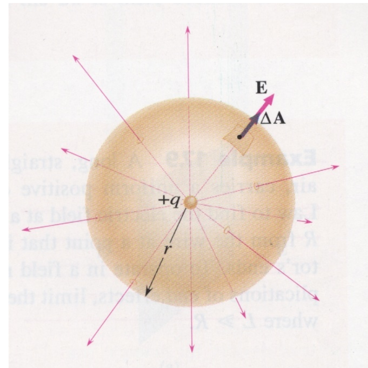
Example 1:

Use Gauss's Law to find the electric field a distance r away from a point charge q.

Example 1:



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint EdA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{enc}}{\varepsilon_0}$$

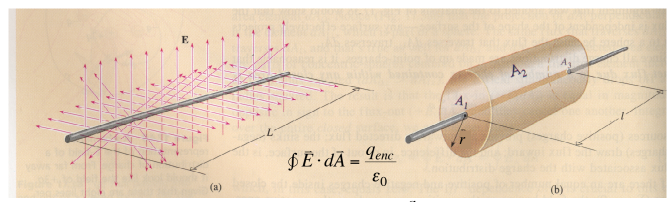
$$E4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

Example 2:

Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ. Find the electric field.

Example 2:



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{\text{cylinder}} \vec{E} \cdot d\vec{A} + \int_{\text{ends}} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int EdA + 0 = \frac{q_{enc}}{\varepsilon_0}$$

$$E\int dA = \frac{q_{enc}}{\varepsilon_0}$$

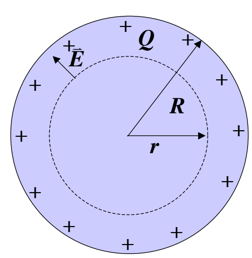
$$E(2\pi r\ell) = \frac{\lambda\ell}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

Example 3:

Positive charge Q is on a solid conducting sphere with radius R. Find the electric field at any point inside or outside the sphere.

Example 3:



$$r < R$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

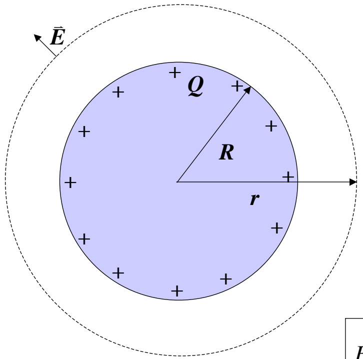
$$\oint EdA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E4\pi r^2 = \frac{0}{\varepsilon_0}$$

$$\boxed{E = 0}$$

Example 3:



$r > R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

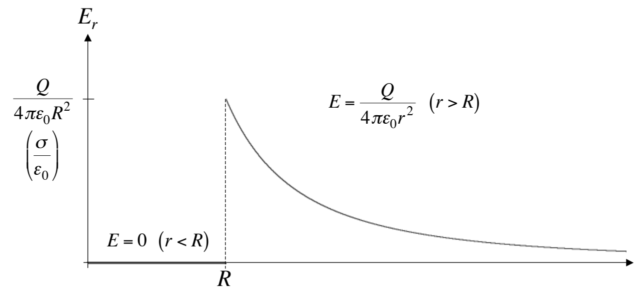
$$\oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2} \left(\sigma = \frac{Q}{4\pi R^2}\right)}$$

Example 3: Charged Spherical Conductor



Example 4:

Positive charge Q is distributed uniformly *throughout the volume* of an *insulating* sphere with radius R. Find the magnitude of the electric field at a point P a distance r from the center of the sphere.

