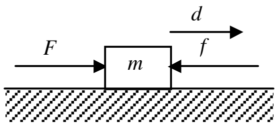
HO 11 Solutions

1.) 

$m = 25\text{ kg}$, $d = 6.0\text{ m}$, $\mu = 0.30$, constant velocity so $a = 0$ and $\Delta K = 0$

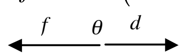
a.) Newton's 2nd Law (in the x-direction)

$$F_{net} = \sum F = ma = 0$$

$$F - f = 0 \text{ so } F = f = \mu_k F_N = \mu_k F_g = \mu_k mg = 0.30(25\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{73.5\text{ N}}$$

b.) $W_F = \vec{F}\cdot\vec{d} = Fd\cos\theta = (73.5\text{ N})(6.0\text{ m})\cos 0 = \boxed{441\text{ J}}$

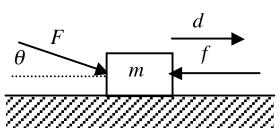
c.) $W_f = \vec{F}\cdot\vec{d} = fd\cos\theta = (73.5\text{ N})(6.0\text{ m})\cos 180° = \boxed{-441\text{ J}}$



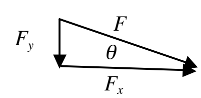
d.) $W_{F_N} = \vec{F}\cdot\vec{d} = F_N d\cos\theta = F_g d\cos\theta = mgd\cos\theta = (25\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(6.0\text{ m})\cos 90° = \boxed{0}$

e.) $W_{net} = W_F + W_f = (441\text{ J}) + (-441\text{ J}) = \boxed{0}$

also since by Work-Energy Theorem $W_{net} = \Delta K = \boxed{0}$

2.) 

$m = 25\text{ kg}$, $\theta = 30°$ components of F



a.) constant velocity so $a = 0$ Newton's 2nd Law (in the x-direction)

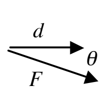
$$F_{net} = \sum F = ma = 0 \text{ and } F_x - f = 0$$

$$F_x = f = \mu_k F_N = \mu_k\left(F_g + F_y\right) = \mu_k\left(mg + F\sin\theta\right)$$

$$F_x = F\cos\theta = \mu_k\left(F_g + F_y\right) = \mu_k\left(mg + F\sin\theta\right) = \mu_k mg + \mu_k F\sin\theta$$

$$F\cos\theta - \mu_k F\sin\theta = \mu_k mg \text{ so } F\left(\cos\theta - \mu_k\sin\theta\right) = \mu_k mg$$

$$F = \frac{\mu_k mg}{\cos\theta - \mu_k\sin\theta} = \frac{0.30(25\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)}{\cos 30° - 0.30\sin 30°} = \boxed{102.65\text{ N}}$$

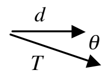
b.) 

$W_F = \vec{F}\cdot\vec{d} = Fd\cos\theta = (102.65\text{ N})(6.0\text{ m})\cos 30° = \boxed{533\text{ J}}$

c.)

$$W_f = \vec{F}\cdot\vec{d} = fd\cos\theta = \mu_k\left(mg + F\sin\theta\right)d\cos\theta = 0.30\left((25\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right) + (102.65\text{ N})\sin 30°\right)(6.0\text{ m})\cos 180° = \boxed{-533\text{ J}}$$

also since constant velocity $W_{net} = W_F + W_f = 0$ and $W_f = -W_F = \boxed{-533\text{ J}}$

3.) 

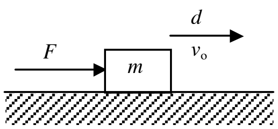
$T = 160\text{ N}$, $\theta = 15.0°$, and $d = 250\text{ m}$

$$W_T = \vec{F}\cdot\vec{d} = Td\cos\theta = (160\text{ N})(250\text{ m})\cos 15.0° = \boxed{38{,}640\text{ J}}$$

4.) $v_o = 0$, $v = 36.0\ \frac{\text{m}}{\text{s}}$, and $m = 0.145\text{ kg}$

Using Work-Energy Theorem

$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}(0.145\text{ kg})\left(36.0\ \frac{\text{m}}{\text{s}}\right)^2 - 0 = \boxed{94\text{ J}}$$

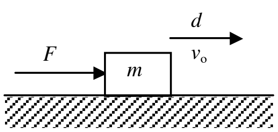
5.) 

$m = 6.00\text{ kg}$, $v_o = 4.0\ \frac{\text{m}}{\text{s}}$, $d = 4.0\text{ m}$, $F = 10.0\text{ N}$ and frictionless

Using Work-Energy Theorem

$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 \text{ and } W_F = \vec{F}\cdot\vec{d} = Fd\cos\theta$$

$$v = \sqrt{\frac{2W_F}{m} + v_o^2} = \sqrt{\frac{2Fd\cos\theta}{m} + v_o^2} = \sqrt{\frac{2(10.0\text{ N})(4.0\text{ m})\cos 0}{(6.00\text{ kg})} + \left(4.0\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{5.42\ \frac{\text{m}}{\text{s}}}$$

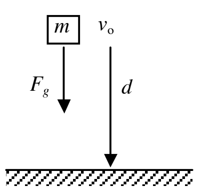
6.) 

$m = 9.00\text{ kg}$, $v_o = 4.0\ \frac{\text{m}}{\text{s}}$, $v = 6.0\ \frac{\text{m}}{\text{s}}$, $d = 3.00\text{ m}$ and frictionless

Using Work-Energy Theorem

$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 \text{ and } W_F = \vec{F}\cdot\vec{d} = Fd\cos\theta$$

$$F = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_o^2}{d} = \frac{m\left(v^2 - v_o^2\right)}{2d} = \frac{(9.00\text{ kg})\left(\left(6.0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(4.0\ \frac{\text{m}}{\text{s}}\right)^2\right)}{2(3.00\text{ m})} = \boxed{30.0\text{ N}}$$

7.) 

$m = 1.20\text{ kg}$, $v_o = 0$, and $d = 30.0\text{ m}$

a.) $$W_{F_g} = \vec{F}\cdot\vec{d} = F_g d\cos\theta = mgd\cos\theta$$

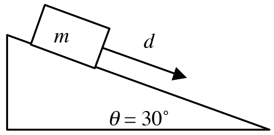
$$W_{F_g} = (1.20\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(30\text{ m})\cos 0 = \boxed{352.8\text{ J}}$$

b.) using Work-Energy Theorem

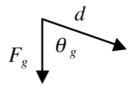
$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right)$$

$$v = \sqrt{\frac{2W_{F_g}}{m} + v_o^2} = \sqrt{\frac{2(352.8\text{ J})}{(1.20\text{ kg})} + 0} = \boxed{24.2\ \frac{\text{m}}{\text{s}}}$$

8.)



inclined plane $\theta = 30°$, $m = 2.00\ \text{kg}$, $d = 0.70\ \text{m}$, $v_o = 0$, and frictionless



$\theta_g = 60°$ $\qquad W_{Fg} = \vec{F}\cdot\vec{d} = F_g d\cos\theta_g = mgd\cos\theta_g$

using Work-Energy Theorem

$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right)$$

$$v = \sqrt{\frac{2W_{F_g}}{m} + v_o^2} = \sqrt{\frac{2mgd\cos\theta_g}{m} + v_o^2} = \sqrt{2gd\cos\theta_g + v_o^2}$$

$$v = \sqrt{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.70\ \text{m})\cos 60° + 0} = \boxed{2.62\ \frac{\text{m}}{\text{s}}}$$

9.) ideal spring $x = 0.040\ \text{m} \Rightarrow W_s = 12\ \text{J}$ $\qquad W_s = \frac{1}{2}kx^2$ so $k = \frac{2W_s}{x^2} = \frac{2(12\ \text{J})}{(0.040\ \text{m})^2} = 15{,}000\ \frac{\text{N}}{\text{m}}$

$$x = 0.030\ \text{m} \Rightarrow W_s = \frac{1}{2}kx^2 = \frac{1}{2}\left(15{,}000\ \frac{\text{N}}{\text{m}}\right)(0.030\ \text{m})^2 = \boxed{6.75\ \text{J}}$$

10.) ideal spring $F = 120\ \text{N} \Rightarrow x = 0.040\ \text{m}$ $\qquad F_s = kx$ so $k = \frac{F_s}{x} = \frac{120\ \text{N}}{0.040\ \text{m}} = 3000\ \frac{\text{N}}{\text{m}}$

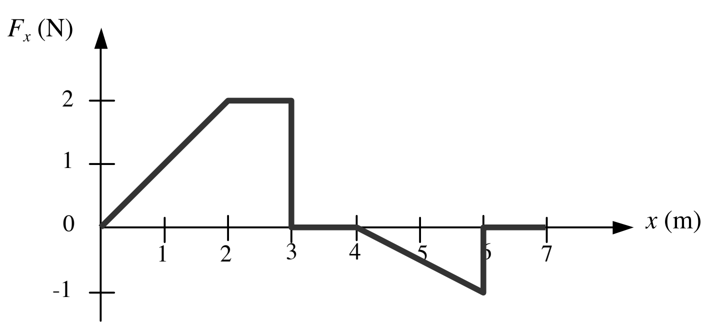
a.) $x = 0.010\ \text{m} \Rightarrow F_s = kx = \left(3000\ \frac{\text{N}}{\text{m}}\right)(0.010\ \text{m}) = \boxed{30\ \text{N}}$

$x = 0.080\ \text{m} \Rightarrow F_s = kx = \left(3000\ \frac{\text{N}}{\text{m}}\right)(0.080\ \text{m}) = \boxed{240\ \text{N}}$

b.) $x = 0.010\ \text{m} \Rightarrow W_s = \frac{1}{2}kx^2 = \frac{1}{2}\left(3000\ \frac{\text{N}}{\text{m}}\right)(0.010\ \text{m})^2 = \boxed{0.15\ \text{J}}$

$x = 0.080\ \text{m} \Rightarrow W_s = \frac{1}{2}kx^2 = \frac{1}{2}\left(3000\ \frac{\text{N}}{\text{m}}\right)(0.080\ \text{m})^2 = \boxed{9.6\ \text{J}}$

11.)



$W = \int \vec{F}\cdot d\vec{r} = Area(F \text{ vs } x)$

a.) $x = 0$ to $3.0\ \text{m}$
$W = \frac{1}{2}(2\ \text{N})(2\ \text{m}) + (2\ \text{N})(1\ \text{m}) = \boxed{4\ \text{J}}$

b.) $x = 3.0\ \text{m}$ to $4.0\ \text{m}$
$W = (0\ \text{N})(1\ \text{m}) = \boxed{0}$

c.) $x = 4.0\ \text{m}$ to $7.0\ \text{m}$
$W = \frac{1}{2}(-1\ \text{N})(2\ \text{m}) + (0\ \text{N})(1\ \text{m}) = \boxed{-1\ \text{J}}$

d.) $x = 0$ to $7.0\ \text{m}$
$W = 4\ \text{J} + 0 - 1\ \text{J} = \boxed{3\ \text{J}}$

12.) $m = 2.00\text{ kg},\ v_o = 0$

using Work-Energy Theorem

$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ so } v = \sqrt{\frac{2W}{m} + v_o^2}$$

a.) $x = 3.0\text{ m}$

$$v = \sqrt{\frac{2(4.0\text{ J})}{(12.0\text{ kg})} + 0} = \boxed{0.82\ \frac{\text{m}}{\text{s}}}$$

b.) $x = 4.0\text{ m}$

$$v = \sqrt{\frac{2(4.0\text{ J})}{(12.0\text{ kg})} + 0} = \boxed{0.82\ \frac{\text{m}}{\text{s}}}$$

c.) $x = 7.0\text{ m}$

$$v = \sqrt{\frac{2(3.0\text{ J})}{(12.0\text{ kg})} + 0} = \boxed{0.71\ \frac{\text{m}}{\text{s}}}$$

