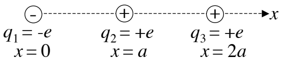
Example 1:

Two point charges are located on the *x*-axis, $q_1 = -e$ at $x = 0$ and $q_2 = +e$ at $x = a$.

a.) Find the work that must be done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$.

b.) Find the total potential energy of the system of three charges.

Example 1:



a.) $W_{force} = ?$

$$W_{field} = -\Delta U = -(U_2 - U_1) = -(U_{13} + U_{23})$$

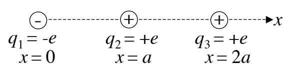
$$r_{13} = r_{23} = \infty$$

$$W_{field} = -\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}\right) = -\frac{1}{4\pi\varepsilon_0}\left(\frac{(-e)(e)}{2a} + \frac{(e)(e)}{a}\right)$$

$$W_{field} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2a} = -\frac{e^2}{8\pi\varepsilon_0 a}$$

$$W_{force} = -W_{field} = \boxed{+\frac{e^2}{8\pi\varepsilon_0 a}}$$

Example 1:



b.) $U_{total} = ?$

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$U_{total} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{(-e)(e)}{a} + \frac{(-e)(e)}{2a} + \frac{(e)(e)}{a}\right)$$

$$\boxed{U_{total} = -\frac{e^2}{8\pi\varepsilon_0 a}}$$

Example 2:

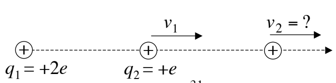
A positron has a mass of 9.11×10^{-31} kg and a charge of $+e = +1.60 \times 10^{-19}$ C. Suppose a positron moves in the vicinity of a stationary alpha particle, which has a charge of $+2e = 3.20 \times 10^{-19}$ C. When the positron is 1.00×10^{-10} m from the alpha particle, it is moving directly away from the alpha particle at a speed of 3.00×10^{6} m/s.

a.) What is the positron's speed when the two particles are 2.00×10^{-10} m apart?

b.) What is the positron's speed when it is very far away from the alpha particle?

c.) Suppose the moving particle were an *electron* heading away the alpha particle. What is the furthest distance away from the alpha particle that the electron reaches?

Example 2a:



$m = 9.11 \times 10^{-31}$ kg $\quad r_2 = 2.00 \times 10^{-10}$ m

$r_1 = 1.00 \times 10^{-10}$ m

$v_1 = 3.00 \times 10^{6}\ \frac{\text{m}}{\text{s}}$

$$K_1 + U_1 = K_2 + U_2$$

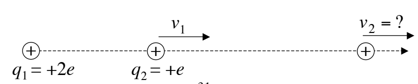
$$\frac{1}{2}mv_1^2 + k\frac{q_1 q_2}{r_1} = \frac{1}{2}mv_2^2 + k\frac{q_1 q_2}{r_2}$$

$$v_2 = \sqrt{v_1^2 + \frac{2kq_1q_2}{m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \sqrt{v_1^2 + \frac{4ke^2}{m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$v_2 = \sqrt{\left(3\times10^6\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{4\left(9\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{9.11\times10^{-31}\ \text{kg}}\left(\frac{1}{1\times10^{-10}\ \text{m}} - \frac{1}{2\times10^{-10}\ \text{m}}\right)}$$

$$\boxed{v_2 = 3.75\times10^6\ \frac{\text{m}}{\text{s}}}$$

Example 2b:



$m = 9.11 \times 10^{-31}$ kg $\quad r_2 = \infty$

$r_1 = 1.00 \times 10^{-10}$ m

$v_1 = 3.00 \times 10^{6}\ \frac{\text{m}}{\text{s}}$

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 + k\frac{q_1 q_2}{r_1} = \frac{1}{2}mv_2^2 + k\frac{q_1 q_2}{r_2}$$

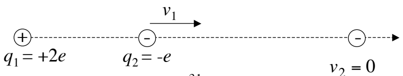
$$r_2 = \infty$$

$$v_2 = \sqrt{v_1^2 + \frac{2kq_1q_2}{mr_1}} = \sqrt{v_1^2 + \frac{4ke^2}{mr_1}}$$

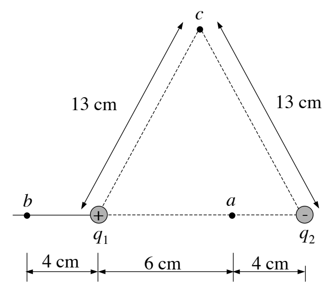
$$v_2 = \sqrt{\left(3\times10^6\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{4\left(9\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1\times10^{-10}\ \text{m}\right)}}$$

$$\boxed{v_2 = 4.37\times10^6\ \frac{\text{m}}{\text{s}}}$$

Example 2c:



