Example 1:

A projectile is launched from ground level with a launch angle of 36.87˚ and speed of 40 m/s.

a.) What is the range of the projectile?

b.) What is the maximum height of the projectile?

c.) What is the velocity of the projectile just before it hits the ground?

Example 1: (Ground-to-ground)

$$\theta_0 = 36.87° \text{ and } v_0 = 40 \ \frac{\text{m}}{\text{s}}$$

a.) $\Delta x =$? when $\Delta y = 0$

$$\Delta y = -\frac{1}{2}gt^2 + v_{y_0}t = 0 = t\left(-\frac{1}{2}gt + v_{y_0}\right)$$

$$t = \frac{2v_{y_0}}{g} = \frac{2v_0\sin\theta_0}{g} = \frac{2\left(40 \ \frac{\text{m}}{\text{s}}\right)\sin(36.87°)}{10 \ \frac{\text{m}}{\text{s}^2}} = 4.8 \text{ s}$$

$$\Delta x = v_{x_0}t = (v_0\cos\theta_0)t = \left(40 \ \frac{\text{m}}{\text{s}}\cos(36.87°)\right)(4.8 \text{ s})$$

$$\boxed{\Delta x = 154 \text{ m}}$$

Example 1: (Ground-to-ground)

$$\theta_0 = 36.87° \text{ and } v_0 = 40 \ \frac{\text{m}}{\text{s}}$$

b.) $y =$? when $v_y = 0$

$$v_y^2 = v_{y_0}^2 - 2g(y - y_0) \qquad \text{so} \quad y = \frac{\left(v_{y_0}^2 - v_y^2\right)}{2g} + y_0$$

$$\text{and} \quad y = \frac{\left((v_0\sin\theta)^2 - v_y^2\right)}{2g} + y_0$$

$$\text{when } v_y = 0 \text{ and } y_0 = 0: \quad y_{max} = \frac{(v_0\sin\theta)^2}{2g} = \frac{\left(40 \ \frac{\text{m}}{\text{s}}\sin(36.87°)\right)^2}{2\left(10 \ \frac{\text{m}}{\text{s}^2}\right)}$$

$$\boxed{y_{max} = 28.8 \text{ m}}$$

Example 1: (Ground-to-ground)

$$\theta_0 = 36.87° \text{ and } v_0 = 40 \ \frac{\text{m}}{\text{s}}$$

b.) $y =$? when $v_y = 0$

$$v_y = -gt + v_{y_0}$$

$$\text{Notice when } v_y = 0: \quad t = \frac{v_{y_0}}{g} = \frac{v_0\sin\theta}{g} = \frac{40 \ \frac{\text{m}}{\text{s}}\sin(36.87°)}{10 \ \frac{\text{m}}{\text{s}^2}} = 2.4 \text{ s}$$

Which is half the time found in part a when $\Delta y = 0$ (symmetry).

Example 1: (Ground-to-ground)

c.) $\vec{v} =$? when $\Delta y = 0$ and $t = 4.8$ s

$$v_x = v_0\cos\theta_0 = 40 \ \frac{\text{m}}{\text{s}}\cos(36.87°) = 32 \ \frac{\text{m}}{\text{s}}$$

$$v_y = -gt + v_{y_0} = \left(-10 \ \frac{\text{m}}{\text{s}^2}\right)(4.8 \text{ s}) + 40 \ \frac{\text{m}}{\text{s}}\sin(36.87°) = -48 \ \frac{\text{m}}{\text{s}} + 24 \ \frac{\text{m}}{\text{s}}$$

$$v_y = -24 \ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(32 \ \frac{\text{m}}{\text{s}}\right)^2 + \left(-24 \ \frac{\text{m}}{\text{s}}\right)^2} \boxed{= 40 \ \frac{\text{m}}{\text{s}}}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-24}{36}\right) = \boxed{-36.87°} \text{ (it is symmetric)}$$

Example 2:

A projectile is launched off a building that is 20 m tall with an initial speed of 30 m/s and launch angle of 53.13˚ with respect to the horizontal.

a.) What is the maximum height with respect to the ground?

b.) How far from the base of the building does the projectile land?

c.) What is the velocity of the projectile just before it hits the ground?