13.) $m = 3.00\text{ kg},\ v_o = 0$, ideal spring $k = 250\ \frac{\text{N}}{\text{m}},\ x = 0.030\text{ m}$, frictionless surface

using Work-Energy Theorem

$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ so } v = \sqrt{\frac{2W}{m} + v_o^2}$$

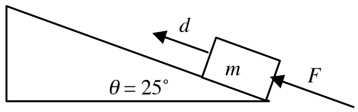
$$v = \sqrt{\frac{2\left(\frac{1}{2}kx^2\right)}{m} + v_o^2} = \sqrt{\frac{kx^2}{m} + v_o^2} = \sqrt{\frac{\left(250\ \frac{\text{N}}{\text{m}}\right)(0.030\text{ m})^2}{3.00\text{ kg}} + 0} = \boxed{0.274\ \frac{\text{m}}{\text{s}}}$$

14.) $F = 175\text{ N},\ v = 9.50\ \frac{\text{m}}{\text{s}}$

$$P = \vec{F}\cdot\vec{v} = Fv\cos\theta = (175\text{ N})\left(9.50\ \frac{\text{m}}{\text{s}}\right)\cos 0 = 1662.5\text{ W}$$

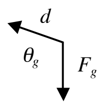
so each cat must supply $\frac{1662.5\text{ W}}{2} = 831.25\text{ W}$

$$P = 831.25\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{1.11\text{ hp}}$$

1.) 

$m = 20.0\text{ kg},\ F = 145\text{ N},\ \mu_k = 0.30$, and $d = 4.60\text{ m}$

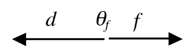
a.) $W_F = \vec{F}\cdot\vec{d} = Fd\cos\theta = (145\text{ N})(4.60\text{ m})\cos 0 = \boxed{667\text{ J}}$

b.) 

$\theta_g = 115°$ $\qquad W_{Fg} = \vec{F}\cdot\vec{d} = F_g d\cos\theta_g = mgd\cos\theta_g$

$$W_{Fg} = (20.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(4.60\text{ m})\cos 115° = \boxed{-381\text{ J}}$$

c.) $W_f = \vec{F}\cdot\vec{d} = fd\cos\theta_f = \mu F_N d\cos\theta_f = \mu mg\cos\theta d\cos\theta_f$



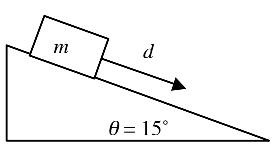
$$W_f = 0.30(20.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\cos 25°(4.60\text{ m})\cos 180° = \boxed{-245\text{ J}}$$

d.) $W_{net} = W_F + W_{F_g} + W_f = 667\text{ J} + (-381\text{ J}) + (-245\text{ J}) = \boxed{41\text{ J}}$

e.) $v_o = 0$

using Work-Energy Theorem

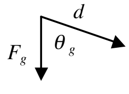
$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ so } v = \sqrt{\frac{2W_{net}}{m} + v_o^2} = \sqrt{\frac{2(41\text{ J})}{(20.0\text{ kg})} + 0} = \boxed{2.02\ \frac{\text{m}}{\text{s}}}$$

2.) 

$m = 4.0\text{ kg},\ d = 2.00\text{ m},\ \mu_k = 0.35$

a.) $W_f = \vec{F}\cdot\vec{d} = fd\cos\theta_f = \mu F_N d\cos\theta_f = \mu mg\cos\theta d\cos\theta_f$

$$W_f = 0.35(4.00\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\cos 15°(2.00\text{ m})\cos 180° = \boxed{-26.5\text{ J}}$$

b.) 

$\theta_g = 75°$ $\qquad W_{Fg} = \vec{F}\cdot\vec{d} = F_g d\cos\theta_g = mgd\cos\theta_g$

$$W_{Fg} = (4.00\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.00\text{ m})\cos 75° = \boxed{20.3\text{ J}}$$

2.) continued

c.) $v_o = 2.4\ \frac{\text{m}}{\text{s}}$

using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right)$$

$$v = \sqrt{\frac{2W_{net}}{m} + v_o^2} = \sqrt{\frac{2\left(W_f + W_{F_g}\right)}{m} + v_o^2} = \sqrt{\frac{2(-26.5\text{ J} + 20.3\text{ J})}{(4.00\text{ kg})} + \left(2.4\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{1.63\ \frac{\text{m}}{\text{s}}}$$

d.) $v_o = 2.4\ \frac{\text{m}}{\text{s}}$ and $v = 0$

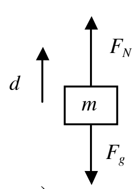
using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 \text{ and } W_{net} = W_f + W_{F_g} = \mu mg\cos\theta d\cos\theta_f + mgd\cos\theta_g = d\left(\mu mg\cos\theta\cos\theta_f + mg\cos\theta_g\right)$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = d\left(\mu mg\cos\theta\cos\theta_f + mg\cos\theta_g\right)$$

$$d = \frac{m\left(v^2 - v_o^2\right)}{2\left(\mu mg\cos\theta\cos\theta_f + mg\cos\theta_g\right)} = \frac{v^2 - v_o^2}{2g\left(\mu\cos\theta\cos\theta_f + \cos\theta_g\right)}$$

$$d = \frac{0 - \left(2.4\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.35\cos 15.0°\cos 180° + \cos 75°)} = \boxed{3.71\text{ m}}$$

3.) 