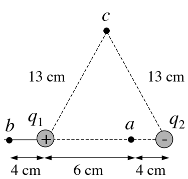
$$m = 9.11\times10^{-31}\ \text{kg}$$
$$r_1 = 1.00\times10^{-10}\ \text{m}$$
$$v_1 = 3.00\times10^{6}\ \frac{\text{m}}{\text{s}}$$
$$K_1 + U_1 = K_2 + U_2$$
$$\frac{1}{2}mv_1^2 + k\frac{q_1q_2}{r_1} = \frac{1}{2}mv_2^2 + k\frac{q_1q_2}{r_2}$$
$$v_2 = 0$$
$$r_2 = \frac{kq_1q_2}{\frac{1}{2}mv_1^2 + k\frac{q_1q_2}{r_1}} = \frac{2kq_1q_2}{mv_1^2 + 2k\frac{q_1q_2}{r_1}} = \frac{-4ke^2}{mv_1^2 - 4k\frac{e^2}{r_1}}$$
$$r_2 = \frac{-4\left(9\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(3\times10^{6}\ \frac{\text{m}}{\text{s}}\right)^2 - 4\left(9\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(1.6\times10^{-19}\ \text{C}\right)^2}{\left(1\times10^{-10}\ \text{m}\right)}}$$
$$\boxed{r_2 = 9.06\times10^{-10}\ \text{m}}$$



Example 3:

An electric dipole consists of two point charges, q_1 = +12 nC and q_2 = -12 nC, placed 10 cm apart. Compute the potentials at points *a*, *b*, and *c* by adding the potentials due to either charge.

Example 3:

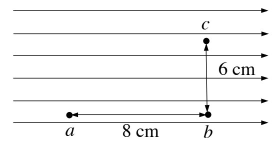


$q_1 = 12$ nC

$q_2 = -12$ nC

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i} = k\sum_i \frac{q_i}{r_i}$$
$$V = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$
$$V_a = 9\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{12\times10^{-9}\ \text{C}}{0.06\ \text{m}} + \frac{-12\times10^{-9}\ \text{C}}{0.04\ \text{m}}\right)$$
$$\boxed{V_a = -900\ \text{V}}$$
$$V_b = 9\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{12\times10^{-9}\ \text{C}}{0.04\ \text{m}} + \frac{-12\times10^{-9}\ \text{C}}{0.14\ \text{m}}\right)$$
$$\boxed{V_b = 1930\ \text{V}}$$
$$V_c = 9\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{12\times10^{-9}\ \text{C}}{0.13\ \text{m}} + \frac{-12\times10^{-9}\ \text{C}}{0.13\ \text{m}}\right)$$
$$\boxed{V_c = 0}$$

E = 10,000 N/C



Example 4:

A charge $q = 2\ \mu$C is moved through a uniform field directed to the right as shown in the figure above. Find the work done by the field when the charge moves from:

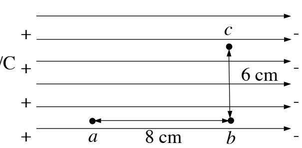
a.) *a* to *b* b.) *b* to *c* c.) *a* to *c* d.) *c* to *a*

Example 4:

E = 10,000 N/C

$q_0 = 2\ \mu$C

$W_{field} = ?$



$W_{field} = -\Delta U = -q_0\Delta V$

For uniform E-fields:

$\Delta V = -Ed$

Where d is the distance in the direction of the field.

so $W_{field} = -q_0\Delta V = q_0Ed$

a.) *a* to *b*

$$W_{field} = q_0Ed$$
$$W_{field} = \left(2\times10^{-6}\ \text{C}\right)\left(10{,}000\ \frac{\text{N}}{\text{C}}\right)(0.08\ \text{m})$$
$$\boxed{W_{field} = 1.6\times10^{-3}\ \text{J}}$$

b.) *b* to *c*

$$W_{field} = q_0Ed$$
$$W_{field} = \left(2\times10^{-6}\ \text{C}\right)\left(10{,}000\ \frac{\text{N}}{\text{C}}\right)(0)$$
$$\boxed{W_{field} = 0}$$

c.) *a* to *c*

$$W_{field} = q_0Ed$$
$$W_{field} = \left(2\times10^{-6}\ \text{C}\right)\left(10{,}000\ \frac{\text{N}}{\text{C}}\right)(0.08\ \text{m})$$
$$\boxed{W_{field} = 1.6\times10^{-3}\ \text{J}}$$

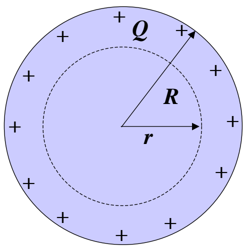
d.) *c* to *a*

$$W_{field} = q_0Ed$$
$$W_{field} = \left(2\times10^{-6}\ \text{C}\right)\left(10{,}000\ \frac{\text{N}}{\text{C}}\right)(-0.08\ \text{m})$$
$$\boxed{W_{field} = -1.6\times10^{-3}\ \text{J}}$$

Example 5:

A solid conducting sphere of radius *R* has a total charge *Q*. Find the potential everywhere, both outside and inside the sphere.