Example 2:

$$y_0 = 20 \text{ m},\ \theta_0 = 53.13°,\ \text{and } v_0 = 30\ \frac{\text{m}}{\text{s}}$$

$$v_x = v_{x_0} = v_0\cos\theta = 30\ \frac{\text{m}}{\text{s}}\cos(53.13°) = 18\ \frac{\text{m}}{\text{s}}$$

$$v_{y_0} = v_0\sin\theta = 30\ \frac{\text{m}}{\text{s}}\sin(53.13°) = 24\ \frac{\text{m}}{\text{s}}$$

a.) $y =$? when $v_y = 0$

$$v_y^2 = v_{y_0}^2 - 2g(y - y_0) \quad \text{so} \quad y = \frac{\left(v_y^2 - v_{y_0}^2\right)}{2g} + y_0$$

$$\text{and} \quad y = \frac{\left(v_y^2 - v_{y_0}^2\right)}{2g} + y_0 = \frac{\left(24\ \frac{\text{m}}{\text{s}}\right)^2 + 0}{2\left(10\ \frac{\text{m}}{\text{s}^2}\right)} + 20 \text{ m}$$

$$\boxed{y = 48.8 \text{ m}}$$

Example 2:

$$y_0 = 20 \text{ m},\ \theta_0 = 53.13°,\ \text{and } v_0 = 30\ \frac{\text{m}}{\text{s}}$$

b.) $\Delta x =$? when $\Delta y = y - y_0 = 0 - 20 \text{ m} = -20 \text{ m}$

$\Delta x = v_x t$ use the y - velocity to find time $v_y^2 = v_{y_0}^2 - 2g(y - y_0)$

$$v_y = -\sqrt{v_{y_0}^2 - 2g(y - y_0)} = -\sqrt{\left(24\ \frac{\text{m}}{\text{s}}\right)^2 - 2\left(10\ \frac{\text{m}}{\text{s}^2}\right)(-20 \text{ m})} = -31.2\ \frac{\text{m}}{\text{s}}$$

$$v_y = -gt + v_{y_0} \text{ so } t = \frac{v_{y_0} - v_y}{g} = \frac{24\ \frac{\text{m}}{\text{s}} - \left(-31.2\ \frac{\text{m}}{\text{s}}\right)}{10\ \frac{\text{m}}{\text{s}^2}} = 5.52 \text{ s}$$

$$\Delta x = v_x t = \left(18\ \frac{\text{m}}{\text{s}}\right)(5.52 \text{ s})$$

$$\boxed{\Delta x = 99 \text{ m}}$$

Example 2:

$$y_0 = 20 \text{ m},\ \theta_0 = 53.13°,\ \text{and } v_0 = 30\ \frac{\text{m}}{\text{s}}$$

c.) $\bar{v} =$? when $\Delta y = y - y_0 = 0 - 20 \text{ m} = -20 \text{ m}$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(18\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-31.2\ \frac{\text{m}}{\text{s}}\right)^2} = 36\ \frac{\text{m}}{\text{s}}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-31.2}{18}\right) = -60°$$

$$\boxed{\bar{v} = 36\ \frac{\text{m}}{\text{s}}\angle -60°}$$

Example 3:

A projectile is launched horizontally off a building that is 40 m tall with an initial speed of 30 m/s.

a.) How far from the base of the building does the projectile land?

b.) What is the velocity of the projectile just before it hits the ground?

c.) What is its speed 20 m above the ground?

Example 3: $y_0 = 40 \text{ m},\ \theta_0 = 0,\ \text{and } v_0 = 30\ \frac{\text{m}}{\text{s}}$

a.) $\Delta x =$? when $\Delta y = y - y_0 = 0 - 40 \text{ m} = -40 \text{ m}$

$v_x = v_0 = 30\ \frac{\text{m}}{\text{s}}$ and $v_{y_0} = 0$ (horizontally launched $\theta_0 = 0$)

$$\Delta x = v_x t$$

$$\Delta y = -\frac{1}{2}gt^2 + \cancel{v_{y_0}t} = -\frac{1}{2}gt^2 \text{ so } t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-40 \text{ m})}{10\ \frac{\text{m}}{\text{s}^2}}} = 2.83 \text{ s}$$

$$\Delta x = v_x t = \left(30\ \frac{\text{m}}{\text{s}}\right)(2.83 \text{ s})$$

$$\boxed{\Delta x = 84.9 \text{ m}}$$

Example 3: $y_0 = 40 \text{ m},\ \theta_0 = 0,\ \text{and } v_0 = 30\ \frac{\text{m}}{\text{s}}$

b.) $\bar{v} =$? when $\Delta y = y - y_0 = 0 - 40 \text{ m} = -40 \text{ m}$

$$v_x = v_0 = 30\ \frac{\text{m}}{\text{s}}$$

$$v_y = -\sqrt{\cancel{v_{y_0}^2} - 2g\Delta y} = -\sqrt{-2g\Delta y} = -\sqrt{-2\left(10\ \frac{\text{m}}{\text{s}^2}\right)(-40 \text{ m})} = -28.3\ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(30\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-28.3\ \frac{\text{m}}{\text{s}}\right)^2} = 41.2\ \frac{\text{m}}{\text{s}}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-28.3}{30}\right) = -43.3°$$