$d = 15.0\text{ m}$, during upward acceleration $W_{F_N} = 8250\text{ J}$ and $W_{F_g} = -7350\text{ J}$

a.) $$W_{F_g} = \vec{F}\cdot\vec{d} = F_g d\cos\theta_g = mgd\cos\theta_g \text{ so } m = \frac{W_{F_g}}{gd\cos\theta_g} = \frac{-7350\text{ J}}{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(15.0\text{ m})\cos 180°} = \boxed{50\text{ kg}}$$

b.) $$W_{F_N} = \vec{F}\cdot\vec{d} = F_N d\cos\theta_N \text{ so } F_N = \frac{W_{F_N}}{d\cos\theta_N} = \frac{8250\text{ J}}{(15.0\text{ m})\cos 0} = \boxed{550\text{ N}}$$

c.) using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ so } v^2 - v_o^2 = \frac{2W_{net}}{m} = \frac{2\left(W_{F_N} + W_{F_g}\right)}{m}$$

assuming uniform acceleration $v^2 = v_o^2 + 2ad$

$$a = \frac{v^2 - v_o^2}{2d} = \frac{2\left(W_{F_N} + W_{F_g}\right)}{2md} = \frac{W_{F_N} + W_{F_g}}{md} = \frac{8250\text{ J} + (-7350\text{ J})}{(50\text{ kg})(15.0\text{ m})} = \boxed{1.2\ \frac{\text{m}}{\text{s}^2}}$$

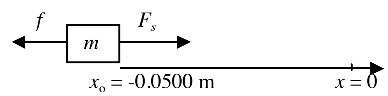
4.) ideal spring $k = 400\ \frac{\text{N}}{\text{m}}$, compressed $\Delta x = -0.0500\text{ m}$, $m = 0.0300\text{ kg}$, barrel is 0.0500 m long

a.) $v_o = 0$ using Work-Energy Theorem

$$W_s = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right)$$

$$v = \sqrt{\frac{2W_s}{m} + v_o^2} = \sqrt{\frac{2\left(\frac{1}{2}k\Delta x^2\right)}{m} + v_o^2} = \sqrt{\frac{k\Delta x^2}{m} + v_o^2} = \sqrt{\frac{\left(400\ \frac{\text{N}}{\text{m}}\right)(-0.0500\text{ m})^2}{(0.0300\text{ kg})} + 0} = \boxed{5.77\ \frac{\text{m}}{\text{s}}}$$

b.) $f = 6.00\text{ N}$ exiting the barrel $x = 0$



the work done by friction is $W_f = \vec{F}\cdot\vec{d} = f\Delta x\cos\theta = f\left(x - x_o\right)\cos\theta = (6.00\text{ N})\left(0 - (-0.0500\text{ m})\right)\cos 180° = -0.300\text{ J}$

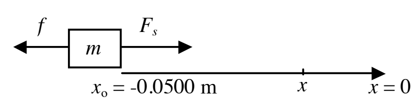
the work done by the spring is $W_s = \frac{1}{2}kx_o^2 - \frac{1}{2}kx^2 = \frac{1}{2}k\left(x_o^2 - x^2\right) = \frac{1}{2}\left(400\ \frac{\text{N}}{\text{m}}\right)\left((-0.0500\text{ m})^2 - 0\right) = 0.500\text{ J}$

the total work is therefore $W_{net} = W_s + W_f = 0.500\text{ J} + (-0.300\text{ J}) = 0.200\text{ J}$

using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ and } v = \sqrt{\frac{2W_{net}}{m} + v_o^2} = \sqrt{\frac{2(0.200\text{ J})}{(0.0300\text{ kg})} + 0} = \boxed{3.65\ \frac{\text{m}}{\text{s}}}$$

c.) $f = 6.00\text{ N}$ and maximum speed occurs when $a = \frac{dv}{dt} = 0$



Newton's 2nd Law (in the x-direction) $F_{net} = \sum F = ma = 0$

$$F_s - f = 0 \text{ and } F_s = -kx \text{ so } -kx - f = 0$$

$$x = \frac{f}{-k} = \frac{6.00\text{ N}}{\left(-400\ \frac{\text{N}}{\text{m}}\right)} = -0.015\text{ m (compressed)}$$

the work done by friction is

$$W_f = \vec{F}\cdot\vec{d} = f\Delta x\cos\theta = f\left(x - x_o\right)\cos\theta = (6.00\text{ N})\left(-0.0150\text{ m} - (-0.0500\text{ m})\right)\cos 180° = -0.210\text{ J}$$

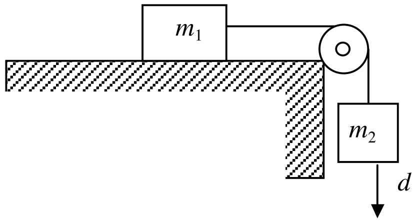
the work done by the spring is

$$W_s = \frac{1}{2}kx_o^2 - \frac{1}{2}kx^2 = \frac{1}{2}k\left(x_o^2 - x^2\right) = \frac{1}{2}\left(400\ \frac{\text{N}}{\text{m}}\right)\left((-0.0500\text{ m})^2 - (-0.015\text{ m})^2\right) = 0.455\text{ J}$$

the total work is therefore $W_{net} = W_s + W_f = 0.455\text{ J} + (-0.21\text{ J}) = 0.245\text{ J}$

using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ and } v = \sqrt{\frac{2W_{net}}{m} + v_o^2} = \sqrt{\frac{2(0.245\text{ J})}{(0.0300\text{ kg})} + 0} = \boxed{4.04\ \frac{\text{m}}{\text{s}}}$$

5.) 

