Example 1:

Rat travels east on a straight road for 40 km at 20 km/h. She then continues in the same direction for 90 km at 30 km/h.

a.) What is Rat's displacement for the entire trip?

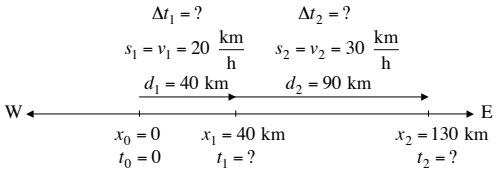
b.) How much distance does Rat cover for the entire trip?

c.) What is Rat's average speed for the entire trip?

d.) What is Rat's average velocity for the entire trip?

Example 1:

$d_1 = 40$ km, $\bar{v}_1 = 20\ \frac{\text{km}}{\text{h}}$ East, $d_2 = 90$ km, and $\bar{v}_2 = 30\ \frac{\text{km}}{\text{h}}$ East



a.) $\Delta x = ?$ (for the entire trip)

$$\Delta x = x_2 - x_0$$

$$\Delta x = 130\text{ km} - 0$$

$$\boxed{\Delta x = 130\text{ km}}$$

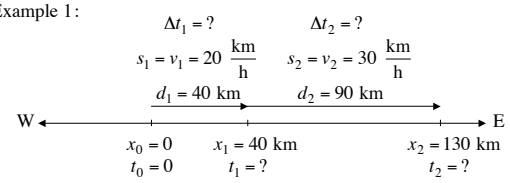
b.) $d = ?$ (for the entire trip)

$$d = d_1 + d_2$$

$$d = 40\text{ km} + 90\text{ km}$$

$$\boxed{d = 130\text{ km}}$$

Example 1:



c.) $s = ?$ (for the entire trip)

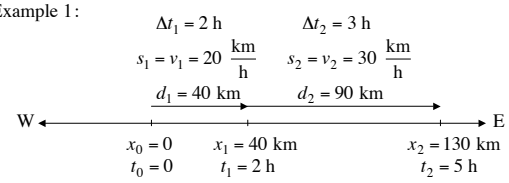
$$s = \frac{d}{t} = \frac{d}{\Delta t_1 + \Delta t_2}$$

$$s_1 = \frac{d_1}{\Delta t_1} \qquad s_2 = \frac{d_2}{\Delta t_2} \qquad s = \frac{d}{\Delta t_1 + \Delta t_2}$$

$$\Delta t_1 = \frac{d_1}{s_1} = \frac{40\text{ km}}{20\ \frac{\text{km}}{\text{h}}} \qquad \Delta t_2 = \frac{d_2}{s_2} = \frac{90\text{ km}}{30\ \frac{\text{km}}{\text{h}}} \qquad s = \frac{130\text{ km}}{2\text{ h} + 3\text{ h}}$$

$$\Delta t_1 = 2\text{ h} \qquad \Delta t_2 = 3\text{ h} \qquad \boxed{s = 26\ \frac{\text{km}}{\text{h}}}$$

Example 1:



d.) $v_{av} = ?$ (for the entire trip)

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_0}{t_2 - t_0}$$

$$v_{av} = \frac{130\text{ km} - 0}{5\text{ h} - 0}$$

$$\boxed{v_{av} = 26\ \frac{\text{km}}{\text{h}}}$$

Example 2:

Larry travels east on a straight road for 40 km at 20 km/h. He then travels west for 90 km at 30 km/h.

a.) What is Larry's displacement for the entire trip?

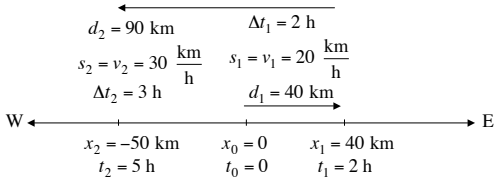
b.) How much distance does Larry cover for the entire trip?

c.) What is Larry's average velocity for the entire trip?

d.) What is Larry's average speed for the entire trip?

