Example 1:

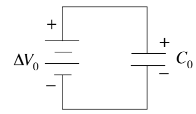
A parallel plate capacitor has plates which have an area of 2000 cm^2 and are 1 cm apart. The potential difference $V_o = 3000$ V. It is then disconnected from the power supply, and a sheet of insulating plastic material is inserted between the plates, completely filling the space between them. The potential difference decreases to 1000 V while the charge on the plates remains constant.

Compute:

a.) the original capacitance C_o
b.) the magnitude of the charge on each plate
c.) the capacitance C after the dielectric is inserted
d.) the dielectric constant κ of the dielectric
e.) the permittivity ε of the dielectric
f.) the magnitude of the induced charge Q_i on each face of the dielectric
g.) the original electric field E_o between the plates
h.) the electric field E after the dielectric is inserted

Example 1:

$A = 0.2\ \text{m}^2,\ d = 0.01\ \text{m},\ \Delta V_0 = 3000\ \text{V}$



a.) $C_0 = ?$

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.2\ \text{m}^2)}{0.01\ \text{m}}$$

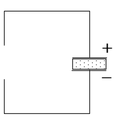
$$\boxed{C_0 = 1.77\times10^{-10}\ \text{F} = 177\ \text{pF}}$$

b.) $Q_0 = ?$ $\quad C = \frac{Q}{\Delta V}$ so $Q = C\Delta V$

$$Q_0 = C_0 V_0 = \left(1.77\times10^{-10}\ \text{F}\right)(3000\ \text{V})$$

$$\boxed{Q_0 = 5.31\times10^{-7}\ \text{C} = 531\ \text{nC}}$$

Example 1:



$+\ Q = Q_0 = 531$ nC
$-\ \Delta V = 1000$ V

c.) $C = ?$

$$C = \frac{Q}{\Delta V} = \frac{5.31\times10^{-7}\ \text{C}}{1000\ \text{V}}$$

$$\boxed{C = 5.31\times10^{-10}\ \text{F} = 531\ \text{pF}}$$

d.) $\kappa = ?$

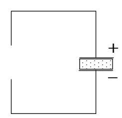
$$\kappa = \frac{C}{C_0} = \frac{531\ \text{pF}}{177\ \text{pF}}$$

$$\boxed{\kappa = 3.0}$$

e.) $\varepsilon = ?$ $\quad \varepsilon = \kappa\varepsilon_0 = 3.0\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)$

$$\boxed{\varepsilon = 2.66\times10^{-11}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}}$$

Example 1:



$+\ Q = Q_0 = 531$ nC
$-\ \Delta V = 1000$ V
$\kappa = 3.0$

f.) $Q_i = ?$

$$\sigma_i = \sigma\left(1 - \frac{1}{\kappa}\right)$$

$$\sigma_i A = \sigma A\left(1 - \frac{1}{\kappa}\right)$$

$$Q_i = Q\left(1 - \frac{1}{\kappa}\right) \quad \left(\sigma = \frac{Q}{A}\right)$$

$$Q_i = 531\ \text{nC}\left(1 - \frac{1}{3.0}\right)$$

$$\boxed{Q_i = 354\ \text{nC}}$$

g.) $E_0 = ?$

$$E_0 = \frac{\Delta V_0}{d} = \frac{3000\ \text{V}}{0.01\ \text{m}}$$

$$\boxed{E_0 = 3\times10^5\ \frac{\text{V}}{\text{m}}}$$

h.) $E = ?$

$$E = \frac{\Delta V}{d} = \frac{1000\ \text{V}}{0.01\ \text{m}}$$

$$\boxed{E = 1\times10^5\ \frac{\text{V}}{\text{m}}}$$

Example 2:

Suppose the capacitor in Example 1 remains connected to the 3000 V power supply while an insulating plastic sheet with $\kappa = 3.0$ is inserted between the plates.

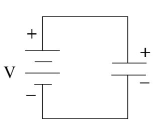
Compute:

a.) the magnitude of the charge Q on each plate after the dielectric is inserted.
b.) the magnitude of the induced charged Q_i on each face of the dielectric.
c.) the electric field E after the dielectric is inserted.
d.) the total energy before and after the dielectric is inserted.

