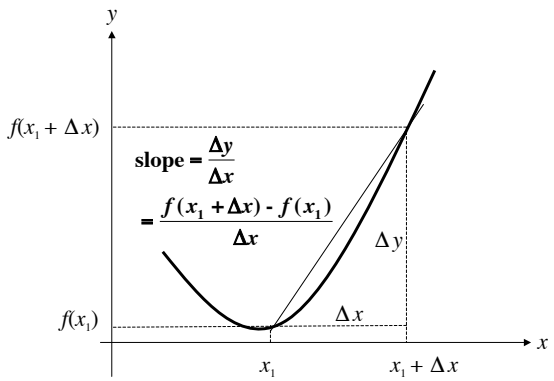
Derivative of a Function

The derivative of a function $y = f(x)$ with respect to x at the point $x = x_1$ is defined as

$$\frac{df(x_1)}{dx} = f'(x_1) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

This essentially determines the slope of a curve at that particular point.

Derivative of a Function



Example

$$f(x) = ax$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a(x + \Delta x) - ax}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{ax + a\Delta x - ax}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} a = \boxed{a}$$

Example

$$f(x) = ax^2$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a(x + \Delta x)^2 - ax^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a(x^2 + 2x\Delta x + (\Delta x)^2) - ax^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{ax^2 + 2ax\Delta x + a(\Delta x)^2 - ax^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2ax\Delta x + a(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 2ax + a\Delta x = \boxed{2ax}$$

Example

$$f(x) = ax^3$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a(x + \Delta x)^3 - ax^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3) - ax^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{ax^3 + 3ax^2\Delta x + 3ax(\Delta x)^2 + a(\Delta x)^3 - ax^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3ax^2\Delta x + 3ax(\Delta x)^2 + a(\Delta x)^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 3ax^2 + 3ax\Delta x + a(\Delta x)^2 = \boxed{3ax^2}$$

To Summarize

$f(x)$	$f'(x)$
ax	a
ax^2	$2ax$
ax^3	$3ax^2$
.	.
.	.
.	.
ax^n	nax^{n-1}
a	0