Capacitance and Dielectrics

Capacitance

A *capacitor* is a charge storage device. It stores energy as potential energy in an electric field. A capacitor consists of two nontouching conductors, which hold equal but opposite charges.

Capacitance

***Capacitance* (*C*) is the ratio of the charge *Q* to the potential difference Δ*V* between the conductors.**

$$C = \frac{Q}{\Delta V} \quad \text{or} \quad Q = C\Delta V \qquad \boxed{\Delta V = \frac{Q}{C}}$$

Capacitance has units of coulombs per volt, which is defined to be a farad (F).

The capacitance is constant for a given capacitor, and depends only upon the structure and dimensions of the device.

Parallel-Plate Capacitors

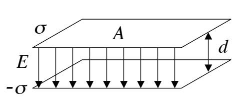
A *parallel-plate capacitor* consists of two conductive plates with area *A*, separated by a distance *d*. The capacitance is given by :

$$C = \frac{\varepsilon_o A}{d}$$

ε_o - the permittivity of a vacuum (ε_o = 8.85 x 10^{-12} $C^2/N\cdot m^2$)
***A* - area of one plate (m^2)**
***d* - spacing between plates (m)**
***C* - capacitance (F)**

Parallel-Plate Capacitors

Parallel conducting sheets total area A, spacing d, and charge Q



$$E = \frac{\Delta V}{d}$$

Between the sheets:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

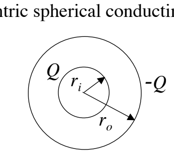
$$\Delta V = V_+ - V_- = Ed \quad \text{(uniform field)}$$

$$\Delta V = \frac{\sigma}{\varepsilon_0}d = \frac{Q}{\varepsilon_0 A}d$$

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

Spherical Capacitors

Concentric spherical conducting shells



Between the spheres:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\Delta V = V_i - V_o = -\int_{r_o}^{r_i} \vec{E}\cdot d\vec{r}$$

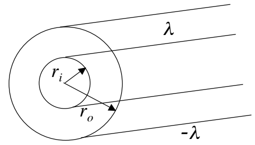
$$\Delta V = -\int_{r_o}^{r_i} \frac{Q}{4\pi\varepsilon_0 r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)$$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_o - r_i}{r_i r_o}\right)$$

$$C = \frac{Q}{\Delta V} = 4\pi\varepsilon_0\left(\frac{r_i r_o}{r_o - r_i}\right)$$

Cylindrical Capacitors

Coaxial cylindrical conducting shells total length L and charge Q



Between the cylinders:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E2\pi r\ell = \frac{\lambda\ell}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

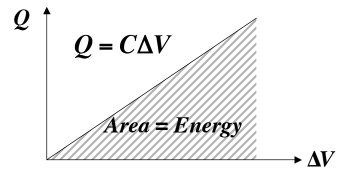
$$\Delta V = V_i - V_o = -\int_{r_o}^{r_i} \vec{E}\cdot d\vec{r}$$

$$\Delta V = -\int_{r_o}^{r_i} \frac{\lambda}{2\pi\varepsilon_0 r}dr \;=\; \frac{\lambda}{2\pi\varepsilon_0}\left(\ln(r_o) - \ln(r_i)\right)$$

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_o}{r_i}\right) \;=\; \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{r_o}{r_i}\right)$$

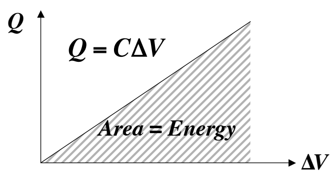
$$C = \frac{Q}{\Delta V} \;=\; \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{r_o}{r_i}\right)}$$

Energy and Capacitance



Check units: $(\mathrm{C})\cdot\left(\frac{\mathrm{J}}{\mathrm{C}}\right)\,[=]\,\mathrm{J}$

Energy and Capacitance



The electric potential energy stored in a charged capacitor is:

$$\boxed{U_C = \frac{1}{2}Q\Delta V \;=\; \frac{1}{2}C(\Delta V)^2} = \frac{1}{2}\frac{Q^2}{C}$$

Capacitors with Dielectric Fillers

When the space between the conductors is filled with a dielectric (nonconducting) material other than air the capacitance increases by a factor κ called the dielectric constant of the material.

$$\kappa = \frac{C}{C_o}$$

For a parallel-plate capacitor, the capacitance becomes:

$$C = \kappa C_o = \frac{\kappa\varepsilon_o A}{d} \;=\; \frac{\varepsilon A}{d} \qquad \boxed{C = \frac{\kappa\varepsilon_o A}{d}}$$

Capacitors with Dielectric Fillers

Dielectrics serve three functions:

1.) **Provide the mechanical support between the plates.**
2.) **Increase the maximum possible potential difference between the plates. (Air ionizes at 3 x 10^6 V/m.)**
3.) **Increase the capacitance of a capacitor with given dimensions.**

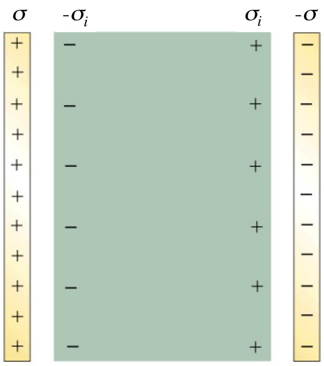
More on Dielectrics

Inserting a dielectric material decreases the electric field between the plates.

$$E = \frac{E_o}{\kappa}$$

Surface charge on the conducting plates does not change, but an *induced* charge of the opposite sign appears on each surface of dielectric.

Capacitors with Dielectric Fillers



$$E_0 = \frac{\sigma}{\varepsilon_0} \qquad E = \frac{\sigma - \sigma_i}{\varepsilon_0}$$

$$E < E_0$$

$$\Delta V = Ed$$

$$\Delta V < \Delta V_0$$

$$C = \frac{Q}{\Delta V}$$

$$C > C_0$$

Still More on Dielectrics

The induced charge can be found by comparing the electric field with and without the dielectric.

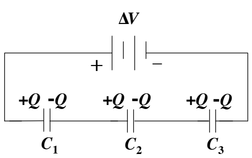
$$E_o = \frac{\sigma}{\varepsilon_o}, \qquad E = \frac{\sigma - \sigma_i}{\varepsilon_o} = \frac{E_o}{\kappa} = \frac{\sigma}{\kappa \varepsilon_o}$$

The induced surface charge density is:

$$\sigma_i = \sigma\left(1 - \frac{1}{\kappa}\right) \text{ and } \kappa = \frac{\sigma}{\sigma - \sigma_i}$$

Capacitors in Series

- **Charge is the same on each capacitor**
- **Voltage drop across each capacitor is different unless the capacitance is the same**



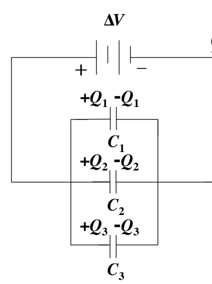
$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$$

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\boxed{\frac{1}{C_s} = \sum_i \frac{1}{C_i}}$$

Capacitors in Parallel

- **Voltage drop is the same across each capacitor**
- **Charge is different on each capacitor, the higher the capacitance the higher the charge**



$$Q = Q_1 + Q_2 + Q_3 = C_1\Delta V + C_2\Delta V + C_3\Delta V$$

$$Q = \left(C_1 + C_2 + C_3\right) \cdot \Delta V$$

$$\Delta V = \frac{Q}{\left(C_1 + C_2 + C_3\right)} = \frac{Q}{C_p}$$

$$\boxed{C_p = \sum_i C_i}$$