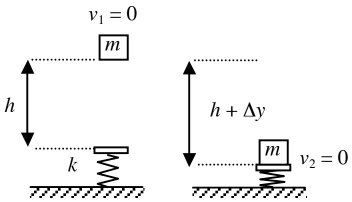
a.) $F = -\frac{dU}{dx} = 0$ or when slope is zero

slope is zero at points **b and d**

b.) point **b** is stable since the slope is zero and the restoring force (negative slope) on either side of the point forces the particle back towards the equilibrium point.

c.) point **d** is unstable since the slope is zero and the restoring force (negative slope) on either side of the point forces the particle away from the equilibrium point.

5.)



$m = 90.0\text{ kg},\ h = 2.50\text{ m},\ \Delta y = 0.200\text{ m}$

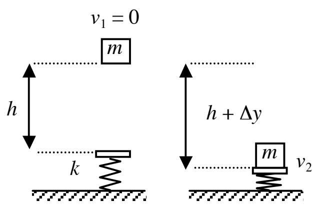
using the lowest point as a reference $y_1 = h + \Delta y$ and $y_2 = 0$

using Conservation of Energy

$$U_{g_1} = U_{e_2}$$

$$mgy_1 = \frac{1}{2}k\Delta y^2 \text{ so } mg(h+\Delta y) = \frac{1}{2}k\Delta y^2 \text{ and } k = \frac{2mg(h+\Delta y)}{\Delta y^2}$$

$$k = \frac{2(90.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.50\text{ m} + 0.200\text{ m})}{(0.200\text{ m})^2} = 1.19 \text{ x } 10^5\ \frac{\text{N}}{\text{m}}$$



$\Delta y = 0.100\text{ m}$

using the lowest point as a reference $y_1 = h + \Delta y$ and $y_2 = 0$

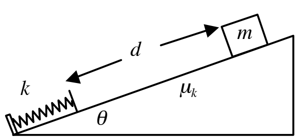
using Conservation of Energy

$$U_{g_1} = K_2 + U_{e_2}$$

$$mgy_1 = \frac{1}{2}mv_2^2 + \frac{1}{2}k\Delta y^2 \text{ so } mg(h+\Delta y) = \frac{1}{2}mv_2^2 + \frac{1}{2}k\Delta y^2 \text{ and } v_2 = \sqrt{\frac{2mg(h+\Delta y) - k\Delta y^2}{m}}$$

$$v_2 = \sqrt{\frac{2(90.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.50\text{ m} + 0.100\text{ m}) - \left(1.19 \text{ x } 10^5\ \frac{\text{N}}{\text{m}}\right)(0.100\text{ m})^2}{90.0\text{ kg}}} = \boxed{6.14\ \frac{\text{m}}{\text{s}}}$$

6.)



$m = 2.00\text{ kg},\ \theta = 53.1°,\ d = 4.00\text{ m},\ k = 140\ \frac{\text{N}}{\text{m}}$, and $\mu_k = 0.20$

a.) $v_1 = 0$ using the lowest point as a reference $y_2 = 0$ and $y_1 = d\sin\theta$

using Conservation of Energy

$$U_{g_1} + W_f = K_2$$

$$mgy_1 + fd\cos\theta_f = \frac{1}{2}mv_2^2 \text{ so } mgd\sin\theta + \mu_k F_N d\cos\theta_f = mgd\sin\theta + \mu_k mg\cos\theta d\cos\theta_f = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gd(\sin\theta + \mu_k\cos\theta\cos\theta_f)} = \sqrt{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(4.00\text{ m})(\sin 53.1° + 0.20\cos 53.1°\cos 180°)} = \boxed{7.30\ \frac{\text{m}}{\text{s}}}$$

6.) continued

b.) maximum compression occurs when $v_2 = 0$

starting from the release of the block at the top of the incline and using the lowest point as a reference $y_2 = 0$ and $y_1 = (d + \Delta x)\sin\theta$ where Δx is the compression of the spring

using Conservation of Energy

$$U_{g_1} + W_f = U_{e_2}$$

$$mgy_1 + fd_f\cos\theta_f = \frac{1}{2}k\Delta x^2 \text{ or } mg(d + \Delta x)\sin\theta + \mu_k mg\cos\theta(d + \Delta x)\cos\theta_f = \frac{1}{2}k\Delta x^2$$

$$mgd\sin\theta + mg\Delta x\sin\theta + \mu_k mgd\cos\theta\cos\theta_f + \mu_k mg\Delta x\cos\theta\cos\theta_f = \frac{1}{2}k\Delta x^2$$

$$mgd(\sin\theta + \mu_k\cos\theta\cos\theta_f) + \Delta x(mg\sin\theta + \mu_k mg\cos\theta\cos\theta_f) = \frac{1}{2}k\Delta x^2$$

$$0 = \frac{1}{2}k\Delta x^2 - mg(\sin\theta + \mu_k\cos\theta\cos\theta_f)\Delta x - mgd(\sin\theta + \mu_k\cos\theta\cos\theta_f)$$

$$0 = \frac{1}{2}\left(140\ \frac{\text{N}}{\text{m}}\right)\Delta x^2 - (2.00\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(\sin 53.1° + 0.20\cos 53.1°\cos 180°)\Delta x$$
$$-(2.00\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(4.00\text{ m})(\sin 53.1° + 0.20\cos 53.1°\cos 180°)$$

$$0 = \left(70\ \frac{\text{N}}{\text{m}}\right)\Delta x^2 - (13.31\text{ N})\Delta x - 53.28\text{ J}$$

using Quadratic Formula $\Delta x = 0.972\text{ m}$ or $\Delta x = -0.783\text{ m}$ so $\Delta x = \boxed{0.972\text{ m}}$

c.) starting from the maximum compression of the spring and letting the lowest point be the reference $y_1 = 0$ $v_1 = v_2 = 0$, $\Delta x = 0.972\text{ m}$, and letting *d* be the distance the block travels up the incline $y_2 = d\sin\theta$

using Conservation of Energy

$$U_{e_1} + W_f = U_{g_2}$$

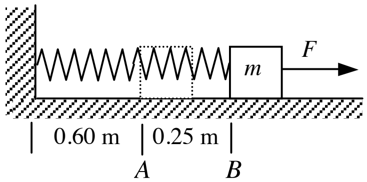
$$\frac{1}{2}k\Delta x^2 + fd\cos\theta_f = mgy_2 \text{ and } \frac{1}{2}k\Delta x^2 + \mu_k mg\cos\theta d\cos\theta_f = mgd\sin\theta$$

$$\frac{1}{2}k\Delta x^2 = d(mg\sin\theta - \mu_k mg\cos\theta\cos\theta_f)$$

$$d = \frac{k\Delta x^2}{2mg(\sin\theta - \mu_k\cos\theta\cos\theta_f)} = \frac{\left(140\ \frac{\text{N}}{\text{m}}\right)(0.972\text{ m})^2}{2(2.00\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(\sin 53.1° - 0.20\cos 53.1°\cos 180°)} = 3.669\text{ m}$$

so the block traveled a distance of 4.971 m down the incline and the 3.669 m back up the incline and returns

$4.972\text{ m} - 3.669\text{ m} = \boxed{1.30\text{ m}}$ from where it started

7.) 

$m = 0.500\text{ kg}$, ideal spring $k = 40.0\ \frac{\text{N}}{\text{m}}$ and length $\ell = 0.60\text{ m}$

$\Delta x_1 = 0$, $v_1 = 0$ and $F = 20.0\text{ N}$ on a frictionless surface

a.) reaching point *B*

using Conservation of Energy

$$W_F = K_2 + U_{e_2}$$

$$Fd\cos\theta_F = \frac{1}{2}mv_2^2 + \frac{1}{2}k\Delta x_2^2 \text{ and } v_2 = \sqrt{\frac{2Fd\cos\theta_F - k\Delta x_2^2}{m}}$$

$$v_2 = \sqrt{\frac{2(20.0\text{ N})(0.25\text{ m})\cos 0 - \left(40.0\ \frac{\text{N}}{\text{m}}\right)(0.25\text{ m})^2}{0.500\text{ kg}}} = \boxed{3.873\ \frac{\text{m}}{\text{s}}}$$

b.) at the maximum displacement from the wall $v_2 = 0$ and starting from the point where the force is removed $v_1 = 3.873\ \frac{\text{m}}{\text{s}}$ and $\Delta x_1 = 0.25\text{ m}$

using Conservation of Energy

$$K_1 + U_{e_1} = U_{e_2}$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}k\Delta x_1^2 = \frac{1}{2}k\Delta x_2^2 \text{ and } \Delta x_2 = \sqrt{\frac{mv_1^2 + k\Delta x_1^2}{k}} = \sqrt{\frac{(0.500\text{ kg})\left(3.873\ \frac{\text{m}}{\text{s}}\right)^2 + \left(40.0\ \frac{\text{N}}{\text{m}}\right)(0.25\text{ m})^2}{40.0\ \frac{\text{N}}{\text{m}}}} = 0.50\text{ m}$$

This is the total stretch of the spring from point *A*, so the displacement from the wall is

$$\Delta x_2 + \ell = 0.50\text{ m} + 0.60\text{ m} = \boxed{1.10\text{ m}}$$

c.) the block is now pulled back to the wall by the spring and starting from the point of maximum displacement $v_1 = 0$, $\Delta x_1 = 0.50\text{ m}$

when the block reaches its closest approach to the wall $v_2 = 0$

using Conservation of Energy

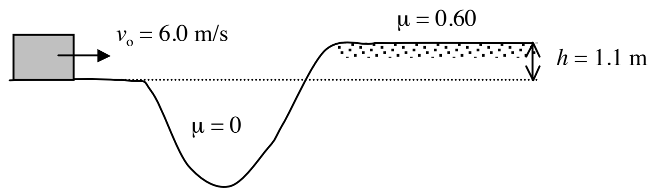
$$U_{e_1} = U_{e_2}$$

$$\frac{1}{2}k\Delta x_1^2 = \frac{1}{2}k\Delta x_2^2 \text{ so } \Delta x_2 = \pm\sqrt{\Delta x_1^2} = \pm 0.50\text{ m}$$

since the spring is compressed $\Delta x_2 = -0.50\text{ m}$

so the block is a distance of $\Delta x_2 + \ell = -0.50\text{ m} + 0.60\text{ m} = \boxed{0.10\text{ m}}$ away from the wall

8.)



Since the valley is frictionless and only conservative forces are present going from the left side to the right side, the valley can ignored when using energy conservation. It only matters that the block loses kinetic energy as the result of the increase in gravitational potential energy due to the height difference on either side of the valley.

$v_2 = 0$, and using the starting height as a reference $y_1 = 0$ and $y_2 = 1.1$ m

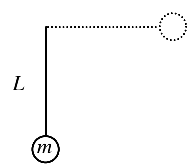
using Conservation of Energy

$$K_1 + W_f = U_{g_2}$$

$$\frac{1}{2}mv_1^2 + fd\cos\theta_f = mgy_2 \text{ and } \frac{1}{2}mv_1^2 + \mu_k mgd\cos\theta_f = mgy_2$$

$$d = \frac{2gy_2 - v_1^2}{2\mu_k g\cos\theta_f} = \frac{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(1.1\text{ m}) - \left(6.0\ \frac{\text{m}}{\text{s}}\right)^2}{2(0.60)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\cos 180°} = \boxed{1.23\text{ m}}$$

9.)



$m = 0.200$ kg, $L = 3.00$ m and using the lowest point as a reference $y_1 = L$ and $y_2 = 0$

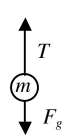
a.) released from rest so $v_1 = 0$

using Conservation of Energy

$$U_{g_1} = K_2$$

$$mgy_1 = \frac{1}{2}mv_2^2 \text{ and } v_2 = \sqrt{2gy_1} = \sqrt{2gL} = \sqrt{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(3.00\text{ m})} = \boxed{7.67\ \frac{\text{m}}{\text{s}}}$$

b.) looking at the forces on the ball at its equilibrium position



Using Newton's 2nd Law $F_{net} = \sum F = ma$

$$T - F_g = ma \text{ or } T = ma + F_g$$

the ball is in circular motion so $a = a_r = \frac{v^2}{r} = \frac{v_2^2}{L}$ so $T = \frac{mv_2^2}{L} + mg = m\left(\frac{v_2^2}{L} + g\right)$

$$T = (0.200\text{ kg})\left(\frac{\left(7.67\ \frac{\text{m}}{\text{s}}\right)^2}{3.00\text{ m}} + 9.8\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{5.88\text{ N}}$$

10.) non-ideal spring $F_x(x) = -\alpha x - \beta x^2$ where $\alpha = 70.0\ \frac{\text{N}}{\text{m}}$ and $\beta = 12.0\ \frac{\text{N}}{\text{m}^2}$

a.) $U = 0$ when $x = 0$

$$F_x = -\frac{dU}{dx} \text{ so } \int_0^x F_x dx = -\int_0^U dU \text{ or } \int_0^x \left(-\alpha x - \beta x^2\right) dx = -U\Big]_0^U = -U - 0$$

$$\left(-\alpha \frac{x^2}{2} - \beta \frac{x^3}{3}\right)\Bigg]_0^x = -U \text{ and } U(x) = \alpha \frac{x^2}{2} + \beta \frac{x^3}{3}$$

$$U(x) = \left(70.0\ \frac{\text{N}}{\text{m}}\right)\frac{x^2}{2} + \left(12.0\ \frac{\text{N}}{\text{m}^2}\right)\frac{x^3}{3} = \boxed{\left(35.0\ \frac{\text{N}}{\text{m}}\right)x^2 + \left(4.0\ \frac{\text{N}}{\text{m}^2}\right)x^3}$$

b.) $m = 0.800$ kg, $x_1 = 1.00$ m, $v_1 = 0$ and $x_2 = 0.50$ m

using Conservation of Energy $K_1 + U_1 = K_2 + U_2$ so $0 + U_1 = K_2 + U_2$ or $K_2 = U_1 - U_2$

$$\frac{1}{2} m v_2^2 = U(x_1) - U(x_2) \text{ and } v_2 = \sqrt{\frac{2\left(U(x_1) - U(x_2)\right)}{m}}$$

$$v_2 = \sqrt{\frac{2\left(\left(35.0\ \frac{\text{N}}{\text{m}}\right)x_1^2 + \left(4.0\ \frac{\text{N}}{\text{m}^2}\right)x_1^3 - \left(\left(35.0\ \frac{\text{N}}{\text{m}}\right)x_2^2 + \left(4.0\ \frac{\text{N}}{\text{m}^2}\right)x_2^3\right)\right)}{m}}$$

$$v_2 = \sqrt{\frac{2\left(\left(35.0\ \frac{\text{N}}{\text{m}}\right)(1.00\text{ m})^2 + \left(4.0\ \frac{\text{N}}{\text{m}^2}\right)(1.00\text{ m})^3 - \left(\left(35.0\ \frac{\text{N}}{\text{m}}\right)(0.50\text{ m})^2 + \left(4.0\ \frac{\text{N}}{\text{m}^2}\right)(0.50\text{ m})^3\right)\right)}{0.800\text{ kg}}} = \boxed{8.62\ \frac{\text{m}}{\text{s}}}$$