Gauss's Law

Electric Flux

For uniform electric fields the *electric flux* (Φ_E) through a flat area *A* is the product of the area and the component of the field perpendicular to the area.

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

In general, for fields which are not uniform the electric flux is:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

Electric Flux Facts (Closed Surfaces)

1.) Whether or not there is a net outward or inward electric flux through a closed surface depends upon the sign of the enclosed charge.

2.) Charges outside of a closed surface do not give a net electric flux through the surface.

3.) The net electric flux is directly proportional to the net amount of charge enclosed within the surface but is otherwise independent of the size of the closed surface.

Gauss's Law

The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ε_o.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

The closed surface is *imaginary* and is often referred to as a *Gaussian surface*.

Choosing Gaussian Surfaces

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

1.) The value of the electric field can be argued by symmetry to be considered constant over the surface.

2.) The dot product in previous equation can be expressed as a simple algebraic product *EdA* because E and *d*A are parallel.

3.) The dot product in previous equation is zero because E and *dA* are perpendicular.

4.) The field can be argued to be zero over the surface.

Conductor Tidbits

1.) When excess charge is placed on an isolated conductor it resides entirely on the surface.

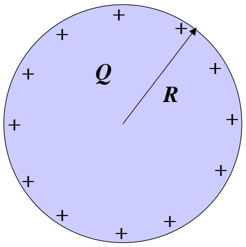
2.) $E = 0$ everywhere inside of an isolated conductor.

3.) The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ε_o, where σ is the surface charge density at this point.

4.) On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is the smallest.

Don't forget about induced charges.

Charged Sphere (Conductor)



$$\sigma = \frac{Q}{A} = \frac{Q}{4\pi R^2}$$

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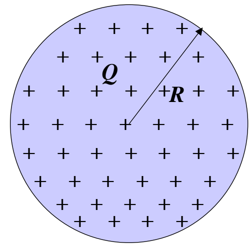
Uniformly Charged Sphere (Insulator)



$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$

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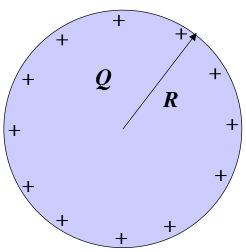
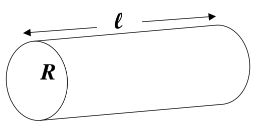
Charged Cylinder (Conductor)



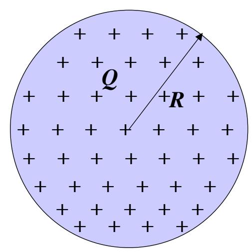


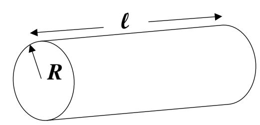
$$\lambda = \frac{Q}{\ell}$$

$$\sigma = \frac{Q}{A} = \frac{Q}{2\pi R\ell} = \frac{\lambda}{2\pi R}$$

$$\sigma = \frac{\lambda}{2\pi R}$$

Uniformly Charged Cylinder (Insulator)





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