Example 5:



$r < R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

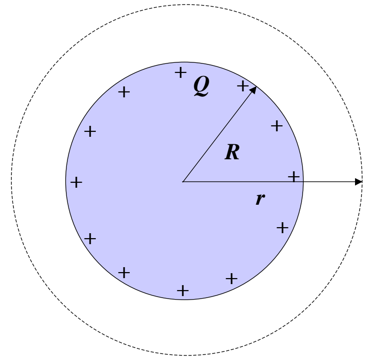
$$\oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{0}{\varepsilon_0}$$

$$E = 0$$

Example 5:



$r > R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

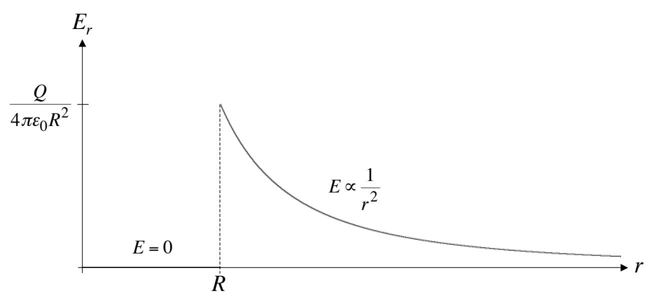
$$\oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{enc}}{\varepsilon_0}$$

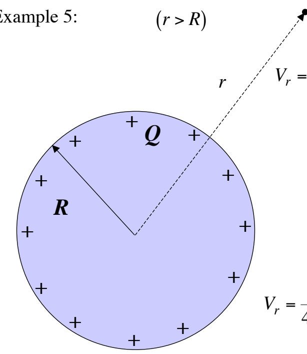
$$E 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Example 5: Charged Spherical Conductor



Example 5: $(r > R)$



$r_2 = \infty$ $V_\infty = 0$

$V_r = ?$

$$V_r - V_\infty = -\int_\infty^r \vec{E} \cdot d\vec{r}$$

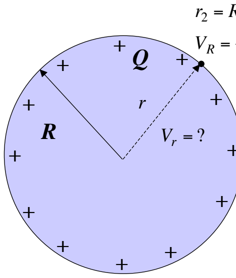
$$V_r - 0 = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$V_r = \frac{-Q}{4\pi\varepsilon_0} \int_\infty^r \frac{dr}{r^2}$$

$$V_r = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r}\Big]_\infty^r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - 0\right)$$

$$\boxed{V_r = \frac{Q}{4\pi\varepsilon_0 r} \quad (r > R)}$$

Example 5: $(r < R)$



$r_2 = R$

$V_R = \frac{Q}{4\pi\varepsilon_0 R}$

$$V_r - V_R = -\int_R^r \vec{E} \cdot d\vec{r}$$

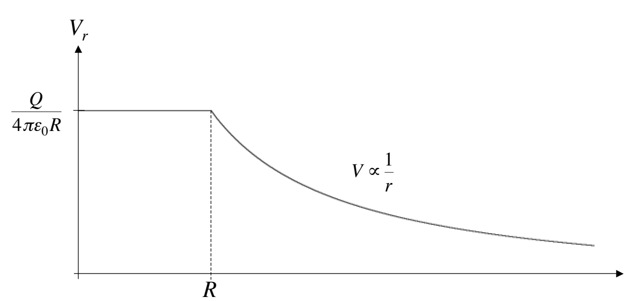
$$V_r - V_R = -\int_R^r 0 dr$$

$$V_r - V_R = 0$$

$$V_r = V_R$$

$$\boxed{V_r = \frac{Q}{4\pi\varepsilon_0 R} \quad (r < R)}$$

Example 5: Charged Spherical Conductor



Example 6:

A solid insulating sphere of radius *R* has a total charge *Q* uniformly distributed throughout its volume. Find the potential everywhere, both outside and inside the sphere.