Example 2:

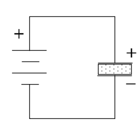
$A = 0.2\ \text{m}^2,\ d = 0.01\ \text{m},\ \Delta V_0 = 3000\ \text{V}$

$\Delta V_0 = 3000$ V



$+\ C_0 = 177$ pF
$-\ Q_0 = 531$ nC

$\Delta V = 3000$ V



$\kappa = 3.0$

a.) $Q = ?$

$$C = \kappa C_0 = 3.0(177\ \text{pF})$$
$$C = 531\ \text{pF}$$

$$Q = C\Delta V = \left(5.31\times10^{-10}\ \text{F}\right)(3000\ \text{V})$$

$$\boxed{Q = 1.593\times10^{-6}\ \text{C} = 1593\ \text{nC}}$$

b.) $Q_i = ?$ $\quad Q_i = Q\left(1 - \frac{1}{\kappa}\right)$

$$Q_i = 1593\ \text{nC}\left(1 - \frac{1}{3.0}\right)$$

$$\boxed{Q_i = 1062\ \text{nC}}$$

c.) $E = ?$ $\quad E = \frac{\Delta V}{d} = \frac{3000\ \text{V}}{0.01\ \text{m}}$

$$\boxed{E = 3\times10^5\ \frac{\text{V}}{\text{m}}}$$

Example 2:

d.) $U_C = ?$

$$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2$$

before dielectric:

$$U_C = \frac{1}{2}C\Delta V^2 = \frac{1}{2}\left(177\times10^{-12}\text{ F}\right)\left(3000\text{ V}\right)^2$$

$$\boxed{U_C = 7.965\times10^{-4}\text{ J} = 0.7965\text{ mJ}}$$

after dielectric:

$$U_C = \frac{1}{2}C\Delta V^2 = \frac{1}{2}\left(531\times10^{-12}\text{ F}\right)\left(3000\text{ V}\right)^2$$

$$\boxed{U_C = 2.39\times10^{-3}\text{ J} = 2.39\text{ mJ}}$$

Example 3:

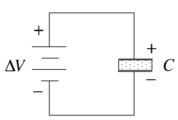
Two parallel plates, each with an area of 40 cm^2, are given opposite charges of magnitude 1.80 x 10^{-7} C. The space between the plates is filled with a dielectric, and the electric field within the dielectric is 3.40 x 10^5 V/m.

a.) What is the dielectric constant?

b.) What is the induced charge on either face of the dielectric?

Example 3:

$$A = 0.40\text{ cm}^2,\ Q = 1.80\times10^{-7}\text{ C},\ E = 3.40\times10^5\ \frac{\text{V}}{\text{m}}$$



a.) $\kappa = ?$

without dielectric:

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{1.80\times10^{-7}\text{ C}}{\left(0.004\text{ m}^2\right)\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}$$

$$E_0 = 5.085\times10^6\ \frac{\text{V}}{\text{m}}$$

$$\kappa = \frac{E_0}{E} = \frac{5.085\times10^6\ \frac{\text{V}}{\text{m}}}{3.40\times10^5\ \frac{\text{V}}{\text{m}}} \quad\text{and}\quad \boxed{\kappa = 15}$$

b.) $Q_i = ?$

$$Q_i = Q\left(1-\frac{1}{\kappa}\right) = \left(1.80\times10^{-7}\text{ C}\right)\left(1-\frac{1}{15}\right)$$

$$\boxed{Q_i = 1.68\times10^{-7}\text{ C}}$$

Example 4:

Two parallel plates have charges Q and $-Q$. When the space between the plates is devoid of matter, the electric field is 2.4 x 10^5 V/m. When the space is filled with a certain dielectric, the field is reduced to 1.2 x 10^5 V/m.

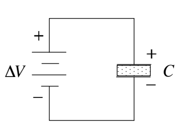
a.) What is the dielectric constant of the dielectric?

b.) If Q = 10 nC, what is the area of the plates?

c.) What is the total induced charge on either face of the dielectric?