$m_1 = 8.00\text{ kg}$, $m_2 = 6.00\text{ kg}$, $\mu_k = 0.35$, and $v_o = 0$

m_2 descends $d = 2.50\text{ m}$

the work done by friction on m_1 is $W_{f1} = \vec{F}\cdot\vec{d} = f_1 d\cos\theta_f = \mu F_{N_1} d\cos\theta_f = \mu m_1 g d\cos\theta_f$

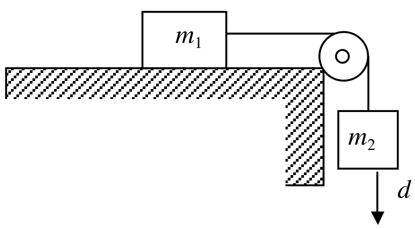
the work done by gravity on m_2 is $W_{F_{g2}} = \vec{F}\cdot\vec{d} = F_{g2} d\cos\theta_g = m_2 g d\cos\theta_g$

the total work is therefore $W_{net} = W_{F_{g2}} + W_{f1} = m_2 g d\cos\theta_g + \mu m_1 g d\cos\theta_f = gd\left(m_2\cos\theta_g + \mu m_1\cos\theta_f\right)$

using Work-Energy Theorem (both blocks are moving at the same velocity)

$$W_{net} = \Delta K = \frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2}(m_1 + m_2)v_o^2 \text{ and } v = \sqrt{\frac{2W_{net}}{m_1 + m_2} + v_o^2} = \sqrt{\frac{2gd\left(m_2\cos\theta_g + \mu m_1\cos\theta_f\right)}{m_1 + m_2} + v_o^2}$$

$$v = \sqrt{\frac{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.50\text{ m})\left((6.00\text{ kg})\cos 0 + 0.30(8.00\text{ kg})\cos 180°\right)}{8.00\text{ kg} + 6.00\text{ kg}} + 0} = \boxed{3.55\ \frac{\text{m}}{\text{s}}}$$

6.) 

$m_1 = 8.00\text{ kg}$, $m_2 = 6.00\text{ kg}$, $\mu_k = 0.35$, $v_o = 2.00\ \frac{\text{m}}{\text{s}}$

after $d = 2.95\text{ m}$ then $v = 0$

the work done by friction on m_1 is $W_{f1} = \vec{F}\cdot\vec{d} = f_1 d\cos\theta_f = \mu F_{N_1} d\cos\theta_f = \mu m_1 g d\cos\theta_f$

the work done by gravity on m_2 is $W_{F_{g2}} = \vec{F}\cdot\vec{d} = F_{g2} d\cos\theta_g = m_2 g d\cos\theta_g$

using Work-Energy Theorem (both blocks are moving at the same velocity)

$$W_{net} = \Delta K = \frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2}(m_1 + m_2)v_o^2 \text{ and } W_{net} = W_{F_{g2}} + W_{f1} = m_2 g d\cos\theta_g + \mu m_1 g d\cos\theta_f$$

so $$\frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2}(m_1 + m_2)v_o^2 = m_2 g d\cos\theta_g + \mu m_1 g d\cos\theta_f$$

$$-\frac{1}{2}(m_1 + m_2)v_o^2 - m_2 g d\cos\theta_g = \mu m_1 g d\cos\theta_f \qquad (v = 0)$$

$$\mu = \frac{-\frac{1}{2}(m_1 + m_2)v_o^2 - m_2 g d\cos\theta_g}{m_1 g d\cos\theta_f} = \frac{-(m_1 + m_2)v_o^2 - 2m_2 g d\cos\theta_g}{2m_1 g d\cos\theta_f}$$

$$\mu = \frac{-(8.00\text{ kg} + 6.00\text{ kg})\left(2.00\ \frac{\text{m}}{\text{s}}\right)^2 - 2(6.00\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.95\text{ m})\cos 0}{2(8.00\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.95\text{ m})\text{co}180°} = \boxed{0.87}$$

7.) ideal spring $k = 250\ \frac{\text{N}}{\text{m}}$, compressed $\Delta x = -0.200\ \text{m}$, $m = 1.50\ \text{kg}$, $\mu_k = 0.30$, $v_o = v = 0$

using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = 0 \text{ and } W_{net} = W_s + W_f = \frac{1}{2}k\Delta x^2 + fd\cos\theta = \frac{1}{2}k\Delta x^2 + \mu_k F_N \cos\theta$$

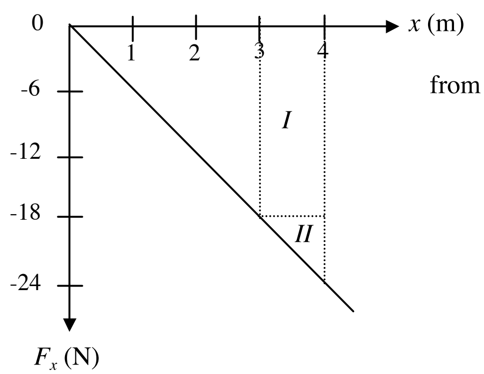
so $$0 = \frac{1}{2}k\Delta x^2 + \mu_k F_N \cos\theta = \frac{1}{2}k\Delta x^2 + \mu_k mgd\cos\theta$$

$$-\mu_k mgd\cos\theta = \frac{1}{2}k\Delta x^2$$

$$d = \frac{\frac{1}{2}k\Delta x^2}{-\mu_k mg\cos\theta} = \frac{-k\Delta x^2}{2\mu_k mg\cos\theta} = \frac{-\left(250\ \frac{\text{N}}{\text{m}}\right)(-0.200\ \text{m})^2}{2(0.30)(1.50\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\cos 180°} = \boxed{1.13\ \text{m}}$$

8.) $F_x = -\left(6\ \frac{\text{N}}{\text{m}}\right)x$, $m = 2.0\ \text{kg}$, and at $x_o = 3.0\ \text{m}$, $v_o = 8{,}0\ \frac{\text{m}}{\text{s}}$

a.) variable force so work is the area under a F versus x graph between $x_o = 3.0\ \text{m}$ and $x = 4.0\ \text{m}$



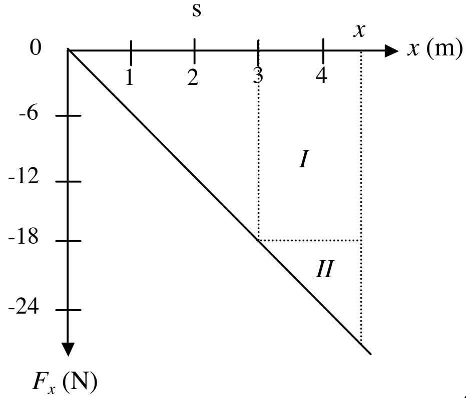
from graph $W_F = Area_I + Area_{II} = (-18\ \text{N})(1\ \text{m}) + \frac{1}{2}(-6\ \text{N})(1\ \text{m}) = -21\ \text{J}$

using Work-Energy Theorem

$$W_F = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right)$$

$$v = \sqrt{\frac{2W_F}{m} + v_o^2} = \sqrt{\frac{2(-21\ \text{J})}{(2.0\ \text{kg})} + \left(8.0\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{6.56\ \frac{\text{m}}{\text{s}}}$$

b.) $v = 5.0\ \frac{\text{m}}{\text{s}}$



using Work-Energy Theorem

$$W_F = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right)$$

$$W_F = \frac{1}{2}(2.0\ \text{kg})\left(\left(5.0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(8.0\ \frac{\text{m}}{\text{s}}\right)^2\right) = -39\ \text{J}$$

$$W_F = Area_I + Area_{II} = (-18\ \text{N})(x - (3\ \text{m})) + \frac{1}{2}\left(\left(-6\ \frac{\text{N}}{\text{m}}\right)x + 18\ \text{N}\right)(x - (3\ \text{m}))$$

$$W_F = (-18\ \text{N})x + 54\ \text{J} + \left(\left(-3\ \frac{\text{N}}{\text{m}}\right)x + 9\ \text{N}\right)(x - (3\ \text{m}))$$

$$W_F = (-18\ \text{N})x + 54\ \text{J} + \left(-3\ \frac{\text{N}}{\text{m}}\right)x^2 + (18\ \text{N})x - 27\ \text{J} = \left(-3\ \frac{\text{N}}{\text{m}}\right)x^2 + 27\ \text{J}$$

so $-39\ \text{J} = \left(-3\ \frac{\text{N}}{\text{m}}\right)x^2 + 27\ \text{J}$ and $x = \sqrt{\frac{-66\ \text{J}}{\left(-3\ \frac{\text{N}}{\text{m}}\right)}} = \boxed{4.69\ \text{m}}$

8.) Alternative solution

$F_x = -\left(6\ \frac{\text{N}}{\text{m}}\right)x$, $m = 2.0$ kg, and at $x_o = 3.0$ m, $v_o = 8{,}0\ \frac{\text{m}}{\text{s}}$

a.) $x = 4.0$ m

$$W = \int \boldsymbol{F}\cdot d\boldsymbol{r} \text{ so } W_F = \int_{x_o}^{x} F_x dx = \int_{3\text{ m}}^{4\text{ m}}\left(-6\ \frac{\text{N}}{\text{m}}\right)x dx = \left(-6\ \frac{\text{N}}{\text{m}}\right)\frac{x^2}{2}\Bigg]_{3\text{ m}}^{4\text{ m}} = \left(-3\ \frac{\text{N}}{\text{m}}\right)\left((4\text{ m})^2 - (3\text{ m})^2\right) = -21\text{ J}$$

using Work-Energy Theorem

$$W_F = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right)$$

$$v = \sqrt{\frac{2W_F}{m} + v_o^2} = \sqrt{\frac{2(-21\text{ J})}{(2.0\text{ kg})} + \left(8.0\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{6.56\ \frac{\text{m}}{\text{s}}}$$

b.) $v = 5.0\ \frac{\text{m}}{\text{s}}$

using Work-Energy Theorem

$$W_F = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right)$$

$$W_F = \frac{1}{2}(2.0\text{ kg})\left(\left(5.0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(8.0\ \frac{\text{m}}{\text{s}}\right)^2\right) = -39\text{ J}$$

$$W_F = \int_{x_o}^{x} F_x dx = \int_{3\text{ m}}^{x}\left(-6\ \frac{\text{N}}{\text{m}}\right)x dx = \left(-6\ \frac{\text{N}}{\text{m}}\right)\frac{x^2}{2}\Bigg]_{3\text{ m}}^{x} = \left(-3\ \frac{\text{N}}{\text{m}}\right)\left(x^2 - (3\text{ m})^2\right) = \left(-3\ \frac{\text{N}}{\text{m}}\right)x^2 + 27\text{ J}$$

so $$\left(-3\ \frac{\text{N}}{\text{m}}\right)x^2 + 27\text{ J} = -39\text{ J} \text{ and } x = \sqrt{\frac{-66\text{ J}}{\left(-3\ \frac{\text{N}}{\text{m}}\right)}} = \boxed{4.69\text{ m}}$$

HO 13 Solutions

1.) $m = 6.00\text{ kg}$, $v_o = 0$, $x(t) = \alpha t^2 + \beta t^3$ where $\alpha = 2.00\ \frac{\text{m}}{\text{s}^2}$ and $\beta = 0.200\ \frac{\text{m}}{\text{s}^3}$

a.) $t = 4.00\text{ s}$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left(\alpha t^2 + \beta t^3\right) = 2\alpha t + 3\beta t^2 = 2\left(2.00\ \frac{\text{m}}{\text{s}^2}\right)t + 3\left(0.200\ \frac{\text{m}}{\text{s}^3}\right)t^2 = \left(4.00\ \frac{\text{m}}{\text{s}^2}\right)t + \left(0.600\ \frac{\text{m}}{\text{s}^3}\right)t^2$$

$$v(4.00\text{ s}) = \left(4.00\ \frac{\text{m}}{\text{s}^2}\right)(4.00\text{ s}) + \left(0.600\ \frac{\text{m}}{\text{s}^3}\right)(4.00\text{ s})^2 = \boxed{25.6\ \frac{\text{m}}{\text{s}}}$$

b.) $t = 4.00\text{ s}$

$$F_{net} = \sum F = ma$$

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\left(4.00\ \frac{\text{m}}{\text{s}^2}\right)t + \left(0.600\ \frac{\text{m}}{\text{s}^3}\right)t^2\right) = 4.00\ \frac{\text{m}}{\text{s}^2} + 2\left(0.600\ \frac{\text{m}}{\text{s}^3}\right)t = 4.00\ \frac{\text{m}}{\text{s}^2} + \left(1.20\ \frac{\text{m}}{\text{s}^3}\right)t$$

$$a(4.00\text{ s}) = 4.00\ \frac{\text{m}}{\text{s}^2} + \left(1.20\ \frac{\text{m}}{\text{s}^3}\right)(4.00\text{ s}) = 8.8\ \frac{\text{m}}{\text{s}^2}$$

$$F_{net} = ma = (6.00\text{ kg})\left(8.8\ \frac{\text{m}}{\text{s}^2}\right) = \boxed{52.8\text{ N}}$$

c.)

using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) = \frac{1}{2}(6.00\text{ kg})\left(\left(25.6\ \frac{\text{m}}{\text{s}}\right)^2 - 0\right) = \boxed{1966\text{ J}}$$

2.) $m = 5.00\text{ kg}$, $v_o = 6.00\ \frac{\text{m}}{\text{s}}$, frictionless surface towards ideal spring $k = 500\ \frac{\text{N}}{\text{m}}$

a.)

using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ and } W_{net} = W_s = -\frac{1}{2}k\Delta x^2 \text{ (compressed)}$$

maximum compression when $v = 0$

$$\frac{1}{2}m\left(v^2 - v_o^2\right) = -\frac{1}{2}k\Delta x^2 \text{ and } \Delta x = \sqrt{\frac{-m\left(v^2 - v_o^2\right)}{k}} = \sqrt{\frac{-(5.00\text{ kg})\left(0 - \left(6.00\ \frac{\text{m}}{\text{s}}\right)^2\right)}{\left(500\ \frac{\text{N}}{\text{m}}\right)}} = \boxed{0.600\text{ m}}$$

b.) $\Delta x = 0.200\text{ m}$

$$\frac{1}{2}m\left(v^2 - v_o^2\right) = -\frac{1}{2}k\Delta x^2 \text{ so } v_o = \sqrt{\frac{k\Delta x^2}{m}} = \sqrt{\frac{\left(500\ \frac{\text{N}}{\text{m}}\right)(0.200\text{ m})^2}{5.00\text{ kg}}} = \boxed{2.00\ \frac{\text{m}}{\text{s}}}$$

3.) $F = 53\text{ kN}$, $m = 9.1 \text{ x } 10^5 \text{ kg}$, $v = 45 \ \frac{\text{m}}{\text{s}}$, and $a = 0$

a.)

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta = (53 \text{ kN})\left(45 \ \frac{\text{m}}{\text{s}}\right)\cos 0 = \boxed{2385 \text{ kW}}$$

b.) $a = 1.0 \ \frac{\text{m}}{\text{s}^2}$

$$F_{net} = ma = (9.1 \text{ x } 10^5 \text{ kg})\left(1.0 \ \frac{\text{m}}{\text{s}^2}\right) = 910 \text{ kN}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta = (910 \text{ kN})\left(45 \ \frac{\text{m}}{\text{s}}\right)\cos 0 = \boxed{41000 \text{ kW}}$$

4.) ideal spring $k = 4000 \ \frac{\text{N}}{\text{m}}$, compressed $\Delta x = 0.375 \text{ m}$, $m = 80.0 \text{ kg}$, $v_\text{o} = 0$, and frictionless

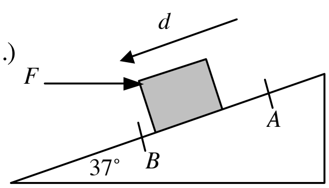
$W_s = \frac{1}{2}k\Delta x^2$ and using Work-Energy Theorem $W_s = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_\text{o}^2 = \frac{1}{2}m\left(v^2 - v_\text{o}^2\right)$

$$v = \sqrt{\frac{2W_s}{m} + v_\text{o}^2} = \sqrt{\frac{2\left(\frac{1}{2}k\Delta x^2\right)}{m} + v_\text{o}^2} = \sqrt{\frac{k\Delta x^2}{m} + v_\text{o}^2} = \sqrt{\frac{\left(4000 \ \frac{\text{N}}{\text{m}}\right)(0.375 \text{ m})^2}{(80.0 \text{ kg})} + 0} = \boxed{2.65 \ \frac{\text{m}}{\text{s}}}$$

5.) $m = 1000 \text{ kg}$, $v = 8.0 \ \frac{\text{m}}{\text{s}}$, and $a = 0$

$$P = \vec{F} \cdot \vec{v} = F_g v\cos\theta = mgv\cos\theta = (1000 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)\left(8.0 \ \frac{\text{m}}{\text{s}}\right)\cos 0 = \boxed{78.4 \text{ kW}}$$

6.)



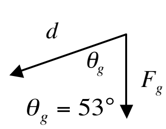
$m = 4.0 \text{ kg}$, $d = 5.0 \text{ m}$, $F = 10 \text{ N}$, $K_A = 10 \text{ J}$, $K_B = 20 \text{ J}$

using Work-Energy Theorem

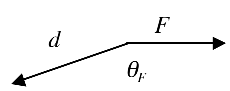
$W_{net} = \Delta K = K_B - K_A$ and $W_{net} = W_F + W_f + W_{F_g}$

so $W_F + W_f + W_{F_g} = K_B - K_A$ and $W_f = K_B - K_A - W_{F_g} - W_F$

$$W_f = K_B - K_A - F_g d\cos\theta_g - Fd\cos\theta_F = K_B - K_A - mgd\cos\theta_g - Fd\cos\theta_F$$



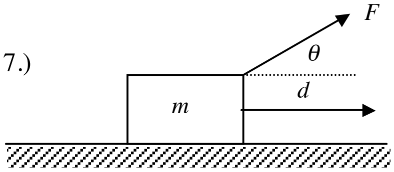
$\theta_g = 53°$



$\theta_F = 143°$

$$W_f = 20 \text{ J} - 10 \text{ J} - (4.0 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(5.0 \text{ m})\cos 53° - (10 \text{ N})(5.0 \text{ m})\cos 143° = \boxed{-68 \text{ J}}$$

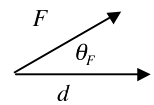
7.)



$$m = 3.0\ \text{kg},\ F = 16\ \text{N},\ d = 5.0\ \text{m},\ \theta = 37°,\ v_o = 4.0\ \frac{\text{m}}{\text{s}},\ v = 6.0\ \frac{\text{m}}{\text{s}}$$

using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ and } W_{net} = W_F + W_f$$

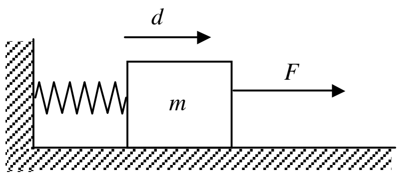


$\theta_F = 37°$

so $$W_f = \frac{1}{2}m\left(v^2 - v_o^2\right) - W_F = \frac{1}{2}m\left(v^2 - v_o^2\right) - Fd\cos\theta_F$$

$$W_f = \frac{1}{2}(3.0\ \text{kg})\left(\left(6.0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(4.0\ \frac{\text{m}}{\text{s}}\right)^2\right) - (16\ \text{N})(5.0\ \text{m})\cos 37° = \boxed{-33.9\ \text{J}}$$

8.)



$$m = 10\ \text{kg},\ k = 800\ \frac{\text{N}}{\text{m}},\ v_o = 0,\ F = 80\ \text{N},\ d = \Delta x = 0.13\ \text{m}$$

frictionless surface

using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ and } W_{net} = W_F + W_s$$

$$v = \sqrt{\frac{2(W_F + W_s)}{m} + v_o^2} = \sqrt{\frac{2\left(Fd\cos\theta_F - \frac{1}{2}k\Delta x^2\right)}{m} + v_o^2}$$

$$v = \sqrt{\frac{2(W_F + W_s)}{m} + v_o^2} = \sqrt{\frac{2\left((80\ \text{N})(0.13\ \text{m})\cos 0 - \frac{1}{2}\left(800\ \frac{\text{N}}{\text{m}}\right)(0.13\ \text{m})^2\right)}{(10\ \text{kg})} + 0} = \boxed{0.85\ \frac{\text{m}}{\text{s}}}$$

9.) $m = 5.0\ \text{kg}$, horizontal force $F = 12\ \text{N}$, $\mu_k = 0.20$, and $v_o = 0$

Newton's 2nd Law (in the x-direction)

$$F_{net} = \sum F = ma$$

$$F - f = ma \text{ and } a = \frac{F - f}{m} = \frac{F - \mu_k F_N}{m} = \frac{F - \mu_k mg}{m} = \frac{12\ \text{N} - 0.20(5.0\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)}{5.0\ \text{kg}} = 0.44\ \frac{\text{m}}{\text{s}^2}$$

when $t = 5.0\ \text{s}$

$$v = at + v_o = \left(0.44\ \frac{\text{m}}{\text{s}^2}\right)(5.0\ \text{s}) + 0 = 2.2\ \frac{\text{m}}{\text{s}}$$

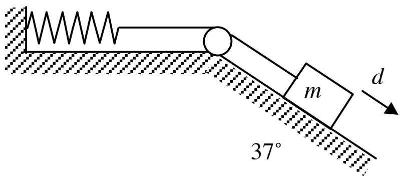
$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta = (12\ \text{N})\left(2.2\ \frac{\text{m}}{\text{s}}\right)\cos 0 = \boxed{26.4\ \text{W}}$$

10.) ideal spring $k = 1000\ \frac{\text{N}}{\text{m}}$, $m = 2.0\text{ kg}$ attached on horizontal frictionless surface, $v_o = 5.0\ \frac{\text{m}}{\text{s}}$

when $\Delta x = 0.20\text{ m}$

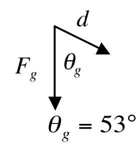
$$W_{net} = \Delta K = K - K_o = K - \frac{1}{2}mv_o^2 \text{ so } K = W_{net} + \frac{1}{2}mv_o^2 = W_s + \frac{1}{2}mv_o^2$$

$$K = -\frac{1}{2}k\Delta x^2 + \frac{1}{2}mv_o^2 = \frac{1}{2}\left(-k\Delta x^2 + mv_o^2\right) = \frac{1}{2}\left(-\left(1000\ \frac{\text{N}}{\text{m}}\right)(0.20\text{ m})^2 + (2.0\text{ kg})\left(5.0\ \frac{\text{m}}{\text{s}}\right)^2\right) = \boxed{5\text{ J}}$$

11.) 

$m = 2.0\text{ kg}$, $k = 100\ \frac{\text{N}}{\text{m}}$, and frictionless surface, $v_o = 0$

$d = \Delta x = 0.20\text{ m}$



using Work-Energy Theorem

$$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m\left(v^2 - v_o^2\right) \text{ and } W_{net} = W_{F_g} + W_s$$

$$v = \sqrt{\frac{2\left(W_{F_g} + W_s\right)}{m} + v_o^2} = \sqrt{\frac{2\left(F_g d\cos\theta_g - \frac{1}{2}k\Delta x^2\right)}{m} + v_o^2} = \sqrt{\frac{2\left(mgd\cos\theta_g - \frac{1}{2}k\Delta x^2\right)}{m} + v_o^2}$$

$$v = \sqrt{\frac{2\left((2.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.20\text{ m})\cos 53° - \frac{1}{2}\left(100\ \frac{\text{N}}{\text{m}}\right)(0.20\text{ m})^2\right)}{2.0\text{ kg}} + 0} = \boxed{0.60\ \frac{\text{m}}{\text{s}}}$$