Example 6:

$$q_{enc} = \int_0^r \rho dV$$

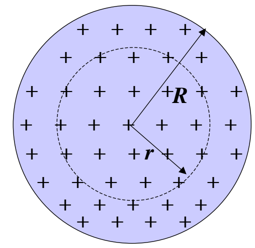
$$q_{enc} = \rho \int_0^r dV$$

$$q_{enc} = \rho V(r)$$

$$q_{enc} = \rho \frac{4}{3}\pi r^3$$

$$\left(\rho = \frac{3Q}{4\pi R^3}\right)$$

$$q_{enc} = Q\frac{r^3}{R^3}$$



r* < *R

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint EdA = \frac{q_{enc}}{\varepsilon_0}$$

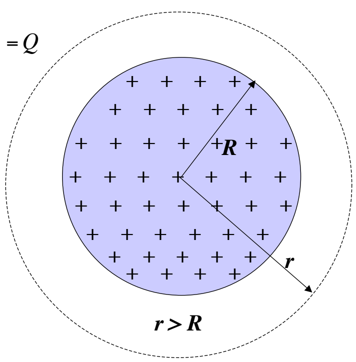
$$E\oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E4\pi r^2 = \frac{\rho\frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E = \frac{\rho r}{3\varepsilon_0} = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

Example 6

$$q_{enc} = Q$$



r* > *R

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

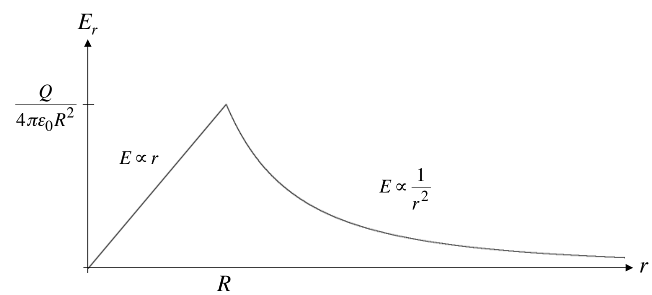
$$\oint EdA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{enc}}{\varepsilon_0}$$

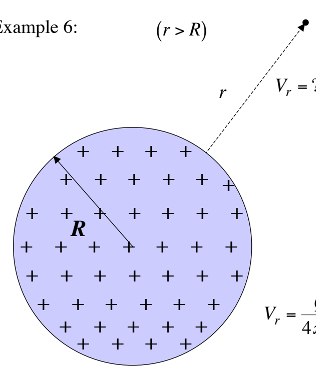
$$E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Example 6: Uniformly Charged Spherical Insulator



Example 6: $(r > R)$



$r_2 = \infty$ $V_\infty = 0$

$V_r = ?$

$$V_r - V_\infty = -\int_\infty^r \vec{E}\cdot d\vec{r}$$

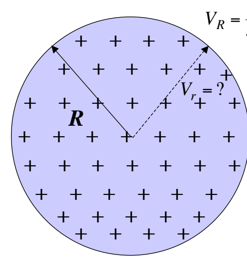
$$V_r - 0 = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2}dr$$

$$V_r = \frac{-Q}{4\pi\varepsilon_0}\int_\infty^r \frac{dr}{r^2}$$

$$V_r = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\Bigg]_\infty^r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - 0\right)$$

$$\boxed{V_r = \frac{Q}{4\pi\varepsilon_0 r} \quad (r > R)}$$

Example 6: $(r < R)$



$r_2 = R$

$$V_R = \frac{Q}{4\pi\varepsilon_0 R}$$

$$V_r - V_R = -\int_R^r \vec{E}\cdot d\vec{r}$$

$$V_r - V_R = -\int_R^r \frac{Qr}{4\pi\varepsilon_0 R^3}dr$$

$$V_r - V_R = \frac{-Q}{4\pi\varepsilon_0 R^3}\int_R^r r dr$$

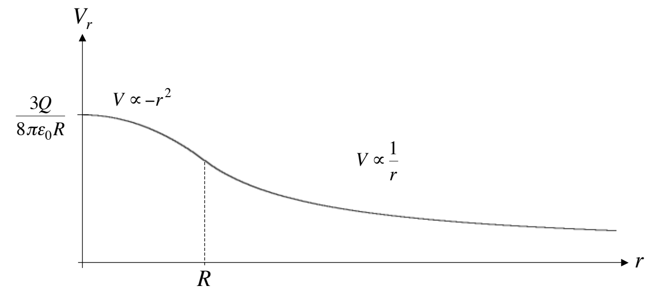
$$V_r - V_R = \frac{-Q}{4\pi\varepsilon_0 R^3}\frac{r^2}{2}\Bigg]_R^r = \frac{-Q}{8\pi\varepsilon_0 R^3}\left(r^2 - R^2\right)$$

$$V_r = \frac{Q}{8\pi\varepsilon_0 R^3}\left(R^2 - r^2\right) + V_R$$

$$V_r = \frac{Q}{8\pi\varepsilon_0 R^3}\left(R^2 - r^2\right) + \frac{Q}{4\pi\varepsilon_0 R}$$

$$\boxed{V_r = \frac{Q}{8\pi\varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right) \quad (r < R)}$$

