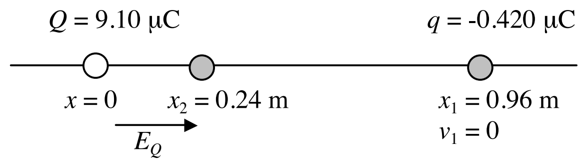
1.) Q = 9.10 μC fixed at origin, q = -0.420 μC and mass of 3.2 x 10^{-4} kg a distance r_1 = 0.960 m from Q.

a.) $$U = k\frac{Qq}{r} = 9 \text{ x } 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\frac{\left(9.1 \text{ x } 10^{-6}\text{C}\right)\left(-0.42 \text{ x } 10^{-6}\text{C}\right)}{0.96 \text{ m}} = \boxed{-0.0358 \text{ J}}$$

b.) r_1 = 0.960 m r_2 = 0.240 m q loses potential energy and gains kinetic energy moving towards Q.



Using Conservation of Energy:

$$K_1 + U_1 = K_2 + U_2$$

$$k\frac{Qq}{x_1} = \frac{1}{2}mv_2^{\,2} + k\frac{Qq}{x_2} \quad \left(K_1 = 0 \text{ since } v_1 = 0\right) \quad \text{so} \quad \frac{1}{2}mv_2^{\,2} = k\frac{Qq}{x_1} - k\frac{Qq}{x_2} = kQq\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$$

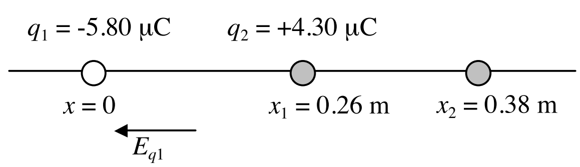
$$v_2 = \sqrt{\frac{2kQq}{m}\left(\frac{1}{x_1} - \frac{1}{x_2}\right)}$$

$$v_2 = \sqrt{\frac{2\left(9 \text{ x } 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(9.1 \text{ x } 10^{-6}\text{C}\right)\left(-0.42 \text{ x } 10^{-6}\text{C}\right)}{3.2 \text{ x } 10^{-4}\text{kg}}\left(\frac{1}{0.96 \text{ m}} - \frac{1}{0.24 \text{ m}}\right)} = \boxed{25.9\ \frac{\text{m}}{\text{s}}}$$

Check units:

$$v_{2f}[=]\sqrt{(\text{m/s})^2 + \frac{\left(\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(\text{C})(\text{C})}{(\text{kg})}\left(\frac{1}{\text{m}}\right)}\ [=]\sqrt{(\text{m/s})^2 + \frac{\left(\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\cdot\text{m}^2}{\text{C}^2}\right)(\text{C})(\text{C})}{(\text{kg})}\left(\frac{1}{\text{m}}\right)}[=]\sqrt{(\text{m/s})^2}\,[=]\ \text{m/s}$$ (units work out)

2.) q_1 = -5.80 μC fixed at x = 0, q_2 = +4.30 μC moves from x= 0.260 m to x = 0.380 m



(q_2 is moving against the field so the field does negative work)

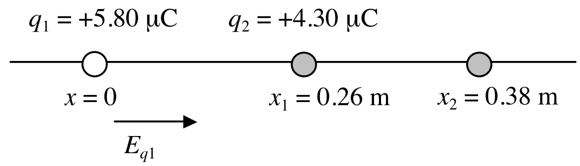
$$W = -\Delta U = -\left(k\frac{q_1q_2}{x_2} - k\frac{q_1q_2}{x_1}\right) = -kq_1q_2\left(\frac{1}{x_2} - \frac{1}{x_1}\right)$$

$$W = -9 \text{ x } 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(-5.8 \text{ x } 10^{-6}\text{C}\right)\left(4.3 \text{ x } 10^{-6}\text{C}\right)\left(\frac{1}{0.38 \text{ m}} - \frac{1}{0.26 \text{ m}}\right) = \boxed{-0.273 \text{ J}}$$

Check units:

$$W\ [=]\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}(\text{C})(\text{C})\left(\frac{1}{\text{m}}\right)\ [=]\ \text{N}\cdot\text{m}\ [=]\ \text{J}$$ (units work out)

3.) $q_1 = +5.80\ \mu\text{C}$ fixed at $x = 0$, $q_2 = +4.30\ \mu\text{C}$ moves from $x = 0.260$ m to $x = 0.380$ m

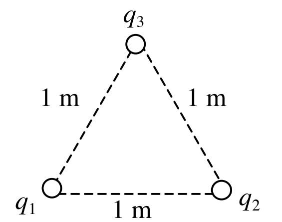


(q_2 is moving with the field so the field does positive work)

$$W = -\Delta U = -\left(k\frac{q_1 q_2}{x_2} - k\frac{q_1 q_2}{x_1}\right) = kq_1q_2\left(\frac{1}{x_2} - \frac{1}{x_1}\right)$$

$$W = -9 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(5.8 \text{ x } 10^{-6}\text{C}\right)\left(4.3 \text{ x } 10^{-6}\text{C}\right)\left(\frac{1}{0.38\text{ m}} - \frac{1}{0.26\text{ m}}\right) = \boxed{+0.273\text{ J}}$$

4.) $q_1 = q_2 = q_3 = 840$ nC fixed on an equilateral triangle with sides of 1.00 m



$$U = \sum_{i<j} k\frac{q_i q_j}{r_{ij}} = k\frac{q_1q_2}{r_{12}} + k\frac{q_1q_3}{r_{13}} + k\frac{q_2q_3}{r_{23}} = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$U = 9 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{\left(840 \text{ x } 10^{-9}\text{C}\right)\left(840 \text{ x } 10^{-9}\text{C}\right)}{1\text{ m}} + \frac{\left(840 \text{ x } 10^{-9}\text{C}\right)\left(840 \text{ x } 10^{-9}\text{C}\right)}{1\text{ m}} + \frac{\left(840 \text{ x } 10^{-9}\text{C}\right)\left(840 \text{ x } 10^{-9}\text{C}\right)}{1\text{ m}}\right) = \boxed{+0.0191\text{ J}}$$

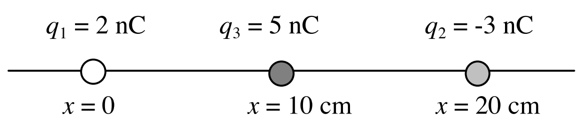
5.) $q_1 = 840$ nC, $q_2 = q_3 = -840$ nC fixed on an equilateral triangle with sides of 1.00 m

$$U = \sum_{i<j} k\frac{q_i q_j}{r_{ij}} = k\frac{q_1q_2}{r_{12}} + k\frac{q_1q_3}{r_{13}} + k\frac{q_2q_3}{r_{23}} = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$U = 9 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{\left(840 \text{ x } 10^{-9}\text{C}\right)\left(-840 \text{ x } 10^{-9}\text{C}\right)}{1\text{ m}} + \frac{\left(840 \text{ x } 10^{-9}\text{C}\right)\left(-840 \text{ x } 10^{-9}\text{C}\right)}{1\text{ m}} + \frac{\left(-840 \text{ x } 10^{-9}\text{C}\right)\left(-840 \text{ x } 10^{-9}\text{C}\right)}{1\text{ m}}\right) = \boxed{-0.00635\text{ J}}$$

6.)

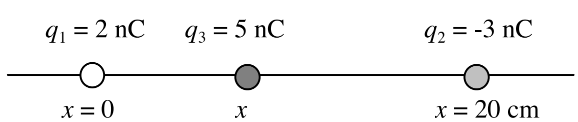
a.)



$$U = \sum_{i<j} k\frac{q_i q_j}{r_{ij}} = k\frac{q_1q_2}{r_{12}} + k\frac{q_1q_3}{r_{13}} + k\frac{q_2q_3}{r_{23}} = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$U = 9 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{\left(2 \text{ x } 10^{-9}\text{C}\right)\left(-3 \text{ x } 10^{-9}\text{C}\right)}{0.2\text{ m}} + \frac{\left(2 \text{ x } 10^{-9}\text{C}\right)\left(5 \text{ x } 10^{-9}\text{C}\right)}{0.1\text{ m}} + \frac{\left(-3 \text{ x } 10^{-9}\text{C}\right)\left(5 \text{ x } 10^{-9}\text{C}\right)}{0.1\text{ m}}\right) = \boxed{-7.2 \text{ x } 10^{-7}\text{ J}}$$

b.)



$$U = \sum_{i<j} k\frac{q_i q_j}{r_{ij}} = k\frac{q_1q_2}{r_{12}} + k\frac{q_1q_3}{r_{13}} + k\frac{q_2q_3}{r_{23}} = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) = 0$$

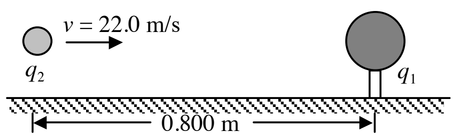
$$9 \text{ x } 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{(2 \text{ x } 10^{-9}\text{C})(-3 \text{ x } 10^{-9}\text{C})}{0.2 \text{ m}} + \frac{(2 \text{ x } 10^{-9}\text{C})(5 \text{ x } 10^{-9}\text{C})}{x} + \frac{(-3 \text{ x } 10^{-9}\text{C})(5 \text{ x } 10^{-9}\text{C})}{(0.2 \text{ m} - x)}\right) = 0$$

$$\left(\frac{-6}{0.2 \text{ m}} + \frac{10}{x} + \frac{-15}{(0.2 \text{ m} - x)}\right) = 0 \quad \text{so} \quad \left(\frac{-6x(0.2 \text{ m} - x) + 10(0.2 \text{ m})(0.2 \text{ m} - x) - 15(0.2 \text{ m})x}{(0.2 \text{ m})x(0.2 \text{ m} - x)}\right) = 0$$

$$\left(\frac{(-1.2 \text{ m})x + 6x^2 + 0.4 \text{ m}^2 - (2 \text{ m})x - (3 \text{ m})x}{(0.04 \text{ m}^2)x - (0.2 \text{ m})x^2}\right) = 0 \quad \text{and} \quad \left(\frac{6x^2 - (6.2 \text{ m})x + 0.4 \text{ m}^2}{-(0.2 \text{ m})x^2 + (0.04 \text{ m}^2)x}\right) = 0$$

$$6x^2 - (6.2 \text{ m})x + 0.4 \text{ m}^2 = 0 \qquad \boxed{x = 0.0691 \text{ m}} \text{ or } x = 0.964 \text{ m} \quad \text{(must be between the charges)}$$