Example 4:

$$E_0 = 2.4\times10^5\ \frac{\text{V}}{\text{m}}\ \text{and}\ E = 1.2\times10^5\ \frac{\text{V}}{\text{m}}$$



a.) $\kappa = ?$

$$\kappa = \frac{E_0}{E} = \frac{2.4\times10^5\ \frac{\text{V}}{\text{m}}}{1.2\times10^5\ \frac{\text{V}}{\text{m}}} \quad\text{and}\quad \boxed{\kappa = 2.0}$$

b.) $Q = 10\text{ nC},\ A = ?$

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}\quad\text{so}\quad A = \frac{Q}{\varepsilon_0 E_0} = \frac{10\times10^{-9}\text{ C}}{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(2.4\times10^5\ \frac{\text{V}}{\text{m}}\right)}$$

$$\boxed{A = 4.71\times10^{-3}\text{ m}^2}$$

$$[=]\frac{\text{C}}{\left(\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{\text{V}}{\text{m}}\right)}$$

$$[=]\frac{\text{C}}{\left(\frac{\text{C}^2}{\text{J}\cdot\text{m}}\right)\left(\frac{\text{J}}{\text{C}\cdot\text{m}}\right)}$$

c.) $Q_i = ?$

$$Q_i = Q\left(1-\frac{1}{\kappa}\right) = \left(10\times10^{-9}\text{ C}\right)\left(1-\frac{1}{2}\right)$$

$$\boxed{Q_i = 5\times10^{-9}\text{ C}}$$

Example 4:

$$E_0 = 2.4\times10^5\ \frac{\text{V}}{\text{m}}\ \text{and}\ E = 1.2\times10^5\ \frac{\text{V}}{\text{m}}$$

c.) $Q_i = ?$

$$E = \frac{\sigma-\sigma_i}{\varepsilon_0}$$

$$E\varepsilon_0 = \sigma - \sigma_i$$

$$\sigma_i = \sigma - E\varepsilon_0$$

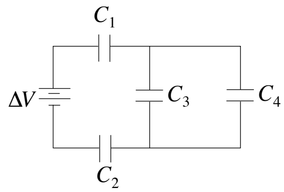
$$\sigma_i A = \sigma A - E\varepsilon_0 A$$

$$Q_i = Q - E\varepsilon_0 A$$

$$Q_i = 10\times10^{-9}\text{ C} - \left(1.2\times10^5\ \frac{\text{V}}{\text{m}}\right)\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(4.71\times10^{-3}\text{ m}^2\right)$$

$$\boxed{Q_i = 5\times10^{-9}\text{ C}}$$

Example 5:



In the above circuit $C_1 = 40\ \mu\text{F}$, $C_2 = 60\ \mu\text{F}$, $C_3 = 50\ \mu\text{F}$, $C_4 = 70\ \mu\text{F}$, and the potential difference $\Delta V = 120$ V.

a.) Find the equivalent capacitance for the circuit.

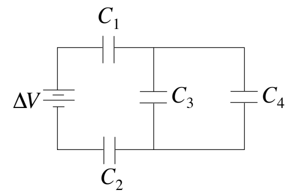
b.) The voltage across and charge stored in each capacitor.

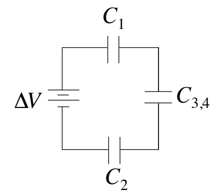
c.) The total electric energy stored in the circuit.

Example 5:

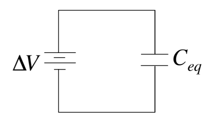
$\Delta V = 120$ V, $C_1 = 40\ \mu\text{F}$, $C_2 = 60\ \mu\text{F}$, $C_3 = 50\ \mu\text{F}$, and $C_4 = 70\ \mu\text{F}$

a.) $C_{eq} = ?$





$$C_{3,4} = C_3 + C_4$$
$$C_{3,4} = 50\ \mu\text{F} + 70\ \mu\text{F}$$
$$C_{3,4} = 120\ \mu\text{F}$$

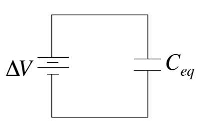


$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_{3,4}}\right)^{-1}$$
$$C_{eq} = \left(\frac{1}{40\ \mu\text{F}} + \frac{1}{60\ \mu\text{F}} + \frac{1}{120\ \mu\text{F}}\right)^{-1}$$
$$\boxed{C_{eq} = 20\ \mu\text{F}}$$

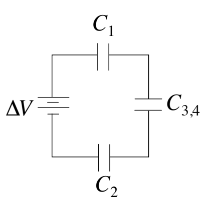
Example 5:

$\Delta V = 120$ V, $C_1 = 40\ \mu\text{F}$, $C_2 = 60\ \mu\text{F}$, $C_3 = 50\ \mu\text{F}$, and $C_4 = 70\ \mu\text{F}$

b.) Q's and ΔV's $= ?$ $\quad C = \frac{Q}{\Delta V}$ so $Q = C\Delta V$ and $\Delta V = \frac{Q}{C}$



$$Q_{eq} = C_{eq}\Delta V$$
$$Q_{eq} = (20\ \mu\text{F})(120\ \text{V})$$
$$Q_{eq} = 2400\ \mu\text{C}$$



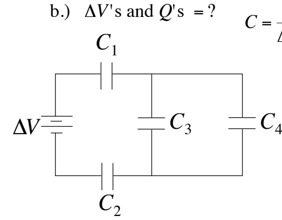
$$Q_1 = Q_2 = Q_{3,4} = Q_{eq}$$
$$\boxed{Q_1 = Q_2 = 2400\ \mu\text{C}} \text{ and } Q_{3,4} = 2400\ \mu\text{C}$$

$$\Delta V_1 = \frac{Q_1}{C_1} \qquad \Delta V_2 = \frac{Q_2}{C_2} \qquad \Delta V_{3,4} = \frac{Q_{3,4}}{C_{3,4}}$$
$$\Delta V_1 = \frac{2400\ \mu\text{C}}{40\ \mu\text{F}} \qquad \Delta V_2 = \frac{2400\ \mu\text{C}}{60\ \mu\text{F}} \qquad \Delta V_{3,4} = \frac{2400\ \mu\text{C}}{120\ \mu\text{F}}$$
$$\boxed{\Delta V_1 = 60\ \text{V}} \qquad \boxed{\Delta V_2 = 40\ \text{V}} \qquad \Delta V_{3,4} = 20\ \text{V}$$

Example 5:

$\Delta V = 120$ V, $C_1 = 40\ \mu\text{F}$, $C_2 = 60\ \mu\text{F}$, $C_3 = 50\ \mu\text{F}$, and $C_4 = 70\ \mu\text{F}$

b.) ΔV's and Q's $= ?$ $\quad C = \frac{Q}{\Delta V}$ so $Q = C\Delta V$ and $\Delta V = \frac{Q}{C}$



$$\Delta V_3 = \Delta V_4 = \Delta V_{3,4}$$
$$\boxed{\Delta V_3 = \Delta V_4 = 20\ \text{V}}$$

$$Q_3 = C_3\Delta V_3$$
$$Q_3 = (50\ \mu\text{F})(20\ \text{V})$$
$$\boxed{Q_3 = 1000\ \mu\text{C}}$$

$$Q_4 = C_4\Delta V_4$$
$$Q_4 = (70\ \mu\text{F})(20\ \text{V})$$
$$\boxed{Q_4 = 1400\ \mu\text{C}}$$

c.) $U_C = ?$

$$U_C = \frac{1}{2}C_{eq}(\Delta V)^2$$
$$U_C = \frac{1}{2}(20\ \mu\text{F})(120\ \text{V})^2$$
$$\boxed{U_C = 0.144\ \text{J}}$$

Example 5:

$\Delta V = 120$ V, $C_1 = 40\ \mu\text{F}$, $C_2 = 60\ \mu\text{F}$, $C_3 = 50\ \mu\text{F}$, and $C_4 = 70\ \mu\text{F}$

c.) $U_C = ?$

$$U_{C_1} = \frac{1}{2}C_1(\Delta V_1)^2 \qquad U_{C_2} = \frac{1}{2}C_2(\Delta V_2)^2$$
$$U_{C_1} = \frac{1}{2}(40\ \mu\text{F})(60\ \text{V})^2 \qquad U_{C_2} = \frac{1}{2}(60\ \mu\text{F})(40\ \text{V})^2$$
$$U_{C_1} = 0.072\ \text{J} \qquad U_{C_2} = 0.048\ \text{J}$$
$$U_{C_3} = \frac{1}{2}C_3(\Delta V_3)^2 \qquad U_{C_4} = \frac{1}{2}C_4(\Delta V_4)^2$$
$$U_{C_3} = \frac{1}{2}(50\ \mu\text{F})(20\ \text{V})^2 \qquad U_{C_4} = \frac{1}{2}(70\ \mu\text{F})(20\ \text{V})^2$$
$$U_{C_3} = 0.010\ \text{J} \qquad U_{C_4} = 0.014\ \text{J}$$

$$U_{C_1} + U_{C_2} + U_{C_3} + U_{C_4} = 0.072\ \text{J} + 0.048\ \text{J} + 0.010\ \text{J} + 0.014\ \text{J} = 0.144\ \text{J}$$