Example 6: Uniformly Charged Spherical Insulator



Example 7:

A solid insulating sphere of radius *R* has a total charge *Q* uniformly distributed throughout its volume and is concentric with a conducting shell of inner radius 2*R* and outer radius 3*R* that has a total charge of -2*Q*. Find the potential for:

a.) $r > 3R$

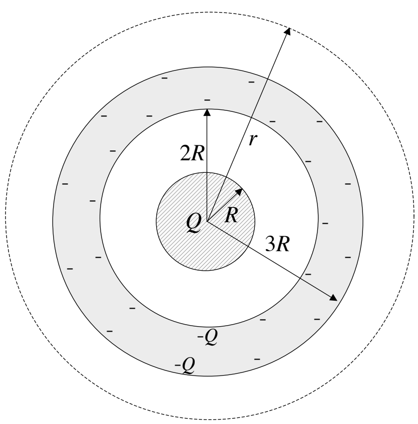
b.) $2R < r < 3R$

c.) $R < r < 2R$

d.) $r < R$

Example 7a:

$r > 3R$



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

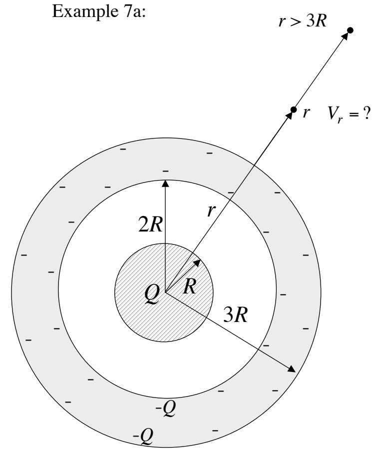
$$E\oint dA = \frac{-2Q+Q}{\varepsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{-Q}{\varepsilon_0}$$

$$E = \frac{-Q}{4\pi\varepsilon_0 r^2}$$

Example 7a:

$r > 3R$ $r_2 = \infty$ $V_\infty = 0$



$$V_r - V_\infty = -\int_\infty^r \vec{E}\cdot d\vec{r}$$

$$V_r - 0 = -\int_\infty^r \frac{-Q}{4\pi\varepsilon_0 r^2}dr$$

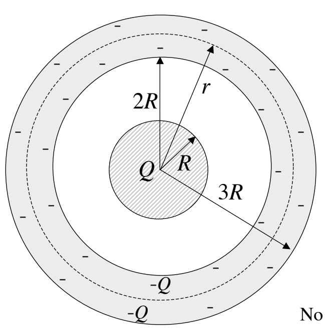
$$V_r = \frac{Q}{4\pi\varepsilon_0}\int_\infty^r \frac{dr}{r^2}$$

$$V_r = \frac{Q}{4\pi\varepsilon_0}\left.\frac{-1}{r}\right]_\infty^r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{-1}{r} - \frac{-1}{\infty}\right)$$

$$V_r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{-1}{r} - 0\right) = \boxed{\frac{-Q}{4\pi\varepsilon_0 r}}$$

Example 7b:

$2R < r < 3R$



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{-Q+Q}{\varepsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{0}{\varepsilon_0}$$

$$E = 0$$

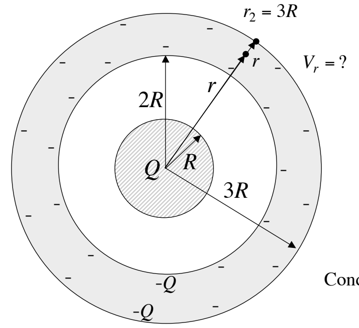
No electric field inside a conductor.

Example 7b:

$2R < r < 3R$

$$V_{3R} = \frac{-Q}{12\pi\varepsilon_0 R}$$

$r_2 = 3R$



$$V_r - V_{3R} = -\int_{3R}^r \vec{E}\cdot d\vec{r}$$

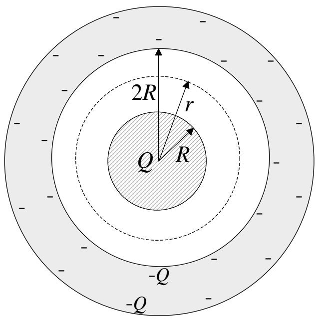
$$V_r - V_{3R} = -\int_{3R}^r 0\cdot d\vec{r}$$

$$V_r - V_{3R} = 0$$

$$V_r = V_{3R} = \boxed{\frac{-Q}{12\pi\varepsilon_0 R}}$$

Conductors are equipotential throughout.

Example 7c: $R < r < 2R$



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

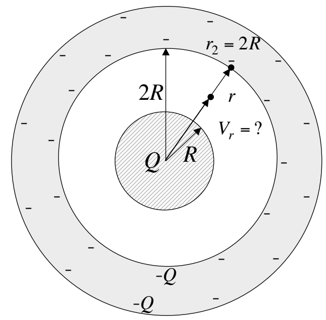
$$E\oint dA = \frac{Q}{\varepsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Example 7c: $R < r < 2R$

$$V_{2R} = V_{3R} = \frac{-Q}{12\pi\varepsilon_0 R}$$



$$V_r - V_{2R} = -\int_{2R}^{r} \vec{E}\cdot d\vec{r}$$

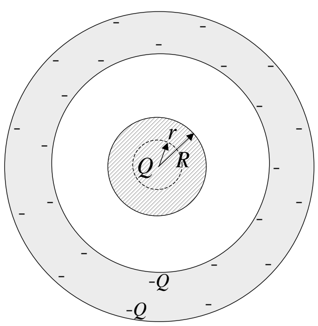
$$V_r - V_{2R} = -\int_{2R}^{r} \frac{Q}{4\pi\varepsilon_0 r^2}dr$$

$$V_r - \frac{-Q}{12\pi\varepsilon_0 R} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\Big]_{2R}^{r}$$

$$V_r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{2R}\right) - \frac{Q}{12\pi\varepsilon_0 R}$$

$$\boxed{V_r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{5}{6R}\right)}$$

Example 7d: $r < R$



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{enc}}{\varepsilon_0}$$

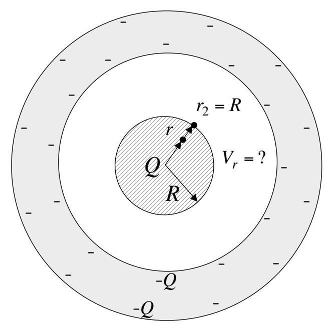
$$E\left(4\pi r^2\right) = \frac{\rho V(r)}{\varepsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{\frac{3Q}{4\pi R^3}\frac{4\pi r^3}{3}}{\varepsilon_0}$$

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

Example 7d: $r < R$

$$V_R = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{5}{6R}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{6R}$$



$$V_r - V_R = -\int_{R}^{r} \vec{E}\cdot d\vec{r}$$

$$V_r - V_R = -\int_{R}^{r} \frac{Qr}{4\pi\varepsilon_0 R^3}dr$$

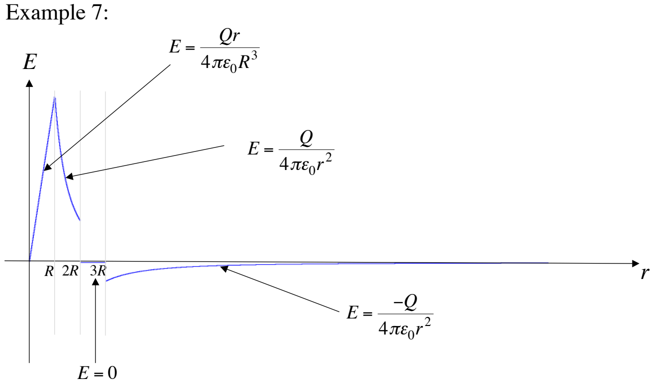
$$V_r - \frac{Q}{4\pi\varepsilon_0}\frac{1}{6R} = \frac{-Q}{4\pi\varepsilon_0 R^3}\frac{r^2}{2}\Big]_{R}^{r}$$

$$V_r - \frac{Q}{4\pi\varepsilon_0}\frac{1}{6R} = \frac{-Q}{4\pi\varepsilon_0 R^3}\left(\frac{r^2}{2} - \frac{R^2}{2}\right)$$

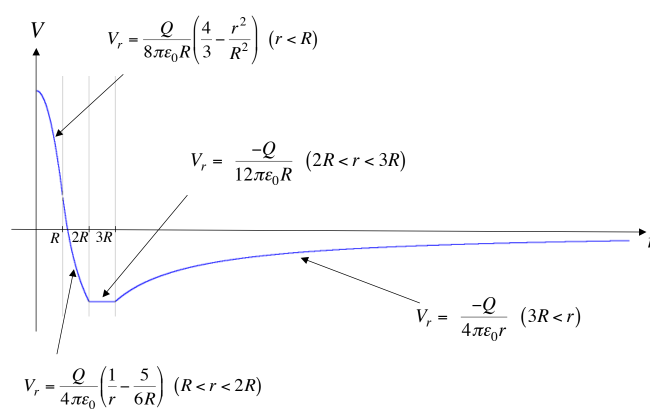
$$V_r = \frac{Q}{4\pi\varepsilon_0 R^3}\left(\frac{R^2}{2} - \frac{r^2}{2}\right) + \frac{Q}{4\pi\varepsilon_0}\frac{1}{6R}$$