7.) $q_1 = +7.50\ \mu\text{C}$
$q_2 = +7.50\ \mu\text{C}$
$m_2 = 2.00$ g
$r_1 = 0.800$ m
$v_{2i} = 22.0$ m/s



a.) when $r_2 = 0.500$ m find v_{2f}

Conservation of Energy

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_{2_i}{}^2 + k\frac{q_1q_2}{r_1} = \frac{1}{2}mv_{2_f}{}^2 + k\frac{q_1q_2}{r_2} \quad \text{and} \quad \frac{1}{2}mv_{2_f}{}^2 = \frac{1}{2}mv_{2_i}{}^2 + k\frac{q_1q_2}{r_1} - k\frac{q_1q_2}{r_2}$$

$$v_{2_f} = \sqrt{v_{2_i}{}^2 + \frac{2kq_1q_2}{m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$v_{2_f} = \sqrt{(22 \text{ m/s})^2 + \frac{2\left(9 \text{ x } 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(7.5 \text{ x } 10^{-6}\text{C})(3 \text{ x } 10^{-6}\text{C})}{(2 \text{ x } 10^{-3}\text{kg})}\left(\frac{1}{0.8 \text{ m}} - \frac{1}{0.5 \text{ m}}\right)} = \boxed{18.2 \ \frac{\text{m}}{\text{s}}}$$

b.) closest when $v_{2f} = 0$

$$K_1 + U_1 = K_2 + U_2 \quad \text{or} \quad \frac{1}{2}mv_{2_i}{}^2 + k\frac{q_1q_2}{r_1} = k\frac{q_1q_2}{r_2}$$

$$\frac{1}{r_2} = \frac{mv_{2_i}{}^2}{2kq_1q_2} + \frac{1}{r_1} = \frac{mv_{2_i}{}^2 r_1 + 2kq_1q_2}{2kq_1q_2r_1} \quad \text{and} \quad r_2 = \frac{2kq_1q_2r_1}{mv_{2_i}{}^2 r_1 + 2kq_1q_2}$$

$$r_2 = \frac{2\left(9 \text{ x } 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(7.5 \text{ x } 10^{-6}\text{C})(3 \text{ x } 10^{-6}\text{C})(0.8 \text{ m})}{(2 \text{ x } 10^{-3} \text{ kg})(22 \text{ m/s})^2(0.8 \text{ m}) + 2\left(9 \text{ x } 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(7.5 \text{ x } 10^{-6}\text{C})(3 \text{ x } 10^{-6}\text{C})} = \boxed{0.275 \text{ m}}$$

Check units:

$$r_2 [=] \frac{\left(\frac{N\cdot m^2}{C^2}\right)(C)(C)(m)}{(kg)(m/s)^2(m)+\left(\frac{N\cdot m^2}{C^2}\right)(C)(C)} [=] \frac{\left(\frac{\frac{kg\cdot m}{s^2}\cdot m^2}{C^2}\right)(C)(C)(m)}{(kg)(m/s)^2(m)+\left(\frac{\frac{kg\cdot m}{s^2}\cdot m^2}{C^2}\right)(C)(C)} [=] \text{ m} \quad \text{(units work out)}$$

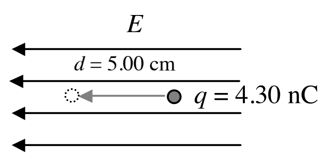
8.) $r = 0.750$ m

$V = 48.0$ V $\qquad V = \dfrac{kq}{r}$ so $q = \dfrac{Vr}{k} = \dfrac{(48\text{ V})(0.75\text{ m})}{\left(9 \times 10^9 \frac{N\cdot m^2}{C^2}\right)} = \boxed{4 \times 10^{-9}\text{ C} = 4\text{ nC}}$

Check units:

$$q [=] \frac{V\cdot m}{\frac{N\cdot m^2}{C^2}} [=] \frac{\frac{J}{C}\cdot m}{\frac{N\cdot m^2}{C^2}} [=] \frac{\frac{N\cdot m}{C}\cdot m}{\frac{N\cdot m^2}{C^2}} [=] \text{ C} \quad \text{(units work out)}$$

9.)



$v_1 = 0$ so $K_1 = 0$

$K_2 = 2.5 \times 10^{-6}$ J

a.) Work Energy Theorem $\quad W = \Delta KE$

$W = K_2 - K_1 = \boxed{2.5 \times 10^{-6}\text{ J}}$

b.) $W = -\Delta U$ and $\Delta V = \dfrac{\Delta U}{q}$ so $\Delta V = \dfrac{-2.5 \times 10^{-6}\text{ J}}{4.3 \times 10^{-9}\text{ C}} = -581$ V

$\Delta V = V_2 - V_1 = -581$ V so $\boxed{V_1 - V_2 = 581\text{ V}}$

c.) $W = \vec{F}\cdot\vec{d} = Fd\cos\theta$ (the field is uniform so the force is uniform)

$W = Fd$ (the force of the field F and the distance moved d are in the same direction, $\theta = 0$)

$W = qEd$ (This is how E is defined)

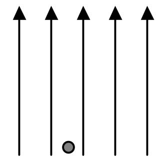
$W = qEd$ so $E = \dfrac{W}{qd} = \dfrac{2.5 \times 10^{-6}\text{ J}}{(4.3 \times 10^{-9}\text{ C})(0.05\text{ m})} = \boxed{1.16 \times 10^4 \frac{V}{m}}$

Check units:

$$E [=] \frac{J}{C\cdot m} [=] \frac{V}{m} \quad \text{(units work out)}$$

10.)

$E = 5.00 \text{ x } 10^4$ N/C



$q = 37.0$ nC

$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$ (the field is uniform so the force is uniform)

$W = qEd\cos\theta$

a.) moving 0.45 m to the right $\theta = 90^\circ$

$$W = qEd\cos\theta = \left(37 \text{ x } 10^{-9}\text{C}\right)\left(5 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\left(0.45 \text{ m}\right)\cos\left(90^\circ\right) = \boxed{0}$$

b.) moving 0.67 m downward $\theta = -180^\circ$

$$W = qEd\cos\theta = \left(37 \text{ x } 10^{-9}\text{C}\right)\left(5 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\left(0.67 \text{ m}\right)\cos\left(180^\circ\right) = \boxed{-1.24 \text{ x } 10^{-3}\text{ J}}$$

c.) moving 0.58 m upward $\theta = 0^\circ$

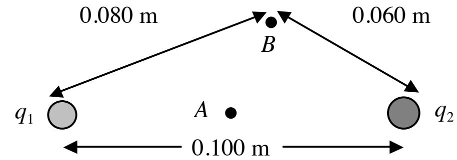
$$W = qEd\cos\theta = \left(37 \text{ x } 10^{-9}\text{C}\right)\left(5 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\left(0.58 \text{ m}\right)\cos\left(0^\circ\right) = \boxed{1.07 \text{ x } 10^{-3}\text{ J}}$$

d.) moving 2.60 m at an angle 45° upward from the horizontal $\theta = 45^\circ$

$$W = qEd\cos\theta = \left(37 \text{ x } 10^{-9}\text{C}\right)\left(5 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\left(2.6 \text{ m}\right)\cos\left(45^\circ\right) = \boxed{3.4 \text{ x } 10^{-3}\text{ J}}$$

HO 32 Solutions

1.)



$q_1 = +6.80$ nC
$q_2 = -5.10$ nC

a.) $$V_A = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = \frac{\left(9 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(6.8 \text{ x } 10^{-9}\text{C}\right)}{0.5 \text{ m}} + \frac{\left(9 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(-5.1 \text{ x } 10^{-9}\text{C}\right)}{0.5 \text{ m}} = \boxed{+306 \text{ V}}$$

Check units:

$$V_A \left[=\right] \frac{\left(\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(\text{C})}{\text{m}} \left[=\right] \frac{\text{N}\cdot\text{m}}{\text{C}} \left[=\right] \frac{\text{J}}{\text{C}} \left[=\right] \text{V} \quad \text{(units work out)}$$

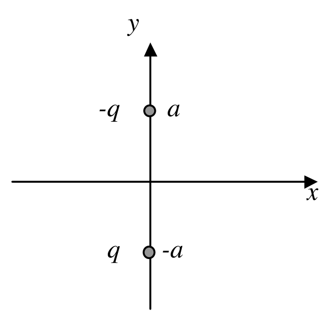
b.) $$V_B = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = \frac{\left(9 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(6.8 \text{ x } 10^{-9}\text{C}\right)}{0.08 \text{ m}} + \frac{\left(9 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(-5.1 \text{ x } 10^{-9}\text{C}\right)}{0.06 \text{ m}} = \boxed{0}$$

c.) $$W = -\Delta U = -q_o \Delta V = -q_o\left(V_A - V_B\right) = -\left(2.5 \text{ x } 10^{-9}\text{C}\right)\left(306 \text{ V} - 0\right) = \boxed{-7.65 \text{ x } 10^{-7}\text{J}}$$

d.) $$W = -\Delta U = -q_o \Delta V = -q_o\left(V_B - V_A\right) = -\left(2.5 \text{ x } 10^{-9}\text{C}\right)\left(0 - 306 \text{ V}\right) = \boxed{7.65 \text{ x } 10^{-7}\text{J}}$$

e.) $$W = -\Delta U = -q_o \Delta V = -q_o\left(V_A - V_B\right) = -\left(-2.5 \text{ x } 10^{-9}\text{C}\right)\left(306 \text{ V} - 0\right) = \boxed{7.65 \text{ x } 10^{-7}\text{J}}$$

2.)



