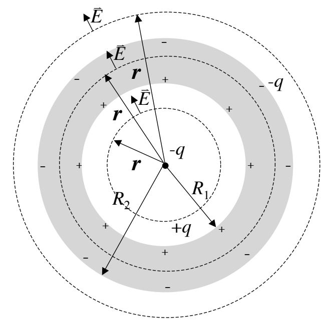
Example 8:

A point charge with charge -q is located in the center of a spherical conductive shell with an inner radius R_1 and outer radius R_2.

a.) Find the magnitude of the electric field at a point P a distance r from the center of the spherical shell.

b.) Repeat (a.) if the outer shell is connected to ground.

Example 8a:



$$E = \frac{q_{enc}}{4\pi\varepsilon_0 r^2}$$

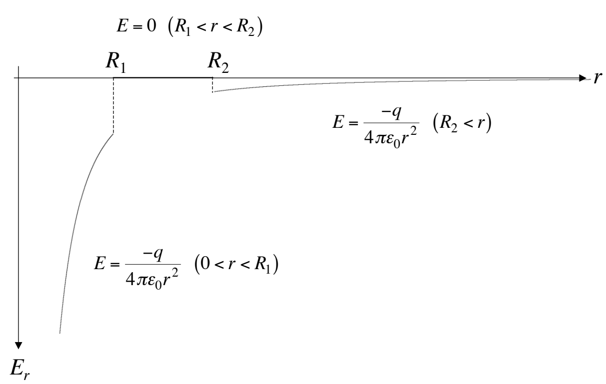
$$(0 < r < R_1) \quad \boxed{E = \frac{-q}{4\pi\varepsilon_0 r^2}}$$

$$(R_1 < r < R_2) \quad \boxed{E = \frac{-q+q}{4\pi\varepsilon_0 r^2} = 0}$$

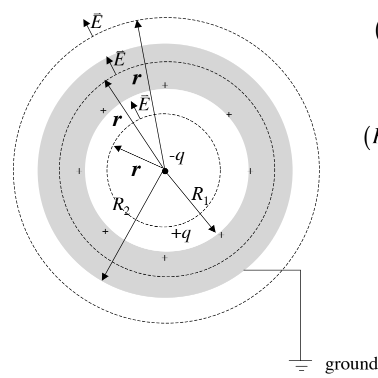
$$(R_2 < r) \quad E = \frac{-q+q-q}{4\pi\varepsilon_0 r^2}$$

$$\boxed{E = \frac{-q}{4\pi\varepsilon_0 r^2}}$$

Example 8a:



Example 8b:



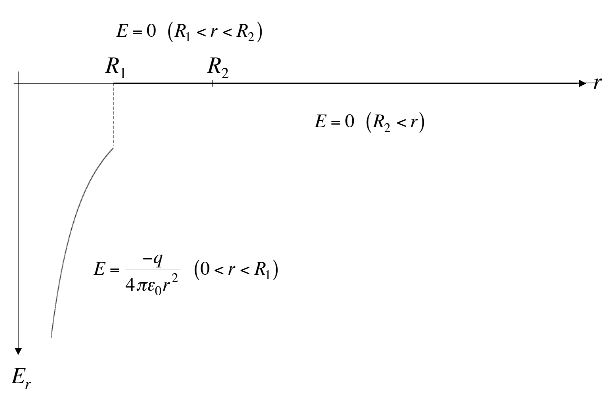
$$E = \frac{q_{enc}}{4\pi\varepsilon_0 r^2}$$

$$(0 < r < R_1) \quad \boxed{E = \frac{-q}{4\pi\varepsilon_0 r^2}}$$

$$(R_1 < r < R_2) \quad \boxed{E = \frac{-q+q}{4\pi\varepsilon_0 r^2} = 0}$$

$$(R_2 < r) \quad \boxed{E = \frac{-q+q}{4\pi\varepsilon_0 r^2} = 0}$$

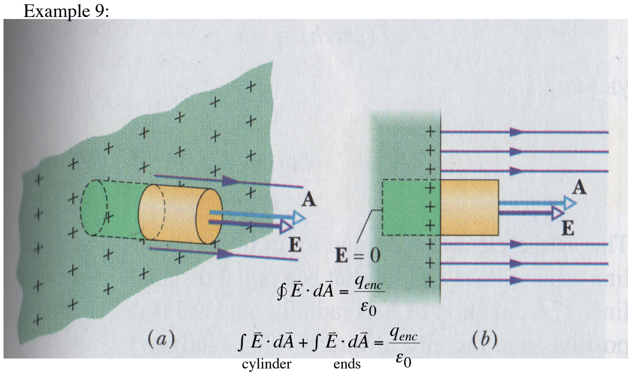
Example 8b:



Example 9:

Use Gauss's Law to find the electric field just outside an infinite conductive surface with a charge density σ.

Example 9:



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\underset{\text{cylinder}}{\int \vec{E}\cdot d\vec{A}} + \underset{\text{ends}}{\int \vec{E}\cdot d\vec{A}} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + E\int dA = \frac{q_{enc}}{\varepsilon_0} \quad \text{so} \quad EA = \frac{\sigma A}{\varepsilon_0} \quad \text{and} \quad \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

For conductive spheres the E-field just above its surface is the electric field outside the sphere evaluated at $r = R$.

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2} = \frac{\sigma}{\varepsilon_0}$$

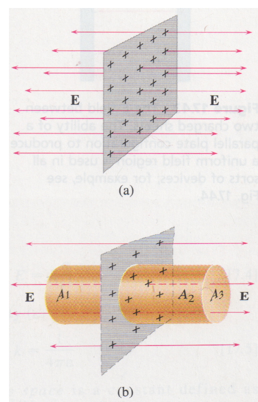
For conductive cylinders the E-field just above its surface is the electric field outside the cylinder evaluated at $r = R$.

$$E = \frac{\sigma R}{\varepsilon_0 r} = \frac{\sigma}{\varepsilon_0}$$

Example 10:

Use Gauss's Law to find the electric field just outside an infinite nonconductive surface with a charge density σ.

Example 10:



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\underset{\text{cylinder}}{\int \vec{E}\cdot d\vec{A}} + \underset{\text{ends}}{\int \vec{E}\cdot d\vec{A}} = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + \int E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$0 + E\int dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E2A = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Example 11:

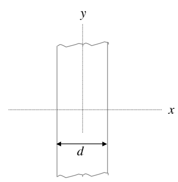
Two large nonconducting parallel plates are given charges of equal magnitude and opposite sign; the charge per unit area is $+\sigma$ for one and $-\sigma$ for the other. Find the electric field in the region between the plates.

Example 11:

Left of $+\sigma$	Between plates	Right of $-\sigma$
$E_+ = \frac{\sigma}{2\varepsilon_0}$ (←)	$E_+ = \frac{\sigma}{2\varepsilon_0}$ (→)	$E_+ = \frac{\sigma}{2\varepsilon_0}$ (→)
$E_- = \frac{\sigma}{2\varepsilon_0}$ (→)	$E_- = \frac{\sigma}{2\varepsilon_0}$ (→)	$E_- = \frac{\sigma}{2\varepsilon_0}$ (←)
$\vec{E}_+ + \vec{E}_- = \frac{-\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$	$\vec{E}_+ + \vec{E}_- = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$	$\vec{E}_+ + \vec{E}_- = \frac{\sigma}{2\varepsilon_0} + \frac{-\sigma}{2\varepsilon_0}$
$\vec{E}_+ + \vec{E}_- = 0$	$\boxed{\vec{E}_+ + \vec{E}_- = \frac{\sigma}{\varepsilon_0}}$	$\vec{E}_+ + \vec{E}_- = 0$

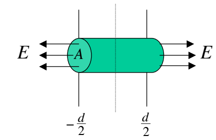
Example 12:

A slab of insulating material has a nonuniform positive charge density of $\rho = Cx^2$, where x is measured from the center of the slab, as shown below. The slab is infinite in the y and z directions. Derive expressions for the electric field in (a) the exterior regions and (b) the interior region of the slab ($-d/2 < x < d/2$).



Example 12:

a.) $x < -\frac{d}{2}$ or $x > \frac{d}{2}$



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

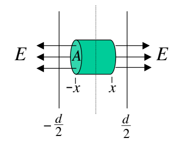
$$q_{enc} = \int \rho dV \qquad \rho = Cx^2 \qquad dV = Adx$$

$$q_{enc} = \int_{-\frac{d}{2}}^{\frac{d}{2}} Cx^2 Adx = CA\frac{x^3}{3}\Bigg]_{-\frac{d}{2}}^{\frac{d}{2}} = \frac{CAd^3}{12}$$

$$E\cdot 2A = \frac{\frac{CAd^3}{12}}{\varepsilon_0}$$

$$\boxed{E = \frac{Cd^3}{24\varepsilon_0}}$$

b.) $-\frac{d}{2} < x < \frac{d}{2}$



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E\cdot 2A = \frac{\frac{2CAx^3}{3}}{\varepsilon_0}$$

$$q_{enc} = \int_{-x}^{x} Cx^2 Adx = CA\frac{x^3}{3}\Bigg]_{-x}^{x} = \frac{2CAx^3}{3}$$

$$\boxed{E = \frac{Cx^3}{3\varepsilon_0}}$$

Example 13:

A solid insulating sphere of radius R has a nonuniform charge density that varies with r according to the expression

$$\rho = Ar^2$$

where A is a constant and $r < R$ is measured from the center of the sphere. Use Gauss's law to determine the magnitude of the electric field at radial distances (a) $r < R$ and (b) $r > R$.

Example 13:

$$q_{enc} = \int \rho dV \qquad \rho = Ar^2 \qquad dV = 4\pi r^2 dr$$

a.) $r < R$

$$q_{enc} = \int_0^r Ar^2 4\pi r^2 dr = 4\pi A\int_0^r r^4 dr = 4\pi A\frac{r^5}{5}\Bigg]_0^r = \frac{4\pi Ar^5}{5}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \text{ so } E\cdot 4\pi r^2 = \frac{4\pi Ar^5}{5\varepsilon_0} \text{ and } \boxed{E = \frac{Ar^3}{5\varepsilon_0}}$$

b.) $r > R$

$$q_{enc} = \int_0^R Ar^2 4\pi r^2 dr = 4\pi A\int_0^R r^4 dr = 4\pi A\frac{r^5}{5}\Bigg]_0^R = \frac{4\pi AR^5}{5}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \text{ so } E\cdot 4\pi r^2 = \frac{4\pi AR^5}{5\varepsilon_0} \text{ and } \boxed{E = \frac{AR^5}{5\varepsilon_0 r^2}}$$

Example 13:

b.) Sometimes the total charge is given as Q.

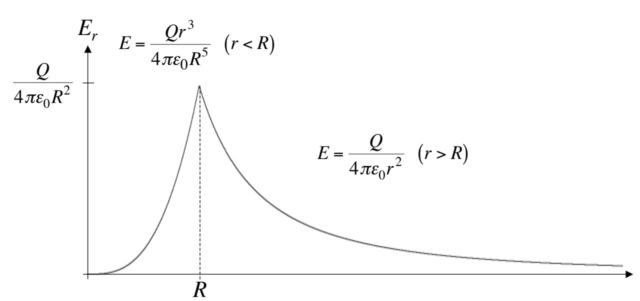
$$q_{enc} = Q = \frac{4\pi AR^5}{5}$$

$$A = \frac{5Q}{4\pi R^5}$$

$$\text{so } E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (r > R)$$

$$\text{and } E = \frac{Qr^3}{4\pi\varepsilon_0 R^5} \quad (r < R)$$

Example 13: Nonuniformly Charged Spherical Insulator



Example 14:

An infinitely long insulating cylinder of radius R has a volume charge density that varies with the radius as

$$\rho = \rho_0\left(a - \frac{r}{b}\right)$$

where ρ_0, a, and b are positive constants and r is the distance from the axis of the cylinder. Use Gauss's law to determine the magnitude of the electric field at radial distances (a) $r < R$ and (b) $r > R$.

Example 14:

$$q_{enc} = \int \rho dV \qquad \rho = \rho_0\left(a - \frac{r}{b}\right) \qquad dV = 2\pi r\ell dr$$

a.) $r < R$

$$q_{enc} = \int_0^r \rho_0\left(a - \frac{r}{b}\right)2\pi r\ell dr = 2\pi\rho_0\ell\int_0^r\left(ar - \frac{r^2}{b}\right)dr = 2\pi\rho_0\ell\left(\left.\frac{ar^2}{2} - \frac{r^3}{3b}\right]_0^r\right)$$

$$q_{enc} = 2\pi\rho_0\ell\left(\frac{ar^2}{2} - \frac{r^3}{3b}\right)$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \quad \text{so } E\cdot 2\pi r\ell = \frac{2\pi\rho_0\ell\left(\frac{ar^2}{2} - \frac{r^3}{3b}\right)}{\varepsilon_0} \quad \text{and } \boxed{E = \frac{\rho_0 r}{2\varepsilon_0}\left(a - \frac{2r}{3b}\right)}$$

b.) $r > R$

$$q_{enc} = \int_0^R \rho_0\left(a - \frac{r}{b}\right)2\pi r\ell dr = 2\pi\rho_0\ell\left(\left.\frac{ar^2}{2} - \frac{r^3}{3b}\right]_0^R\right) = 2\pi\rho_0\ell\left(\frac{aR^2}{2} - \frac{R^3}{3b}\right)$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \quad \text{so } E\cdot 2\pi r\ell = \frac{2\pi\rho_0\ell\left(\frac{aR^2}{2} - \frac{R^3}{3b}\right)}{\varepsilon_0} \quad \text{and } \boxed{E = \frac{\rho_0 R^2}{2\varepsilon_0 r}\left(a - \frac{2R}{3b}\right)}$$

Example 14: Nonuniformly Charged Cylindrical Insulator