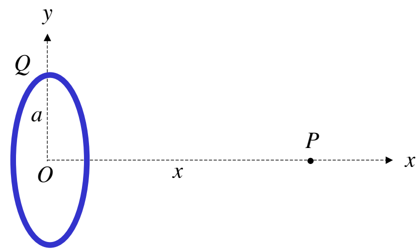
$$\boxed{V_r = \frac{Q}{8\pi\varepsilon_0 R}\left(\frac{4}{3} - \frac{r^2}{R^2}\right)}$$

Example 7:



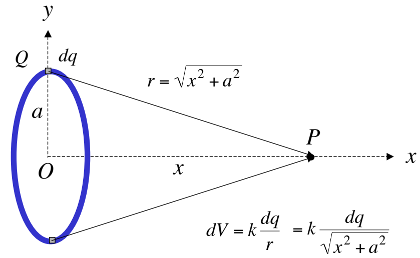
Example 7:





Example 8:
A ring-shaped conductor with radius *a* in the *y*-*z* plane carries a total charge *Q* uniformly distributed around it. Find the electric potential at a point *P* that lies on the axis of the ring at a distance *x* from the origin.

Example 8:

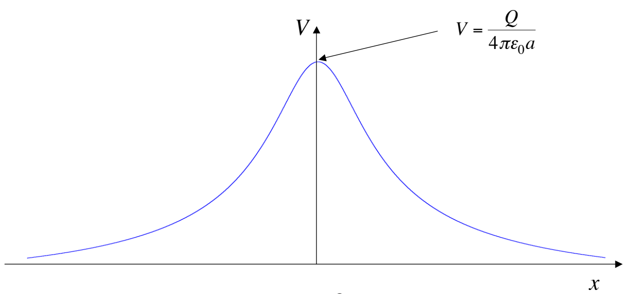


$$dV = k\frac{dq}{r} = k\frac{dq}{\sqrt{x^2+a^2}}$$

$$V = \int k\frac{dq}{\sqrt{x^2+a^2}} = \frac{k}{\sqrt{x^2+a^2}}\int dq = \frac{k}{\sqrt{x^2+a^2}}Q$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+a^2}}}$$

Example 8:



$$V = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+a^2}}$$

Example 8:

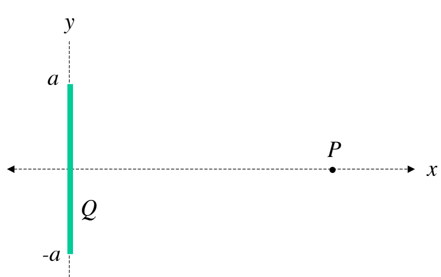
$$V = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+a^2}}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{Q}{4\pi\varepsilon_0\sqrt{x^2+a^2}}\right)$$

$$E_x = -\frac{Q}{4\pi\varepsilon_0}\frac{d}{dx}\left(x^2+a^2\right)^{-1/2}$$

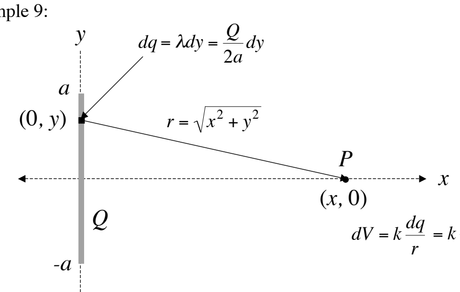
$$E_x = \frac{Q}{8\pi\varepsilon_0}\left(x^2+a^2\right)^{-3/2}(2x)$$

$$E_x = \frac{Qx}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}$$



Example 9:
Positive electric charge *Q* is distributed uniformly along a line with length 2*a*, lying along the *y*-axis between *y* = -*a* and *y* = *a*. Find the electric potential at point *P* on the *x*-axis at a distance of *x* from the origin.