Must consider 3 cases. (At each charge there will be an asymptote.)

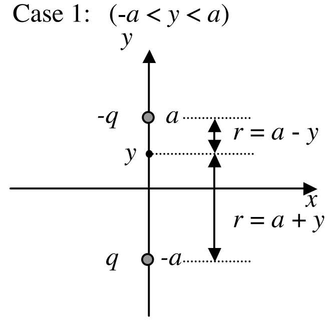
1.) $-a < y < a$

2.) $y > a$

3.) $y < -a$

a.)

Case 1: ($-a < y < a$)

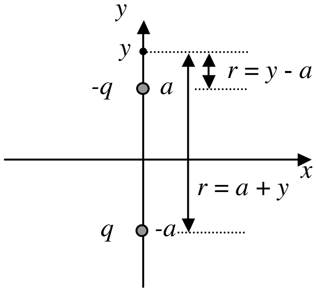


$$V = \sum \frac{kq}{r} = k\left(\frac{-q}{a-y} + \frac{q}{a+y}\right) = k\left(\frac{-q(a+y) + q(a-y)}{(a-y)(a+y)}\right)$$

$$V = k\left(\frac{-qa - qy + qa - qy}{(a-y)(a+y)}\right) = k\left(\frac{-2qy}{(a-y)(a+y)}\right) = \frac{-2kqy}{\left(a^2 - y^2\right)}$$

$$\boxed{V = \frac{2kqy}{\left(y^2 - a^2\right)} \quad (a < y < a)}$$

Case 2: ($y > a$)

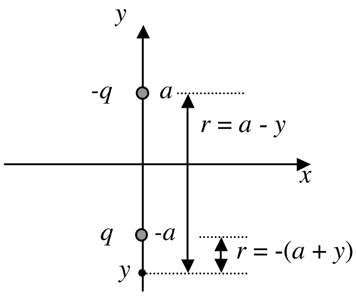


$$V = \sum \frac{kq}{r} = k\left(\frac{-q}{y-a} + \frac{q}{a+y}\right) = k\left(\frac{-q(a+y) + q(y-a)}{(y-a)(a+y)}\right)$$

$$V = k\left(\frac{-qa - qy + qy - qa}{(y-a)(a+y)}\right) = k\left(\frac{-2qa}{(y-a)(y+a)}\right) = \frac{-2kqa}{(y^2 - a^2)}$$

$$\boxed{V = \frac{-2kqa}{(y^2 - a^2)} \qquad (y > a)}$$

Case 3: ($y < -a$)

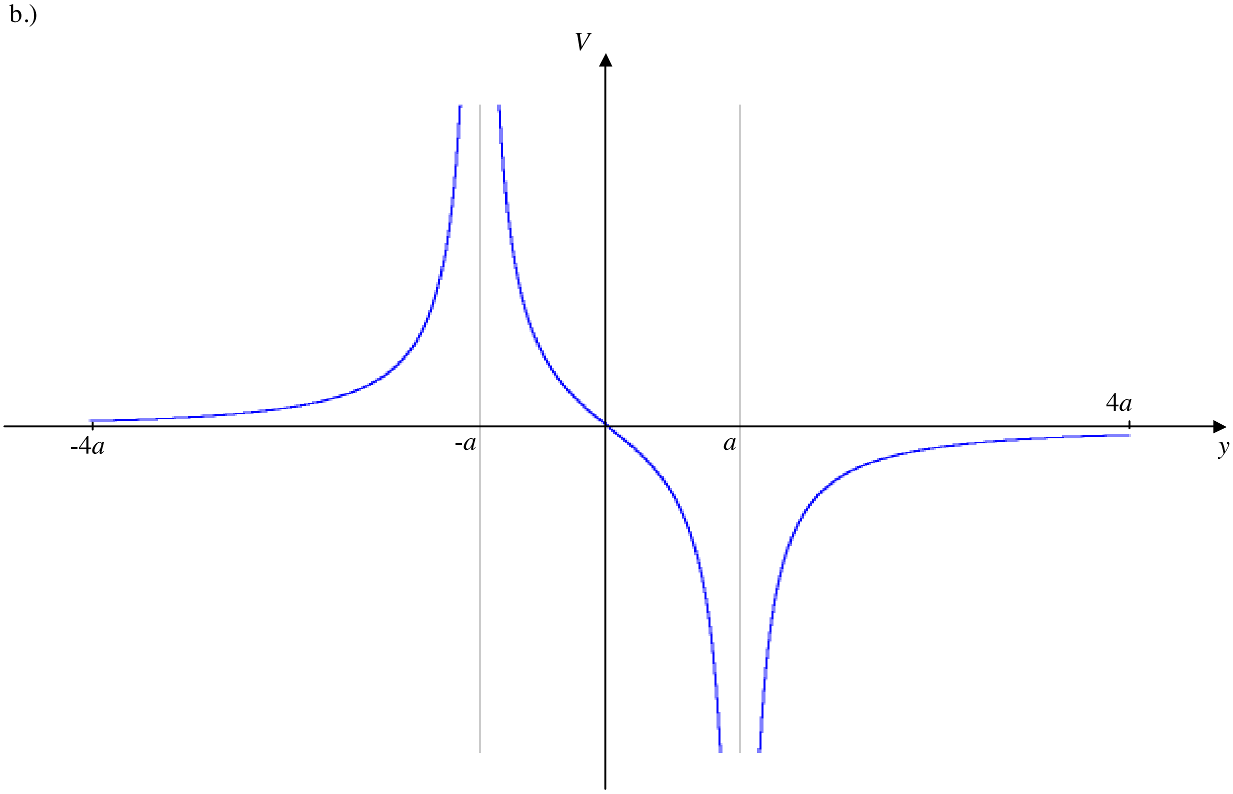


$$V = \sum \frac{kq}{r} = k\left(\frac{-q}{a-y} + \frac{q}{-(a+y)}\right) = k\left(\frac{-q}{(a-y)} + \frac{-q}{(a+y)}\right)$$

$$V = k\left(\frac{-q(a+y) - q(a-y)}{(a-y)(a+y)}\right) = k\left(\frac{-qa - qy - qa + qy}{(a-y)(a+y)}\right) = k\left(\frac{-2qa}{(a^2 - y^2)}\right)$$

$$\boxed{V = \frac{2kqa}{(y^2 - a^2)} \qquad (y < -a)}$$

b.)



The potential approaches infinity from either side at the location of the two charges. It is zero between the charges at $x = 0$. The potential also approaches zero for $x = \pm\infty$.

c.) reversing charges

Case 1: $(-a < y < a)$

$$V = k\left(\frac{q}{a-y} + \frac{-q}{a+y}\right)$$

$$V = k\left(\frac{q(a+y) - q(a-y)}{(a-y)(a+y)}\right)$$

$$V = k\left(\frac{qa + qy - qa + ay}{(a-y)(a+y)}\right)$$

$$V = \frac{2kqy}{(a^2 - y^2)}$$

$$\boxed{V = \frac{-2kqy}{(y^2 - a^2)}}$$

Case 2: $(a < y)$

$$V = k\left(\frac{q}{y-a} + \frac{-q}{a+y}\right)$$

$$V = k\left(\frac{q(a+y) - q(y-a)}{(y-a)(a+y)}\right)$$

$$V = k\left(\frac{qa + qy - qy + qa}{(y-a)(a+y)}\right)$$

$$\boxed{V = \frac{2kqa}{(y^2 - a^2)}}$$

Case 3: $(y < -a)$

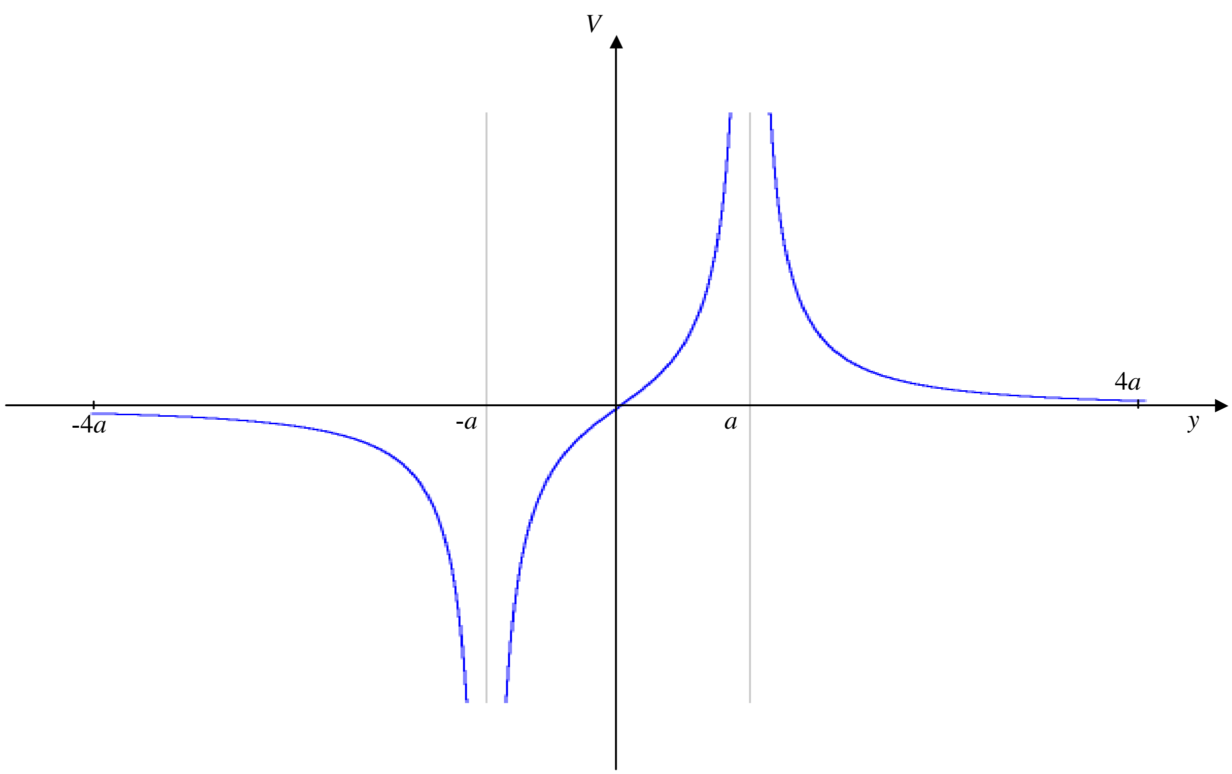
$$V = k\left(\frac{q}{a-y} + \frac{-q}{-(a+y)}\right)$$

$$V = k\left(\frac{q}{a-y} + \frac{q}{(a+y)}\right)$$

$$V = k\left(\frac{q(a+y) + q(a-y)}{(a-y)(a+y)}\right)$$

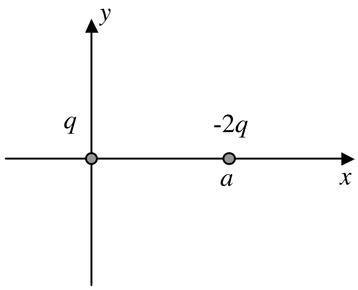
$$V = k\left(\frac{qa + qy + qa - qy}{(a^2 - y^2)}\right)$$

$$\boxed{V = \frac{-2kqa}{(y^2 - a^2)}}$$



Mirror image of the previous part.

3.)



Again must consider 3 cases: (At each charge there will be an asymptote.)

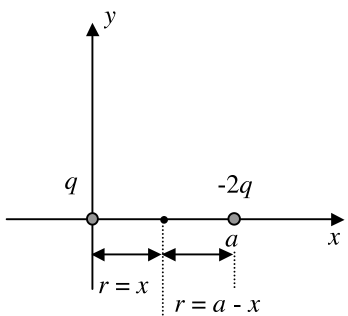
1.) $0 < x < a$

2.) $x > a$

3.) $x < 0$

a.)

Case 1: $(0 < x < a)$

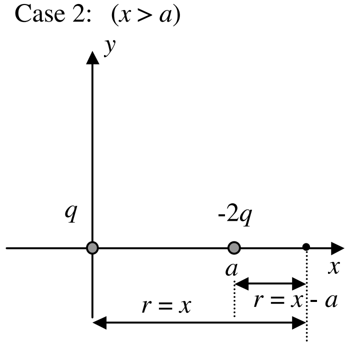


$$V = \sum \frac{kq}{r} = k\left(\frac{q}{x} + \frac{-2q}{a-x}\right) = k\left(\frac{q(a-x) - 2qx}{x(a-x)}\right)$$

$$V = k\left(\frac{qa - qx - 2qx}{x(a-x)}\right) = k\left(\frac{qa - 3qx}{x(a-x)}\right)$$

$$\boxed{V = \frac{kq(3x-a)}{x(x-a)}}$$

Case 2: $(x > a)$

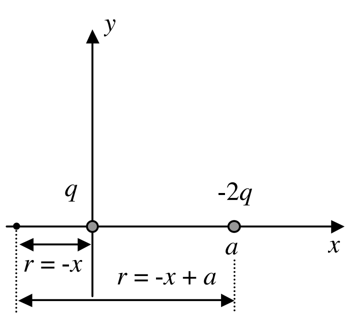


$$V = \sum \frac{kq}{r} = k\left(\frac{q}{x} + \frac{-2q}{x-a}\right) = k\left(\frac{q(x-a) - 2qx}{x(x-a)}\right)$$

$$V = k\left(\frac{qx - qa - 2qx}{x(x-a)}\right) = k\left(\frac{-qa - qx}{x(x-a)}\right)$$

$$\boxed{V = \frac{-kq(x+a)}{x(x-a)}}$$

Case 3: $(x < 0)$

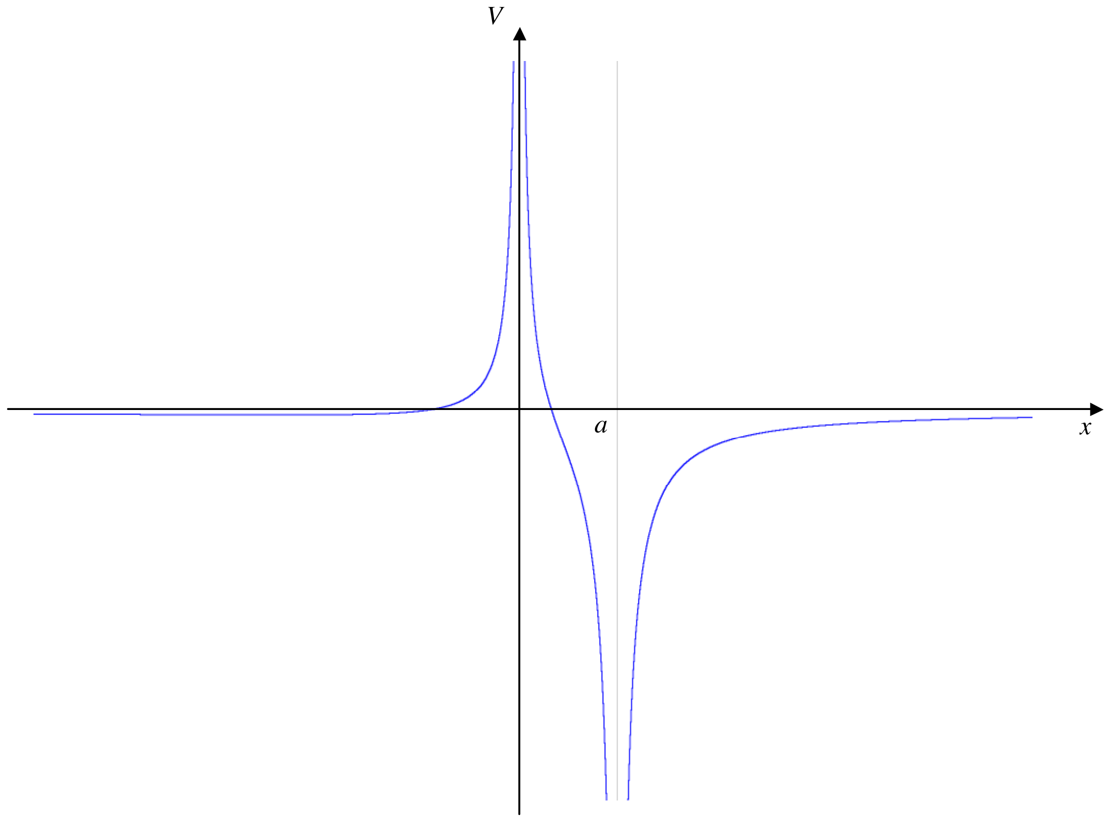


$$V = \sum \frac{kq}{r} = k\left(\frac{q}{-x} + \frac{-2q}{-x+a}\right) = k\left(\frac{-q}{x} + \frac{2q}{x-a}\right) = k\left(\frac{-q(x-a) + 2qx}{x(x-a)}\right)$$

$$V = k\left(\frac{-qx + qa + 2qx}{x(x-a)}\right) = k\left(\frac{qa + qx}{x(x-a)}\right)$$

$$\boxed{V = \frac{kq(x+a)}{x(x-a)}}$$

b.)



c.)

for $0 < x < a$: $V = \frac{kq(3x - a)}{x(x - a)} = 0$ when $3x - a = 0$ or $\boxed{x = \frac{a}{3}}$

for $x < 0$: $V = \frac{kq(x + a)}{x(x - a)} = 0$ when $x + a = 0$ or $\boxed{x = -a}$

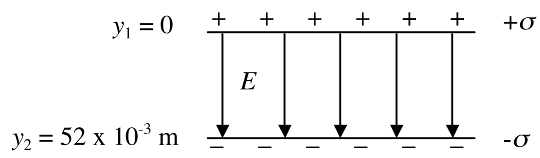
d.) for $x >> a$

$$V = \frac{-kq(x + a)}{x(x - a)} \quad (x > a)$$

$$V = \frac{-kq(x + a)}{x(x - a)} \approx \frac{-kq(x)}{x(x)} = \boxed{\frac{-kq}{x}}$$

(Looks like a point charge with $q_o = (-2q + q) = -q$.)

4.)



For a sheet charge $E = \dfrac{\sigma}{2\varepsilon_o}$ and is directed away from (+) sheets and towards (-) sheets.

$$E = E_+ + E_- = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o} = 670\ \frac{\text{N}}{\text{C}}$$

a.) $V_a - V_b = -\int_b^a \vec{E}.d\vec{r}$ so $V_+ - V_- = -\int_{y_2}^{y_1} Edy = -\int_{y_2}^{y_1} \frac{\sigma}{\varepsilon_o} dy = -\frac{\sigma}{\varepsilon_o}\int_{y_2}^{y_1} dy = -\frac{\sigma}{\varepsilon_o} y\Big]_{y_2}^{y_1}$

$$V_+ - V_- = -\frac{\sigma}{\varepsilon_o}(y_1 - y_2) = \frac{\sigma}{\varepsilon_o}(y_2 - y_1)$$

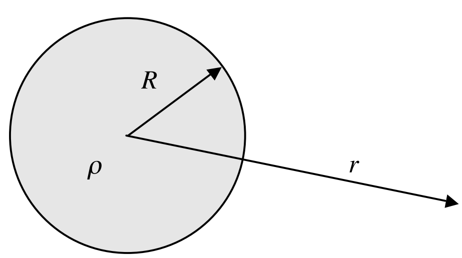
$$V_+ - V_- = 670\ \frac{\text{N}}{\text{C}}(0.0052\ \text{m} - 0) = \boxed{34.84\ \text{V}}$$

Check units:

$V_+ - V_- [=]\ \frac{\text{N}}{\text{C}}(\text{m})\ [=]\ \frac{\text{J}}{\text{C}}\ [=]\ \text{V}$ (units work out)

b.) $E = \dfrac{\sigma}{\varepsilon_o}$ so $\sigma = E\varepsilon_o = \left(670\ \frac{\text{N}}{\text{C}}\right)\left(8.854\ \text{x}\ 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right) = \boxed{5.93\ \text{x}\ 10^{-9}\ \frac{\text{C}}{\text{m}^2}}$

5.)



for $r > R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

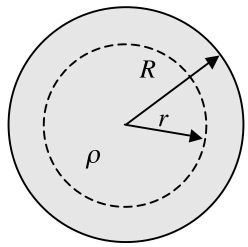
$E4\pi r^2 = \dfrac{Q}{\varepsilon_o}$ and $E = \dfrac{Q}{4\pi\varepsilon_o r^2}$

Total charge Q so $\rho = \dfrac{Q}{V} = \dfrac{Q}{\frac{4}{3}\pi R^3} = \dfrac{3Q}{4\pi R^3}$

Integration starts at $r = \infty$ (where $V = 0$) to a point r which is outside the charged sphere.

$V_a - V_b = -\int_b^a \vec{E}.d\vec{r}$ and $V_r - V_\infty = -\int_\infty^r \frac{Q}{4\pi\varepsilon_o r^2} dr = -\frac{Q}{4\pi\varepsilon_o}\int_\infty^r \frac{1}{r^2} dr$

$V_r - 0 = -\dfrac{Q}{4\pi\varepsilon_o}\int_\infty^r r^{-2}dr = -\dfrac{Q}{4\pi\varepsilon_o}\dfrac{r^{-1}}{(-1)}\Big]_\infty^r = \dfrac{Q}{4\pi\varepsilon_o}\left(\dfrac{1}{r} - \dfrac{1}{\infty}\right)$ and $\boxed{V_r = \dfrac{Q}{4\pi\varepsilon_o r} \quad (r > R)}$



For $r < R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$E4\pi r^2 = \frac{\rho V}{\varepsilon_o} = \frac{\left(\frac{3Q}{4\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_o} = \frac{\frac{Q}{R^3}r^3}{\varepsilon_o}$$

$$E = \frac{Qr}{4\pi\varepsilon_o R^3}$$

Integration starts at $r = R$ to a point r which is inside the charged sphere.

when $r = R$, $\quad V_R = \dfrac{Q}{4\pi\varepsilon_o R}$ (using expression for the potential outside the sphere when $r = R$)

$$V_a - V_b = -\int_b^a \vec{E}.d\vec{r} \quad \text{and} \quad V_r - V_R = -\int_R^r \frac{Qr}{4\pi\varepsilon_o R^3}dr = -\frac{Q}{4\pi\varepsilon_o R^3}\int_R^r r\,dr$$

$$V_r - \frac{Q}{4\pi\varepsilon_o R} = -\frac{Q}{4\pi\varepsilon_o R^3}\left.\frac{r^2}{2}\right]_R^r = -\frac{Q}{4\pi\varepsilon_o R^3}\left(\frac{r^2}{2} - \frac{R^2}{2}\right)$$

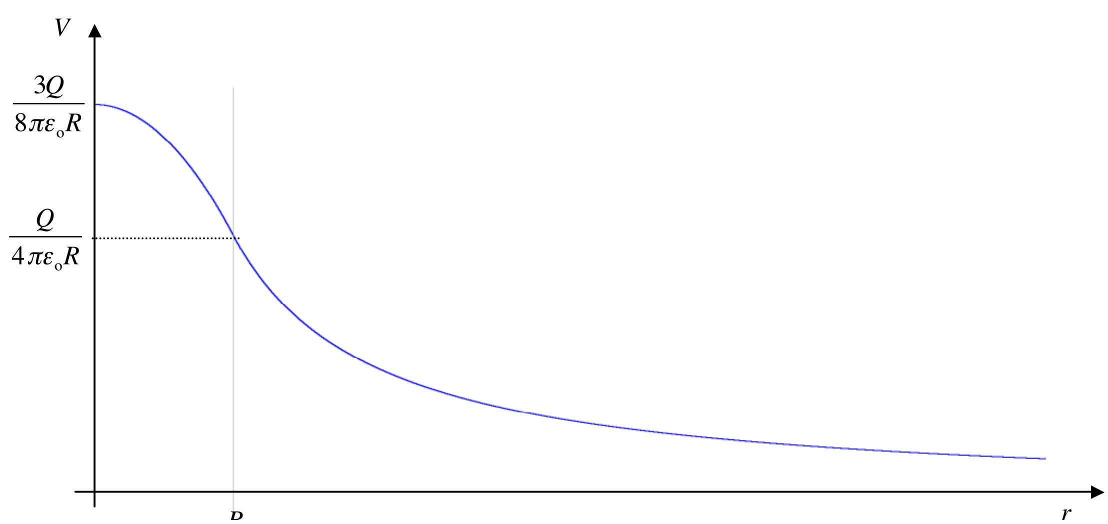
$$V_r - \frac{Q}{4\pi\varepsilon_o R} = -\frac{Q}{8\pi\varepsilon_o R^3}r^2 + \frac{Q}{8\pi\varepsilon_o R^3}R^2$$

$$V_r = -\frac{Q}{8\pi\varepsilon_o R^3}r^2 + \frac{Q}{8\pi\varepsilon_o R} + \frac{Q}{4\pi\varepsilon_o R}$$

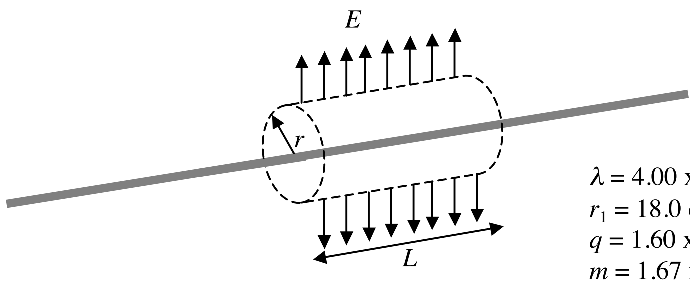
$$\boxed{V_r = -\frac{Q}{8\pi\varepsilon_o R^3}r^2 + \frac{3Q}{8\pi\varepsilon_o R} = \frac{Q}{8\pi\varepsilon_o R}\left(3 - \frac{r^2}{R^2}\right) \quad r < R}$$

Notice that there is continuity between the expressions for potential when $r = R$.

$$V_R(r < R) = -\frac{Q}{8\pi\varepsilon_o R^3}R^2 + \frac{3Q}{8\pi\varepsilon_o R} = -\frac{Q}{8\pi\varepsilon_o R} + \frac{3Q}{8\pi\varepsilon_o R} = \frac{2Q}{8\pi\varepsilon_o R} = \frac{Q}{4\pi\varepsilon_o R} = V_R(r > R)$$



6.)



$\lambda = 4.00 \times 10^{-12}$ C/m
$r_1 = 18.0$ cm
$q = 1.60 \times 10^{-19}$ C
$m = 1.67 \times 10^{-27}$ kg
$v_1 = 2.50 \times 10^{3}$ m/s

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o} \quad \text{so} \quad E = \frac{\lambda}{2\pi\varepsilon_o r}$$

$$K_1 + U_1 = K_2 + U_2 \quad \text{or} \quad K_1 + qV_1 = K_2 + qV_2$$

At closest approach, the velocity $v_2 = 0$ and $K_2 = 0$.

$$K_1 + qV_1 = K_2 + qV_2 \quad \text{so} \quad \frac{1}{2}mv_1^{\,2} = qV_2 - qV_1 = q(V_2 - V_1)$$

$$V_a - V_b = -\int_b^a \vec{E}.d\vec{r} \quad \text{so} \quad V_2 - V_1 = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_o r}dr = \frac{-\lambda}{2\pi\varepsilon_o}\int_{r_1}^{r_2}\frac{1}{r}dr$$

$$V_2 - V_1 = \frac{-\lambda}{2\pi\varepsilon_o}\ln r\Big]_{r_1}^{r_2} = \frac{\lambda}{2\pi\varepsilon_o}\ln r\Big]_{r_2}^{r_1} = \frac{\lambda}{2\pi\varepsilon_o}(\ln r_1 - \ln r_2)$$

$$V_2 - V_1 = \frac{\lambda}{2\pi\varepsilon_o}\ln\left(\frac{r_1}{r_2}\right)$$

$$\frac{1}{2}mv_1^{\,2} = q(V_2 - V_1) = \frac{q\lambda}{2\pi\varepsilon_o}\ln\left(\frac{r_1}{r_2}\right)$$

$$\frac{\pi\varepsilon_o m v_1^{\,2}}{q\lambda} = \ln\left(\frac{r_1}{r_2}\right) \quad \text{and} \quad \exp\left(\frac{\pi\varepsilon_o m v_1^{\,2}}{q\lambda}\right) = \frac{r_1}{r_2}$$

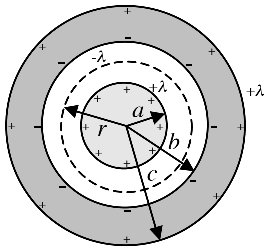
$$r_2 = \frac{r_1}{\exp\left(\frac{\pi\varepsilon_o m v_1^{\,2}}{q\lambda}\right)}$$

$\lambda = 4.00 \times 10^{-12}$ C/m
$r_1 = 18.0$ cm
$q = 1.60 \times 10^{-19}$ C
$m = 1.67 \times 10^{-27}$ kg
$v_1 = 2.50 \times 10^{3}$ m/s

$$r_2 = \frac{0.18\text{ m}}{\exp\left(\frac{\pi\left(8.85\text{x}10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(1.67\text{x}10^{-27}\text{ kg}\right)\left(2.5\text{x}10^{3}\text{ m/s}\right)^2}{\left(1.6\text{x}10^{-19}\text{C}\right)\left(4\text{x}10^{-12}\,\frac{\text{C}}{\text{m}}\right)}\right)}$$

$$\boxed{r_2 = 0.11\text{ m}}$$

7.) Looking at a cross-section of the cable:



The inner cylinder is a conductor so all the charge +λ will be on its outer surface. Since the outer cylinder is also a conductor a charge -λ will induced on its inner surface due to the field of the charge on the inner cylinder. Since the net charge on the outer surface is zero, then there must be a charge +λ on its outer surface. The electric field between the cylinders is due to the charge on the inner cylinder only. Using a cylindrical surface radius r and length L:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$E 2\pi r L = \frac{\lambda L}{\varepsilon_o} \quad \text{so} \quad E = \frac{\lambda}{2\pi\varepsilon_o r} \quad \text{and points radially outward.}$$

The potential difference is: $V_a - V_b = -\int_b^a \vec{E} . d\vec{r}$

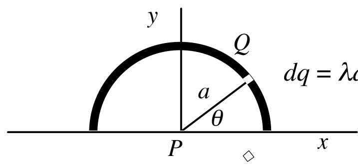
$$V_a - V_b = -\int_b^a \frac{\lambda}{2\pi\varepsilon_o r} dr = \frac{-\lambda}{2\pi\varepsilon_o} \int_b^a \frac{1}{r} dr$$

$$V_a - V_b = \frac{-\lambda}{2\pi\varepsilon_o} \ln r \Big]_b^a = \frac{\lambda}{2\pi\varepsilon_o} \ln r \Big]_a^b = \frac{\lambda}{2\pi\varepsilon_o} (\ln b - \ln a)$$

$$\boxed{V_a - V_b = \frac{\lambda}{2\pi\varepsilon_o} \ln\left(\frac{b}{a}\right)}$$

HO 33 Solutions

1.)



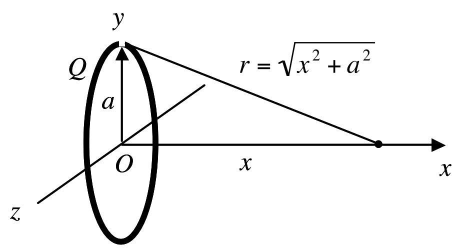
$$dq = \lambda ds = \lambda a d\theta \qquad \left(\lambda = \frac{Q}{\pi a}\right)$$

$$dV = k\frac{dq}{r} = k\frac{dq}{a}$$

$$V = \int k\frac{dq}{a} = \frac{k}{a}\int dq = \frac{k}{a}Q$$

$$\boxed{V = \frac{kQ}{a} = \frac{\lambda\pi a}{4\pi\varepsilon_o a} = \frac{\lambda}{4\varepsilon_o}}$$

2.)



a.)

$$dV = k\frac{dq}{r} = k\frac{dq}{\sqrt{x^2+a^2}}$$

$$V = \int k\frac{dq}{\sqrt{x^2+a^2}} = \frac{k}{\sqrt{x^2+a^2}}\int dq = \frac{k}{\sqrt{x^2+a^2}}Q$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{\sqrt{x^2+a^2}}}$$

b.)

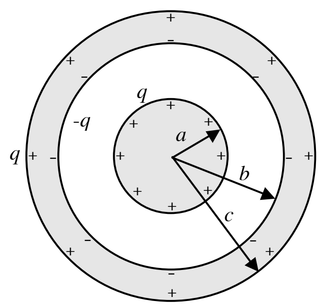
$$E_x = k\frac{Qx}{\left(x^2+a^2\right)^{3/2}} \quad \text{so} \quad V_{x_2} - V_{x_1} = -\int_{x_1}^{x_2} E_x dx = -\int_{x_1}^{x_2} k\frac{Qx}{\left(x^2+a^2\right)^{3/2}}dx = -\int_{x_1}^{x_2} k\frac{Q}{2\left(x^2+a^2\right)^{3/2}}(2xdx)$$

$$V_{x_2} - V_{x_1} = -\frac{kQ}{2}\int_{x_1}^{x_2}\left(x^2+a^2\right)^{-3/2}(2xdx) = -\frac{kQ}{2}\left.\frac{\left(x^2+a^2\right)^{-1/2}}{-1/2}\right]_{x_1}^{x_2}$$

$$V_{x_2} - V_{x_1} = kQ\left(\frac{1}{\left(x_2^2+a^2\right)^{1/2}} - \frac{1}{\left(x_1^2+a^2\right)^{1/2}}\right)$$

$$\left(V_{x_1} = 0 \text{ as } x_1 \to \infty\right) \qquad V_{x_2} - 0 = kQ\left(\frac{1}{\left(x_2^2+a^2\right)^{1/2}} - 0\right) = k\frac{Q}{\sqrt{x_2^2+a^2}} \qquad \boxed{V_{x_2} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{\sqrt{x_2^2+a^2}}}$$

3.)



$V_a - V_b = -\int_b^a \vec{E}.d\vec{r}$ and get E using Gauss's Law.

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$ and for spherical Gaussian surfaces $E 4\pi r^2 = \frac{q_{enclosed}}{\varepsilon_o}$

$E = \frac{q_{enclosed}}{4\pi\varepsilon_o r^2}$

a.) for $r > c$, $q_{enclosed} = +q - q + q = q$ and $E = \frac{q}{4\pi\varepsilon_o r^2}$

Integration starts at $r = \infty$ (where $V = 0$) to a point r which is outside the charged spheres.

$$V_r - V_\infty = -\int_\infty^r \frac{q}{4\pi\varepsilon_o r^2} dr = -\frac{q}{4\pi\varepsilon_o} \int_\infty^r \frac{1}{r^2} dr$$

$$V_r - 0 = -\frac{q}{4\pi\varepsilon_o} \int_\infty^r r^{-2} dr = -\frac{q}{4\pi\varepsilon_o} \frac{r^{-1}}{(-1)}\Bigg]_\infty^r = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$V_r = \frac{q}{4\pi\varepsilon_o r} \quad (r > c) \quad \text{so} \quad \boxed{V_c = \frac{q}{4\pi\varepsilon_o c}}$$

b.) for $b < r < c$, $q_{enclosed} = +q - q = 0$ so $E = \frac{0}{4\pi\varepsilon_o r^2} = 0$ (inside a conductor)

$$V_r - V_c = -\int_c^r 0 dr = 0 \text{ and } V_r = V_c$$

$$V_r = \frac{q}{4\pi\varepsilon_o c} \quad (b < r < c) \quad \text{so} \quad \boxed{V_b = \frac{q}{4\pi\varepsilon_o c}}$$ (a conductor is an equipotential surface)

c.) for $a < r < b$, $q_{enclosed} = +q$ so $E = \frac{q}{4\pi\varepsilon_o r^2}$

$$V_r - V_b = -\int_b^r \frac{q}{4\pi\varepsilon_o r^2} dr = -\frac{q}{4\pi\varepsilon_o} \int_b^r \frac{1}{r^2} dr$$

$$V_r - \frac{q}{4\pi\varepsilon_o c} = -\frac{q}{4\pi\varepsilon_o} \int_b^r r^{-2} dr = -\frac{q}{4\pi\varepsilon_o} \frac{r^{-1}}{(-1)}\Bigg]_b^r = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{b}\right)$$

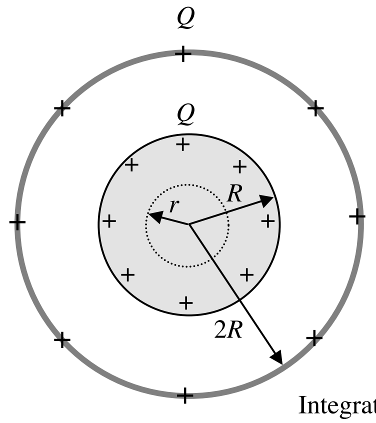
$$V_r = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{b} + \frac{1}{c}\right) \quad (a < r < b) \quad \text{so} \quad \boxed{V_a = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c}\right)}$$

d.) for $0 < r < a$, $q_{enclosed} = 0$ so $E = 0$ (inside a conductor)

$$V_r - V_a = -\int_a^r 0 dr = 0 \text{ and } V_r = V_a \quad (0 < r < a)$$

$$V_r = V_a = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c}\right) \text{ and } \boxed{V_0 = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c}\right)}$$ (a conductor is an equipotential surface)

4.)



$V_a - V_b = -\int_b^a \vec{E}.d\vec{r}$ and get E using Gauss's Law.

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$ and for spherical Gaussian surfaces $E4\pi r^2 = \frac{q_{enclosed}}{\varepsilon_o}$

$$E = \frac{q_{enclosed}}{4\pi\varepsilon_o r^2}$$

For $r > 2R$, $q_{enclosed} = Q + Q = 2Q$ and $E = \frac{2Q}{4\pi\varepsilon_o r^2}$

Integration starts at $r = \infty$ (where $V = 0$) to a point r which is outside the charged spheres.

$$V_r - V_\infty = -\int_\infty^r \frac{2Q}{4\pi\varepsilon_o r^2}dr = -\frac{2Q}{4\pi\varepsilon_o}\int_\infty^r \frac{1}{r^2}dr$$

$$V_r - 0 = -\frac{2Q}{4\pi\varepsilon_o}\int_\infty^r r^{-2}dr = -\frac{2Q}{4\pi\varepsilon_o}\frac{r^{-1}}{(-1)}\Bigg]_\infty^r = \frac{2Q}{4\pi\varepsilon_o}\left(\frac{1}{r}-\frac{1}{\infty}\right)$$

$$V_r = \frac{Q}{2\pi\varepsilon_o r}\quad (r > 2R) \quad \text{so} \quad V_{2R} = \frac{Q}{4\pi\varepsilon_o R}$$

for $R < r < 2R$, $q_{enclosed} = Q$ so $E = \frac{Q}{4\pi\varepsilon_o r^2}$

$$V_r - V_{2R} = -\int_{2R}^r \frac{Q}{4\pi\varepsilon_o r^2}dr = -\frac{Q}{4\pi\varepsilon_o}\int_{2R}^r \frac{1}{r^2}dr$$

$$V_r - \frac{Q}{4\pi\varepsilon_o R} = -\frac{Q}{4\pi\varepsilon_o}\int_{2R}^r r^{-2}dr = -\frac{Q}{4\pi\varepsilon_o}\frac{r^{-1}}{(-1)}\Bigg]_{2R}^r = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r}-\frac{1}{2R}\right)$$

$$V_r = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r}-\frac{1}{2R}\right)+\frac{Q}{4\pi\varepsilon_o R} = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r}-\frac{1}{2R}+\frac{1}{R}\right) = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r}+\frac{1}{2R}\right)$$

$$\text{so} \quad V_R = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{R}+\frac{1}{2R}\right) = \frac{Q}{4\pi\varepsilon_o}\left(\frac{2}{2R}+\frac{1}{2R}\right) \quad \text{and} \quad V_R = \frac{3Q}{8\pi\varepsilon_o R}$$

$$V_R - V_{2R} = \frac{3Q}{8\pi\varepsilon_o R} - \frac{Q}{4\pi\varepsilon_o R} = \frac{3Q}{8\pi\varepsilon_o R} - \frac{2Q}{8\pi\varepsilon_o R} = \boxed{\frac{Q}{8\pi\varepsilon_o R}}$$

Since we are only looking for the potential difference between the outer and inner spheres and not the value for the potential everywhere, we really only need to look at the field between the spheres.

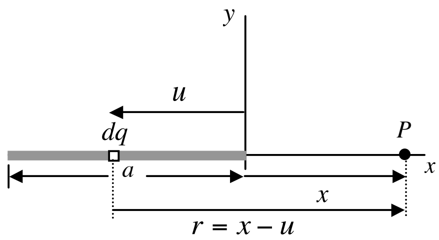
$$V_R - V_{2R} = -\int_{2R}^R \frac{Q}{4\pi\varepsilon_o r^2}dr = -\frac{Q}{4\pi\varepsilon_o}\int_{2R}^R \frac{1}{r^2}dr$$

$$V_R - V_{2R} = -\frac{Q}{4\pi\varepsilon_o}\int_{2R}^R r^{-2}dr = -\frac{Q}{4\pi\varepsilon_o}\frac{r^{-1}}{(-1)}\Bigg]_{2R}^R = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{R}-\frac{1}{2R}\right) = \frac{Q}{4\pi\varepsilon_o}\left(\frac{2}{2R}-\frac{1}{2R}\right)$$

$$\boxed{V_R - V_{2R} = \frac{Q}{8\pi\varepsilon_o R}}$$

5.)

a.)



$$dq = \lambda du = \frac{Q}{a} du$$

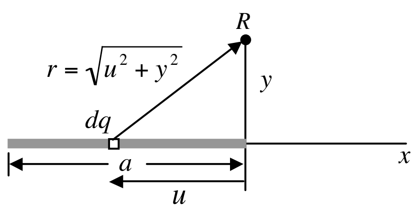
$$dV = k\frac{dq}{r} = k\frac{\lambda du}{(x-u)}$$

$$V = \int_{-a}^{0} k\frac{\lambda du}{(x-u)} = k\lambda \int_{-a}^{0} \frac{du}{(x-u)} = -k\lambda \int_{-a}^{0} \frac{-du}{(x-u)}$$

$$V = -k\lambda \ln(x-u)\Big]_{-a}^{0} = -k\lambda\big(\ln x - \ln(x+a)\big) = k\lambda\big(\ln(x+a) - \ln x\big)$$

$$V = k\lambda \ln\frac{(x+a)}{x} \quad \text{and} \quad \boxed{V = \frac{\lambda}{4\pi\varepsilon_o}\ln\frac{(x+a)}{x} \qquad (x>0)}$$

b.)



$$dq = \lambda du = \frac{Q}{a} du$$

$$dV = k\frac{dq}{r} = k\frac{\lambda du}{\sqrt{u^2+y^2}}$$

$$V = \int_{-a}^{0} k\frac{\lambda du}{\sqrt{u^2+y^2}} = k\lambda\int_{-a}^{0}\frac{du}{\sqrt{u^2+y^2}}$$

$$\int \frac{dx}{\sqrt{x^2+y^2}} = \ln\left(x+\sqrt{x^2+y^2}\right) + C \quad \text{(from integral tables)}$$

$$V = k\lambda\int_{-a}^{0}\frac{du}{\sqrt{u^2+y^2}} = k\lambda \ln\left(u+\sqrt{u^2+y^2}\right)\Big]_{-a}^{0}$$

$$V = k\lambda\left(\ln\left(0+\sqrt{0^2+y^2}\right) - \ln\left(-a+\sqrt{a^2+y^2}\right)\right) = k\lambda \ln\frac{y}{\left(-a+\sqrt{a^2+y^2}\right)}$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_o a}\ln\frac{y}{\left(-a+\sqrt{a^2+y^2}\right)}}$$

Aside: (for fun) let $x = y\tan\theta$ and $dx = y\sec^2\theta d\theta$

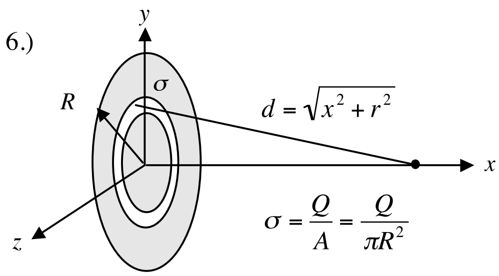
$$\int\frac{dx}{\sqrt{x^2+y^2}} = \int\frac{y\sec^2\theta d\theta}{\sqrt{y^2\left(\tan^2\theta+1\right)}} = \int\frac{y\sec^2\theta d\theta}{y\sqrt{\sec^2\theta}} = \int\sec\theta d\theta = \int\sec\theta\left(\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}\right)d\theta = \int\left(\frac{\sec^2\theta+\sec\theta\tan\theta}{\sec\theta+\tan\theta}\right)d\theta$$

$$= \int\left(\frac{\sec^2\theta+\sec\theta\tan\theta}{\tan\theta+\sec\theta}\right)d\theta = \ln(\tan\theta+\sec\theta)+C$$

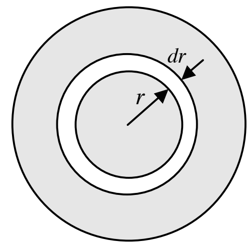
since $\tan\theta = \frac{x}{y}$ and $\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{x}{y}\right)^2}$ $\qquad \ln(\tan\theta+\sec\theta)+C = \ln\left(\frac{x}{y}+\sqrt{1+\left(\frac{x}{y}\right)^2}\right)+C = \ln\left(\frac{x}{y}+\frac{1}{y}\sqrt{y^2+x^2}\right)+C$

$$= \ln\left(\frac{x+\sqrt{y^2+x^2}}{y}\right)+C = \ln\left(x+\sqrt{y^2+x^2}\right)-\ln(y)+C = \ln\left(x+\sqrt{y^2+x^2}\right)+C' \quad \text{where} \quad C' = -\ln(y)+C$$

6.)



$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$$



$$dq = \sigma dA$$
$$dA = 2\pi r dr$$
$$dq = \sigma 2\pi r dr$$

a.)

$$dV = k\frac{dq}{d} = k\frac{\sigma 2\pi r dr}{\sqrt{x^2 + r^2}}$$

$$V = \int_0^R k\frac{\sigma 2\pi r dr}{\sqrt{x^2 + r^2}} = k\sigma\pi\int_0^R \frac{2r dr}{\sqrt{x^2 + r^2}} = k\sigma\pi\int_0^R \left(x^2 + r^2\right)^{-1/2} 2r dr$$

$$V = k\sigma\pi \left.\frac{\left(x^2 + r^2\right)^{1/2}}{1/2}\right]_0^R = 2k\sigma\pi\left(\left(x^2 + R^2\right)^{1/2} - \left(x^2 + 0^2\right)^{1/2}\right)$$

$$V = 2k\sigma\pi\left(\left(x^2 + R^2\right)^{1/2} - x\right) = 2k\sigma\pi\left(\sqrt{x^2 + R^2} - x\right)$$

$$V = \frac{2Q\pi}{4\pi\varepsilon_o \pi R^2}\left(\sqrt{x^2 + R^2} - x\right) = \frac{Q}{2\pi\varepsilon_o R^2}\left(\sqrt{x^2 + R^2} - x\right)$$

$$\boxed{V = \frac{Q}{2\pi\varepsilon_o R^2}\left(\sqrt{x^2 + R^2} - x\right)}$$ (for points on the *x*-axis)

Note that when $x \to \infty$ $\quad V = \frac{Q}{2\pi\varepsilon_o R^2}\left(\sqrt{x^2} - x\right) = \frac{Q}{2\pi\varepsilon_o R^2}(x - x) = 0$

b.)

$$E_x = -\frac{dV}{dx}$$

$$E_x = -\frac{d}{dx}\left(\frac{Q}{2\pi\varepsilon_o R^2}\left(\sqrt{x^2 + R^2} - x\right)\right) = -\frac{Q}{2\pi\varepsilon_o R^2}\frac{d}{dx}\left(\left(\sqrt{x^2 + R^2} - x\right)\right)$$

$$E_x = -\frac{Q}{2\pi\varepsilon_o R^2}\left(\frac{2x}{2\sqrt{x^2 + R^2}} - 1\right) = -\frac{Q}{2\pi\varepsilon_o R^2}\left(\frac{x}{\sqrt{x^2 + R^2}} - 1\right)$$

$$\boxed{E_x = \frac{Q}{2\pi\varepsilon_o R^2}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)}$$

(This is the same result found in class using when finding the electric field of a uniform disk for points on the *x*-axis.)

7.) The electric field outside a conductive sphere can be obtained using Gauss's Law.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} \quad \text{and for spherical Gaussian surfaces} \quad E4\pi r^2 = \frac{q_{enclosed}}{\varepsilon_o}$$

$$E = \frac{q_{enclosed}}{4\pi\varepsilon_o r^2} = \frac{Q}{4\pi\varepsilon_o r^2} \qquad \text{for both spheres}$$

a.)

To get the electric potential: $V_a - V_b = -\int_b^a \vec{E}.d\vec{r}$

Integration starts at $r = \infty$ (where $V = 0$) to a point r which is outside the charged sphere.

$$V_r - V_\infty = -\int_\infty^r \frac{Q}{4\pi\varepsilon_o r^2}\,dr = -\frac{Q}{4\pi\varepsilon_o}\int_\infty^r \frac{1}{r^2}\,dr$$

$$V_r - 0 = -\frac{Q}{4\pi\varepsilon_o}\int_\infty^r r^{-2}dr = -\frac{Q}{4\pi\varepsilon_o}\left.\frac{r^{-1}}{(-1)}\right]_\infty^r = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$V_r = \frac{Q}{4\pi\varepsilon_o r} \qquad \text{for both spheres}$$

For smaller sphere at the surface $r = R$ and $V_R = \frac{Q_B}{4\pi\varepsilon_o R}$

For larger sphere at the surface $r = 3R$ and $V_{3R} = \frac{Q_A}{4\pi\varepsilon_o 3R} = \frac{Q_A}{12\pi\varepsilon_o R}$

The electric potential is the same for both on the surface so $V_R = V_{3R}$ and

$$\frac{Q_B}{4\pi\varepsilon_o R} = \frac{Q_A}{12\pi\varepsilon_o R} \quad \text{and} \quad \frac{Q_B}{Q_A} = \frac{4\pi\varepsilon_o R}{12\pi\varepsilon_o R} = \frac{1}{3}$$

$$\boxed{\frac{Q_B}{Q_A} = \frac{1}{3}}$$

b.)

For smaller sphere at the surface $r = R$ and $E_R = \frac{Q_B}{4\pi\varepsilon_o R^2} = E_B$

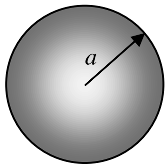
For larger sphere at the surface $r = 3R$ and $E_{3R} = \frac{Q_A}{4\pi\varepsilon_o (3R)^2} = \frac{Q_A}{36\pi\varepsilon_o R^2} = E_A$

Since $\frac{Q_B}{Q_A} = \frac{1}{3}$ it follows that $\frac{E_B}{E_A} = \frac{\frac{Q_B}{4\pi\varepsilon_o R^2}}{\frac{Q_A}{36\pi\varepsilon_o R^2}} = \frac{Q_B}{Q_A}\frac{36\pi\varepsilon_o R^2}{4\pi\varepsilon_o R^2} = \frac{1}{3}\left(\frac{9}{1}\right)$

$$\boxed{\frac{E_B}{E_A} = 3}$$

8.)

a.)



$$\rho(r) = \rho_o \left(\frac{r}{a}\right)^3$$

$$V_a - V_b = -\int_b^a \vec{E}.d\vec{r} \quad \text{and get } E \text{ using Gauss's Law} \quad E4\pi r^2 = \frac{q_{enclosed}}{\varepsilon_o}$$

$$E = \frac{q_{enclosed}}{4\pi\varepsilon_o r^2}$$

($r > a$)

Outside the sphere the field is due to the entire charge.

$$q_{enclosed} = q_{total} = \int \rho(r) dV$$ this is more difficult because charge density is a function of r

for a sphere: $V = \frac{4}{3}\pi r^3$ so $\frac{dV}{dr} = 4\pi r^2$ and $dV = 4\pi r^2 dr$

The total charge is: $q_{total} = \int \rho(r) dV = \int_0^a \rho_o \left(\frac{r}{a}\right)^3 4\pi r^2 dr = \int_0^a \frac{4\pi\rho_o}{a^3} r^5 dr$

$$q_{total} = \frac{4\pi\rho_o}{a^3}\int_0^a r^5 dr = \frac{4\pi\rho_o}{a^3}\frac{r^6}{6}\Big]_0^a = \frac{4\pi\rho_o}{a^3}\left(\frac{a^6}{6} - 0\right) = \frac{4\pi\rho_o a^6}{6a^3}$$

$$q_{total} = \frac{2\pi\rho_o a^3}{3}$$

Therefore: $E = \frac{q_{enclosed}}{4\pi\varepsilon_o r^2} = \frac{\left(\frac{2\pi\rho_o a^3}{3}\right)}{4\pi\varepsilon_o r^2} = \frac{\rho_o a^3}{6\varepsilon_o r^2}$

Integration starts at $r = \infty$ (where $V = 0$) to a point r which is outside the charged sphere.

$$V_r - V_\infty = -\int_\infty^r \frac{\rho_o a^3}{6\varepsilon_o r^2} dr = -\frac{\rho_o a^3}{6\varepsilon_o}\int_\infty^r \frac{1}{r^2} dr = -\frac{\rho_o a^3}{6\varepsilon_o}\int_\infty^r r^{-2} dr$$

$$V_r - 0 = -\frac{\rho_o a^3}{6\varepsilon_o}\left(\frac{r^{-1}}{-1}\right)\Bigg]_\infty^r = \frac{\rho_o a^3}{6\varepsilon_o}\left(\frac{1}{r}\right)\Bigg]_\infty^r = \frac{\rho_o a^3}{6\varepsilon_o}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{\rho_o a^3}{6\varepsilon_o}\left(\frac{1}{r} - 0\right)$$

$$\boxed{V_r = \frac{\rho_o a^3}{6\varepsilon_o r} \qquad (r > a)}$$

($r < a$)

Inside the sphere the field is only due the portion of the charge that is enclosed by the Gaussian surface.

$$q_{enclosed} = \int \rho(r) dV = \int_0^r \rho_o \left(\frac{r}{a}\right)^3 4\pi r^2 dr = \int_0^r \frac{4\pi\rho_o}{a^3} r^5 dr$$

$$q_{enclosed} = \frac{4\pi\rho_o}{a^3}\int_0^r r^5 dr = \frac{4\pi\rho_o}{a^3}\frac{r^6}{6}\Big]_0^r = \frac{4\pi\rho_o}{a^3}\left(\frac{r^6}{6} - 0\right) = \frac{4\pi\rho_o r^6}{6a^3}$$

$$q_{enclosed} = \frac{2\pi\rho_o r^6}{3a^3}$$

Using Gauss's Law: $$E = \frac{q_{enclosed}}{4\pi\varepsilon_o r^2} = \frac{\left(\frac{2\pi\rho_o r^6}{3a^3}\right)}{4\pi\varepsilon_o r^2}$$

$$E = \frac{\rho_o r^4}{6\varepsilon_o a^3} \qquad (r < a)$$

Integration starts at $r = a$ (where $V_a = \frac{\rho_o a^3}{6\varepsilon_o a} = \frac{\rho_o a^2}{6\varepsilon_o}$) to a point r which is inside the charged sphere.

$$V_r - V_a = -\int_a^r \frac{\rho_o r^4}{6\varepsilon_o a^3} dr = -\frac{\rho_o}{6\varepsilon_o a^3}\int_a^r r^4 dr$$

$$V_r - \frac{\rho_o a^2}{6\varepsilon_o} = -\frac{\rho_o}{6\varepsilon_o a^3}\left(\frac{r^5}{5}\right)\Bigg]_a^r = -\frac{\rho_o}{6\varepsilon_o a^3}\left(\frac{r^5}{5} - \frac{a^5}{5}\right)$$

$$V_r = -\frac{\rho_o}{6\varepsilon_o a^3}\left(\frac{r^5}{5} - \frac{a^5}{5}\right) + \frac{\rho_o a^2}{6\varepsilon_o} = -\frac{\rho_o}{6\varepsilon_o}\left(\frac{r^5}{5a^3} - \frac{a^2}{5} - a^2\right) = -\frac{\rho_o}{6\varepsilon_o}\left(\frac{r^5}{5a^3} - \frac{a^2}{5} - \frac{5a^2}{5}\right)$$

$$V_r = -\frac{\rho_o}{6\varepsilon_o}\left(\frac{r^5}{5a^3} - \frac{6a^2}{5}\right) = -\frac{\rho_o}{30\varepsilon_o}\left(\frac{r^5}{a^3} - 6a^2\right)$$

$$\boxed{V_r = \frac{\rho_o}{30\varepsilon_o}\left(6a^2 - \frac{r^5}{a^3}\right) \quad (r < a)}$$

At $r = a$ $$V_a = \frac{\rho_o}{30\varepsilon_o}\left(6a^2 - \frac{a^5}{a^3}\right) = \frac{\rho_o}{30\varepsilon_o}\left(6a^2 - a^2\right) = \frac{\rho_o}{6\varepsilon_o}a^2$$

b.)