Example 2:

$d_1 = 40$ km, $\bar{v}_1 = 20\ \frac{\text{km}}{\text{h}}$ East, $d_2 = 90$ km, and $\bar{v}_2 = 30\ \frac{\text{km}}{\text{h}}$ West



a.) $\Delta x = ?$ (for the entire trip)

$$\Delta x = x_2 - x_0$$

$$\Delta x = -50\text{ km} - 0$$

$$\boxed{\Delta x = -50\text{ km}}$$

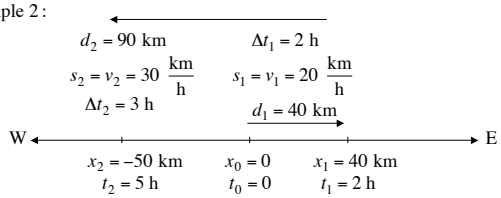
b.) $d = ?$ (for the entire trip)

$$d = d_1 + d_2$$

$$d = 40\text{ km} + 90\text{ km}$$

$$\boxed{d = 130\text{ km}}$$

Example 2 :



c.) s = ? (for the entire trip)

$$s = \frac{d}{\Delta t_1 + \Delta t_2}$$

$$s = \frac{130 \text{ km}}{2 \text{ h} + 3 \text{ h}}$$

$$\boxed{s = 26 \frac{\text{km}}{\text{h}}}$$

d.) v_{av} = ? (for the entire trip)

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_0}{t_2 - t_0}$$

$$v_{av} = \frac{-50 \text{ km} - 0}{5 \text{ h} - 0}$$

$$\boxed{v_{av} = -10 \frac{\text{km}}{\text{h}}}$$

Example 3:

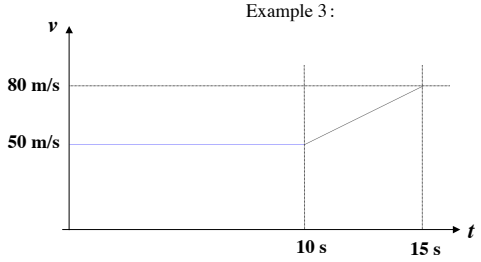
A particle travels in the positive x direction for 10 s at a constant speed of 50 m/s. It then accelerates uniformly to a speed of 80 m/s in the next 5 s.

a.) Find its average acceleration in the interval $t = 0$ s to $t = 15$ s.

b.) Find the total displacement of the particle between $t = 0$ s and $t = 15$ s.

c.) Draw v-t, a-t, and x-t profiles for the entire motion.

Example 3 :



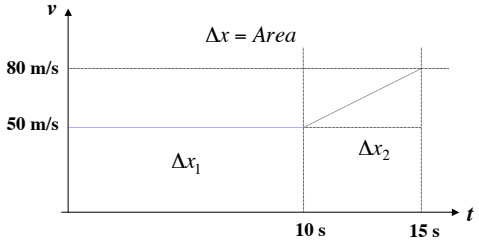
a.) a_{av} = ? ($0 < t < 15$ s)

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{80 \frac{\text{m}}{\text{s}} - 50 \frac{\text{m}}{\text{s}}}{15 \text{ s} - 0}$$

$$\boxed{a_{av} = 2 \frac{\text{m}}{\text{s}^2}}$$

Example 3 :

b.) Δx = ? ($0 < t < 15$ s)



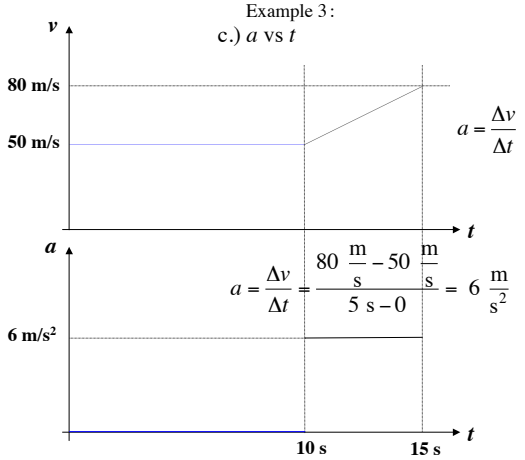
$$\Delta x_1 = (10 \text{ s})\left(50 \frac{\text{m}}{\text{s}}\right) = 500 \text{ m}$$

$$\Delta x_2 = \frac{1}{2}(5 \text{ s})\left(30 \frac{\text{m}}{\text{s}}\right) + (5 \text{ s})\left(50 \frac{\text{m}}{\text{s}}\right) = 325 \text{ m}$$

$$\Delta x = 500 \text{ m} + 325 \text{ m} = \boxed{825 \text{ m}}$$

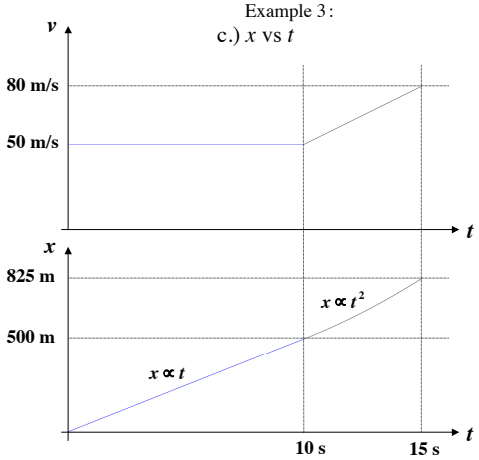
Example 3 :

c.) a vs t



Example 3 :

c.) x vs t



Example 4:

A hockey puck sliding on a frozen lake comes to rest after traveling 200 m. Its initial velocity is 3 m/s.

a.) What is its acceleration if its acceleration is assumed constant?

b.) How long is the puck in motion?

c.) What is its speed after traveling 150 m?

Example 4 :

$$v_0 = 3\ \frac{\text{m}}{\text{s}},\ v = 0,\ \text{and}\ \Delta x = 200\ \text{m}$$

a.) $a = ?$

$$v^2 = v_0^2 + 2a\Delta x$$

$$a = \frac{v^2 - v_0^2}{2\Delta x}$$

$$a = \frac{0 - \left(3\ \frac{\text{m}}{\text{s}}\right)^2}{2(200\ \text{m})}$$

$$\boxed{a = -0.0225\ \frac{\text{m}}{\text{s}^2}}$$

Example 4 :

$$v_0 = 3\ \frac{\text{m}}{\text{s}},\ v = 0,\ \text{and}\ \Delta x = 200\ \text{m}$$

b.) $t = ?$

$$\Delta x = \left(\frac{v + v_0}{2}\right)t$$

$$t = \frac{2\Delta x}{v + v_0}$$

$$t = \frac{2(200\ \text{m})}{0 + 3\ \frac{\text{m}}{\text{s}}}$$

$$\boxed{t = 133\ \text{s}}$$

Example 4 :

$$v_0 = 3\ \frac{\text{m}}{\text{s}},\ a = -0.0225\ \frac{\text{m}}{\text{s}^2},\ \text{and}\ \Delta x = 150\ \text{m}$$

c.) $v = ?$

$$v^2 = v_0^2 + 2a\Delta x$$

$$v = \sqrt{v_0^2 + 2a\Delta x}$$

$$v = \sqrt{\left(3\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(-0.0225\ \frac{\text{m}}{\text{s}^2}\right)(150\ \text{m})}$$

$$\boxed{v = 1.5\ \frac{\text{m}}{\text{s}}}$$

Example 5:

A cart with an initial velocity of 5.0 m/s experiences a constant acceleration of 2.0 m/s^2. What is the cart's displacement during the first 6.0 s of its motion?

Example 5 :

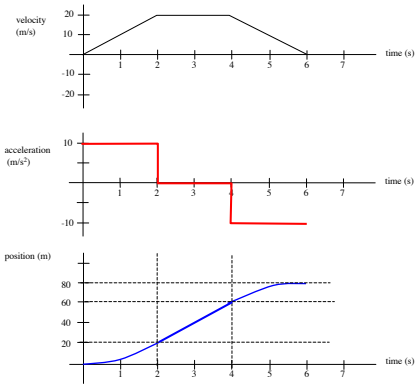
$$v_0 = 5.0\ \frac{\text{m}}{\text{s}},\ a = 2.0\ \frac{\text{m}}{\text{s}^2},\ \text{and}\ t = 6.0\ \text{s}$$

$$\Delta x = ?$$

$$\Delta x = \frac{1}{2}at^2 + v_0 t$$

$$\Delta x = \frac{1}{2}\left(2.0\ \frac{\text{m}}{\text{s}^2}\right)(6.0\ \text{s})^2 + \left(5.0\ \frac{\text{m}}{\text{s}}\right)(6.0\ \text{s})$$

$$\boxed{\Delta x = 66\ \text{m}}$$



Example 6:

An object's position varies with time and is given by:

$$x(t) = 2t^3 + 3t + 5$$

a.) Find the average velocity between 0 and 1 second.

b.) Find the average acceleration between 0 and 1 second.

c.) Find the instantaneous velocity at $t = 3$ s.

d.) Find the instantaneous acceleration at $t = 6$ s.

Example 6:

$$x(t) = 2t^3 + 3t + 5$$

a.) $v_{av} = ? \ (0 < t < 1 \text{ s})$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x(1) - x(0)}{t_1 - 0}$$

$$x(1) = 2(1)^3 + 3(1) + 5 = 10 \text{ m}$$

$$x(0) = 2(0)^3 + 3(0) + 5 = 5 \text{ m}$$

$$v_{av} = \frac{10 \text{ m} - 5 \text{ m}}{1 \text{ s} - 0}$$

$$\boxed{v_{av} = 5 \ \frac{\text{m}}{\text{s}}}$$

Example 6:

$$x(t) = 2t^3 + 3t + 5$$

b.) $a_{av} = ? \ (0 < t < 1 \text{ s})$

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v(1) - v(0)}{t_1 - 0}$$

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(2t^3 + 3t + 5\right) = 6t^2 + 3$$

$$v(1) = 6(1)^2 + 3 = 9 \ \frac{\text{m}}{\text{s}}$$

$$v(0) = 6(0)^2 + 3 = 3 \ \frac{\text{m}}{\text{s}}$$

$$a_{av} = \frac{9 \ \frac{\text{m}}{\text{s}} - 3 \ \frac{\text{m}}{\text{s}}}{1 \text{ s} - 0}$$

$$\boxed{a_{av} = 6 \ \frac{\text{m}}{\text{s}^2}}$$

Example 6:

$$x(t) = 2t^3 + 3t + 5$$

c.) $v(3) = ?$

$$v(t) = 6t^2 + 3$$

$$v(3) = 6(3)^2 + 3$$

$$\boxed{v(3) = 57 \ \frac{\text{m}}{\text{s}}}$$

d.) $a(6) = ?$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(6t^2 + 3\right) = 12t$$

$$a(6) = 12(6)$$

$$\boxed{a(6) = 72 \ \frac{\text{m}}{\text{s}^2}}$$

Example 7:

An object's acceleration varies with time and is given by:

$$a(t) = 12t$$

If $v(0) = 3$ m/s and $x(0) = 5$ m, find:

a.) $v(t)$

b.) $x(t)$

Example 7 :

$$a(t) = 12t$$

$$v(0) = 3\ \frac{\text{m}}{\text{s}} \text{ and } x(0) = 5\ \text{m}$$

a.) $v(t) = ?$

$$v(t) = \int a\,dt = \int (12t)\,dt$$

$$v(t) = \frac{12t^2}{2} + C_1 = 6t^2 + C_1$$

$$v(0) = 3\ \frac{\text{m}}{\text{s}} = 6(0)^2 + C_1$$

$$C_1 = 3\ \frac{\text{m}}{\text{s}}$$

$$\boxed{v(t) = 6t^2 + 3}$$

b.) $x(t) = ?$

$$x(t) = \int v\,dt = \int \left(6t^2 + 3\right) dt$$

$$x(t) = \frac{6t^3}{3} + \frac{3t^1}{1} + C_2 = 2t^3 + 3t + C_2$$

$$x(0) = 5\ \text{m} = 2(0)^3 + 3(0) + C_2$$

$$C_2 = 5\ \text{m}$$

$$\boxed{x(t) = 2t^3 + 3t + 5}$$

Example 7 :

Check Results :

$$x(t) = 2t^3 + 3t + 5$$

$$x(0) = 2(0)^3 + 3(0) + 5 = 5\ \text{m}\ \surd$$

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left(2t^3 + 3t + 5\right)$$

$$v(t) = 6t^2 + 3\ \surd$$

$$v(0) = 6(0)^2 + 3 = 3\ \frac{\text{m}}{\text{s}}\ \surd$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left(6t^2 + 3\right)$$

$$a(t) = 12t\ \surd$$