Example 9:



$$dV = k\frac{dq}{r} = k\frac{\lambda dy}{\sqrt{x^2+y^2}}$$

$$V = \int_{-a}^{a} k\frac{\lambda dy}{\sqrt{x^2+y^2}} = k\lambda\int_{-a}^{a}\frac{dy}{\sqrt{x^2+y^2}}$$

(from integral tables) $\int\frac{dy}{\sqrt{x^2+y^2}} = \ln\left(\sqrt{x^2+y^2}+y\right)+C$

Example 9:

(from integral tables) $\int \frac{dy}{\sqrt{x^2+y^2}} = \ln\left(\sqrt{x^2+y^2}+y\right)+C$

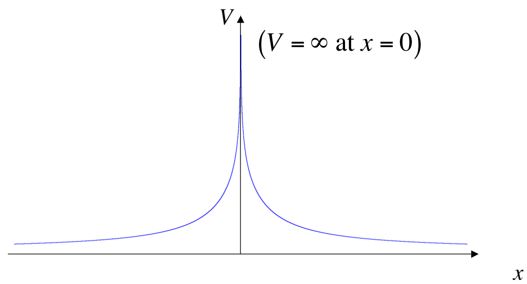
$$V = k\lambda \int_{-a}^{a} \frac{dy}{\sqrt{x^2+y^2}} = k\lambda \ln\left(\sqrt{x^2+y^2}+y\right)\Big]_{-a}^{a}$$

$$V = k\lambda\left(\ln\left(\sqrt{x^2+a^2}+a\right)-\ln\left(\sqrt{x^2+a^2}-a\right)\right)$$

$$V = k\lambda \ln\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right)$$

$$\boxed{V = \frac{Q}{8\pi\varepsilon_0 a}\ln\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right)}$$

Example 9:



$$V = \frac{Q}{8\pi\varepsilon_0 a}\ln\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right)$$

Example 9:

$$V = \frac{Q}{8\pi\varepsilon_0 a}\ln\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right)$$

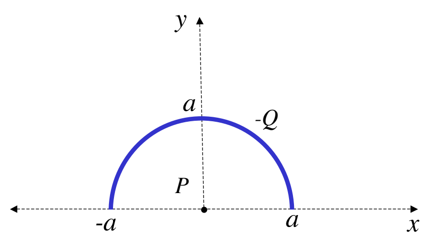
$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{Q}{8\pi\varepsilon_0 a}\ln\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right)\right)$$

$$E_x = -\frac{dV}{dx} = \frac{-Q}{8\pi\varepsilon_0 a}\left(\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+a^2}+a}\right)\frac{\left(\sqrt{x^2+a^2}-a\right)\left(\tfrac{1}{2}\right)\left(x^2+a^2\right)^{-1/2}(2x)-\left(\sqrt{x^2+a^2}+a\right)\left(\tfrac{1}{2}\right)\left(x^2+a^2\right)^{-1/2}(2x)}{\left(\sqrt{x^2+a^2}-a\right)^2}$$

$$E_x = \frac{-Q}{8\pi\varepsilon_0 a}\frac{\left(\sqrt{x^2+a^2}-a\right)\left(x^2+a^2\right)^{-1/2}(x)-\left(\sqrt{x^2+a^2}+a\right)\left(x^2+a^2\right)^{-1/2}(x)}{\left(\sqrt{x^2+a^2}+a\right)\left(\sqrt{x^2+a^2}-a\right)}$$

$$E_x = \frac{-Q}{8\pi\varepsilon_0 a}\frac{x\sqrt{x^2+a^2}x-ax-x\sqrt{x^2+a^2}-ax}{\sqrt{x^2+a^2}\left(x^2+a^2-a^2\right)}$$

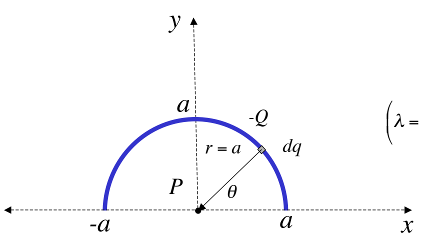
$$E_x = \frac{-Q}{8\pi\varepsilon_0 a}\frac{(-2ax)}{\sqrt{x^2+a^2}\left(x^2\right)} = \frac{Q}{4\pi\varepsilon_0 x\sqrt{x^2+a^2}}$$



Example 10:

A negative charge -*Q* is uniformly distributed around a semicircle of radius *a*. Find the electric potential at the center of curvature *P*. Express your answer in terms of *a*, λ, and ε_o.

Example 10:



$$\left(\lambda = \frac{-Q}{\pi a}\right)$$

$$dV = k\frac{dq}{r} = k\frac{dq}{a}$$

$$V = \int k\frac{dq}{a} = \frac{k}{a}\int dq = \frac{k}{a}(-Q)$$

$$\boxed{V = \frac{-kQ}{a} = \frac{-(-\lambda\pi a)}{4\pi\varepsilon_0 a} = \frac{\lambda}{4\varepsilon_0}}$$