$$\boxed{\bar{v} = 41.2\ \frac{\text{m}}{\text{s}}\angle -43.3°}$$

Example 3: $y_0 = 40\text{ m},\ \theta_0 = 0,\text{ and } v_0 = 30\ \frac{\text{m}}{\text{s}}$

c.) $v = ?$ when $\Delta y = y - y_0 = 20\text{ m} - 40\text{ m} = -20\text{ m}$

$$v_x = v_0 = 30\ \frac{\text{m}}{\text{s}}$$

$$v_y = -\sqrt{v_{y_0}^2 - 2g\Delta y} = -\sqrt{-2g\Delta y} = -\sqrt{-2\left(10\ \frac{\text{m}}{\text{s}^2}\right)(-20\text{ m})} = -20\ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(30\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-20\ \frac{\text{m}}{\text{s}}\right)^2}$$

$$\boxed{v = 36.1\ \frac{\text{m}}{\text{s}}}$$

Example 4:

An object's acceleration while moving around a circular path of radius 2.0 m is described by the equation $a = 2t$, where a is in m/s^2. If the objects initial velocity is zero, find the total acceleration of the object at $t = 2.0$ s.

Example 4:

$$r = 2\text{ m},\ v_0 = 0,\ a_{tot} = ?\text{ when } t = 2\text{ s}$$

$$a = \sqrt{a_{rad}^2 + a_{tan}^2} \quad a_{tan} = \left(2\ \frac{\text{m}}{\text{s}^3}\right)t \text{ and } a_{rad} = \frac{v^2}{r}$$

$$a_{tan} = \left(2\ \frac{\text{m}}{\text{s}^3}\right)t = \left(2\ \frac{\text{m}}{\text{s}^3}\right)(2\text{ s}) = 4\ \frac{\text{m}}{\text{s}^2}$$

$$v = \int a_{tan}\,dt = \frac{2t^2}{2} + C = t^2 + C \quad v_0 = 0 = 0 + C \text{ so } C = 0$$

$$v = \left(1\ \frac{\text{m}}{\text{s}^3}\right)t^2 \text{ so } v(2\text{ s}) = \left(1\ \frac{\text{m}}{\text{s}^3}\right)(2\text{ s})^2 = 4\ \frac{\text{m}}{\text{s}}$$

$$a_{rad} = \frac{v^2}{r} \qquad a = \sqrt{a_{rad}^2 + a_{tan}^2}$$

$$\text{so } a_{rad} = \frac{\left(4\ \frac{\text{m}}{\text{s}}\right)^2}{2\text{ m}} = 8\ \frac{\text{m}}{\text{s}^2} \qquad a = \sqrt{\left(8\ \frac{\text{m}}{\text{s}^2}\right)^2 + \left(4\ \frac{\text{m}}{\text{s}^2}\right)^2} = \boxed{8.9\ \frac{\text{m}}{\text{s}^2}}$$

Example 5:

A car accelerates uniformly around a circular path of radius 100 m. If the car accelerates from 10 m/s to 20 m/s in 2.0 s, find the total acceleration of the car at $t = 1.0$ s.

Example 5:

$$r = 100\text{ m},\ v_0 = 10\ \frac{\text{m}}{\text{s}},\ v = 20\ \frac{\text{m}}{\text{s}},\text{ when } t = 2\text{ s},\ a_{tot} = ?\text{ when } t = 1\text{ s}$$

$$v = at + v_0 \text{ so } a_{tan} = \frac{v - v_0}{t} = \frac{20\ \frac{\text{m}}{\text{s}} - 10\ \frac{\text{m}}{\text{s}}}{2\text{ s}} = 5\ \frac{\text{m}}{\text{s}^2}$$

$$v = at + v_0 \text{ so } t = 1\text{ s},\ v = \left(5\ \frac{\text{m}}{\text{s}^2}\right)(1\text{ s}) + 10\ \frac{\text{m}}{\text{s}} = 15\ \frac{\text{m}}{\text{s}}$$

$$a_{rad} = \frac{v^2}{r} \text{ so } a_{rad} = \frac{\left(15\ \frac{\text{m}}{\text{s}}\right)^2}{100\text{ m}} = 2.25\ \frac{\text{m}}{\text{s}^2}$$

$$a = \sqrt{a_{rad}^2 + a_{tan}^2} \qquad a = \sqrt{\left(2.25\ \frac{\text{m}}{\text{s}^2}\right)^2 + \left(5\ \frac{\text{m}}{\text{s}^2}\right)^2} = \boxed{5.5\ \frac{\text{m}}{\text{s}^2}}$$