Example 4:

$$q_{enc} = \int_0^r \rho dV$$

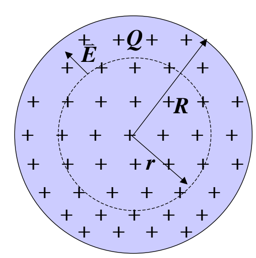
$$q_{enc} = \rho \int_0^r dV$$

$$q_{enc} = \rho V(r)$$

$$q_{enc} = \rho \frac{4}{3}\pi r^3$$

$$\left(\rho = \frac{3Q}{4\pi R^3}\right)$$

$$q_{enc} = Q\frac{r^3}{R^3}$$



$r < R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

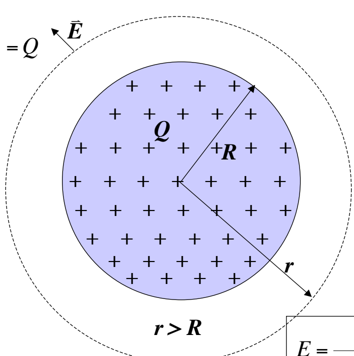
$$E \oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho r}{3\varepsilon_0} = \frac{Qr}{4\pi\varepsilon_0 R^3}}$$

Example 4:

$q_{enc} = Q$



$r > R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

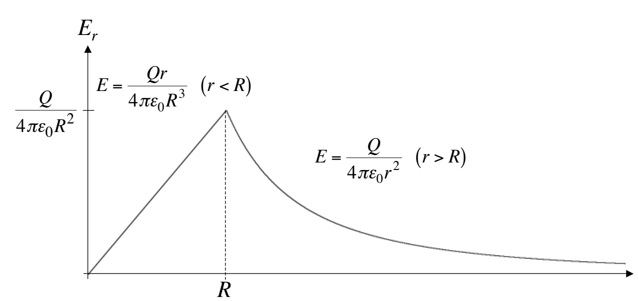
$$\oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2} \left(\rho = \frac{3Q}{4\pi R^3}\right)}$$

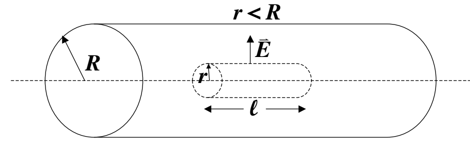
Example 4: Uniformly Charged Spherical Insulator



Example 5:

Positive charge is distributed uniformly a long *conducting* cylinder with radius R. Find the magnitude of the electric field at a point P a distance r from the center of the cylinder. Express in terms of surface charge density σ and linear charge density λ.

Example 5:



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

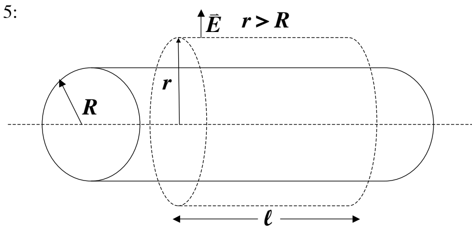
$$\int_{\text{cylinder}} \vec{E}\cdot d\vec{A} + \int_{\text{ends}} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int EdA + 0 = \frac{q_{enc}}{\varepsilon_0}$$

$$E\int dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E2\pi r\ell = \frac{0}{\varepsilon_0} \text{ and } \boxed{E = 0}$$

Example 5:



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{\text{cylinder}} \vec{E}\cdot d\vec{A} + \int_{\text{ends}} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

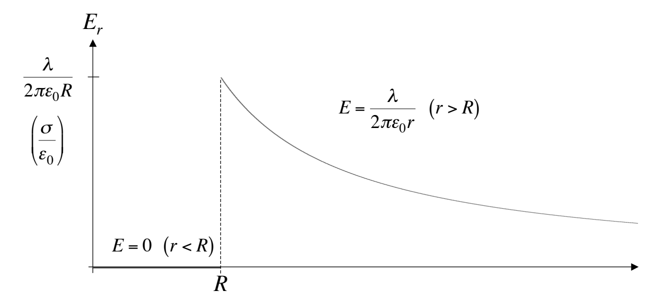
$$\int EdA + 0 = \frac{q_{enc}}{\varepsilon_0}$$

$$E\int dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E2\pi r\ell = \frac{\lambda\ell}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma R}{\varepsilon_0 r}} \quad \left(\sigma = \frac{\lambda}{2\pi R}\right)$$

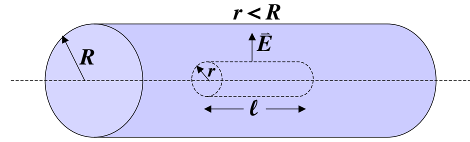
Example 5: Charged Cylindrical Conductor



Example 6:

Positive charge is distributed uniformly *throughout the volume* of a long *insulating* cylinder with radius R. Find the magnitude of the electric field at a point P a distance r from the center of the cylinder. Express in terms of volume charge density ρ and linear charge density λ.

Example 6:



$$q_{enc} = \rho V(r)$$

$$q_{enc} = \rho\pi r^2\ell$$

$$\left(\rho = \frac{\lambda}{\pi R^2}\right) \quad q_{enc} = \lambda\frac{r^2}{R^2}\ell$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

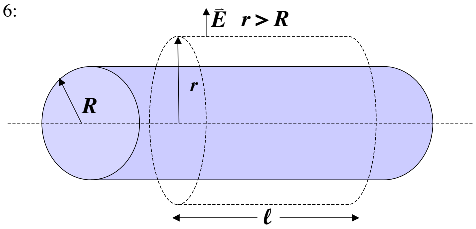
$$\int_{\text{cylinder}} \vec{E}\cdot d\vec{A} + \int_{\text{ends}} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int EdA + 0 = \frac{q_{enc}}{\varepsilon_0}$$

$$E\int dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E2\pi r\ell = \frac{\rho\pi r^2\ell}{\varepsilon_0} \text{ and } \boxed{E = \frac{\rho r}{2\varepsilon_0} = \frac{\lambda r}{2\pi\varepsilon_0 R^2}}$$

Example 6:



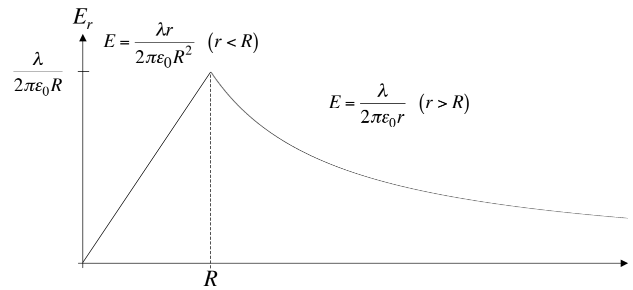
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\int_{\text{cylinder}} \vec{E}\cdot d\vec{A} + \int_{\text{ends}} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E2\pi r\ell = \frac{\lambda\ell}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\rho R^2}{2\varepsilon_0 r} \quad \left(\rho = \frac{\lambda}{\pi R^2}\right)}$$

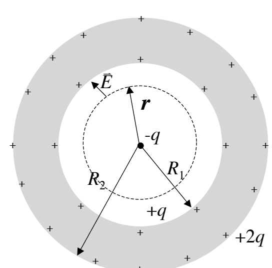
Example 6: Uniformly Charged Cylindrical Insulator



Example 7:

A point charge with charge $-q$ is located in the center of a spherical conductive shell with an inner radius R_1 and outer radius R_2. The outer shell has a net charge of $+3q$. Find the magnitude of the electric field at a point P a distance r from the center of the spherical shell.

Example 7:



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

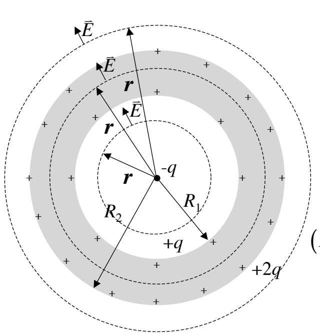
$$\oint E dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{q_{enc}}{4\pi\varepsilon_0 r^2}$$

Example 7:



$$E = \frac{q_{enc}}{4\pi\varepsilon_0 r^2}$$

$$(0 < r < R_1) \quad \boxed{E = \frac{-q}{4\pi\varepsilon_0 r^2}}$$

$$(R_1 < r < R_2) \quad \boxed{E = \frac{-q+q}{4\pi\varepsilon_0 r^2} = 0}$$

$$(R_2 < r) \quad E = \frac{-q+q+2q}{4\pi\varepsilon_0 r^2} = \frac{2q}{4\pi\varepsilon_0 r^2}$$

$$\boxed{E = \frac{q}{2\pi\varepsilon_0 r^2}}$$

Example 7: