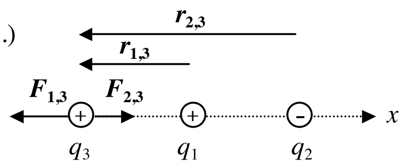
1.)



$q_1 = 1.0 \text{ x } 10^{-9}\ \text{C}$, $q_2 = -3.0 \text{ x } 10^{-9}\ \text{C}$, and $q_3 = 5.0 \text{ x } 10^{-9}\ \text{C}$

$r_{1,3} = 0.020\ \text{m}$ and $r_{2,3} = 0.040\ \text{m}$

$$\vec{F}_{1,2} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{1,2}^{\ 2}}\hat{r}_{1,2} \text{ and } \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi\left(8.85 \text{ x } 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = k = 9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$$

$$\vec{F}_{1,3} = k\frac{q_1 q_3}{r_{1,3}^2}\hat{r}_{1,3} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(1.0 \text{ x } 10^{-9}\ \text{C}\right)\left(5.0 \text{ x } 10^{-9}\ \text{C}\right)}{\left(0.020\ \text{m}\right)^2}\left(-\hat{i}\right) = \left(-1.125 \text{ x } 10^{-4}\ \text{N}\right)\hat{i}$$

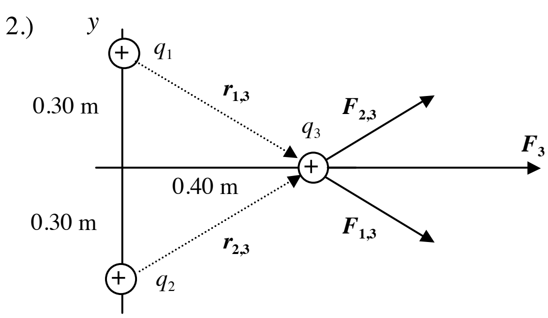
$$\vec{F}_{1,3} = \left(-1.125 \text{ x } 10^{-4}\ \text{N}\right)\hat{i}$$

$$\vec{F}_{2,3} = k\frac{q_2 q_3}{r_{2,3}^2}\hat{r}_{2,3} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-3.0 \text{ x } 10^{-9}\ \text{C}\right)\left(5.0 \text{ x } 10^{-9}\ \text{C}\right)}{\left(0.040\ \text{m}\right)^2}\left(-\hat{i}\right) = \left(8.438 \text{ x } 10^{-5}\ \text{N}\right)\hat{i}$$

$$\vec{F}_{2,3} = \left(8.438 \text{ x } 10^{-5}\ \text{N}\right)\hat{i}$$

$$\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = \left(-1.125 \text{ x } 10^{-4}\ \text{N}\right)\hat{i} + \left(8.438 \text{ x } 10^{-5}\ \text{N}\right)\hat{i} = \boxed{\left(-2.81 \text{ x } 10^{-5}\ \text{N}\right)\hat{i}}$$

2.)



$q_1 = q_2 = 2.0 \text{ x } 10^{-6}\ \text{C}$ and $q_3 = 4.0 \text{ x } 10^{-6}\ \text{C}$

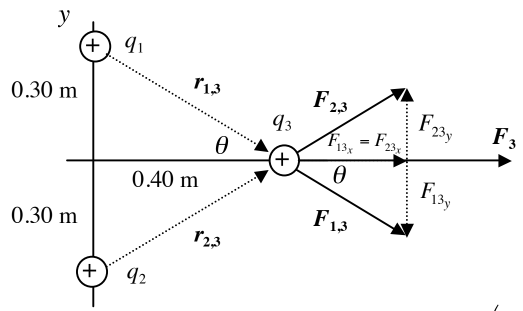
$r_{1,3} = r_{2,3} = 0.50\ \text{m}$

$$\vec{F}_{1,3} = k\frac{q_1 q_3}{r_{13}^2}\hat{r}_{1,3} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(2.0 \text{ x } 10^{-6}\ \text{C}\right)\left(4.0 \text{ x } 10^{-6}\ \text{C}\right)}{\left(0.50\ \text{m}\right)^2}\frac{\left(0.40\ \text{m}\right)\hat{i} + \left(-0.30\ \text{m}\right)\hat{j}}{0.50\ \text{m}} = \left(0.230\ \text{N}\right)\hat{i} + \left(-0.173\ \text{N}\right)\hat{j}$$

$$\vec{F}_{2,3} = k\frac{q_1 q_3}{r_{23}^2}\hat{r}_{2,3} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(2.0 \text{ x } 10^{-6}\ \text{C}\right)\left(4.0 \text{ x } 10^{-6}\ \text{C}\right)}{\left(0.50\ \text{m}\right)^2}\frac{\left(0.40\ \text{m}\right)\hat{i} + \left(0.30\ \text{m}\right)\hat{j}}{0.50\ \text{m}} = \left(0.230\ \text{N}\right)\hat{i} + \left(0.173\ \text{N}\right)\hat{j}$$

$$\boxed{\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = \left(0.460\ \text{N}\right)\hat{i}}$$

2.) (alternative solution)



$q_1 = q_2 = 2.0 \text{ x } 10^{-6}\text{ C}$ and $q_3 = 4.0 \text{ x } 10^{-6}\text{ C}$

$r_{1,3} = r_{2,3} = 0.50\text{ m}$

$$\cos\theta = \frac{0.4}{0.5} = 0.80$$

$$F_{2,3} = F_{1,3} = k\frac{q_1 q_3}{r_{13}^2} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(2.0 \text{ x } 10^{-6}\text{ C}\right)\left(4.0 \text{ x } 10^{-6}\text{ C}\right)}{\left(0.50\text{ m}\right)^2} = 0.288\text{ N}$$

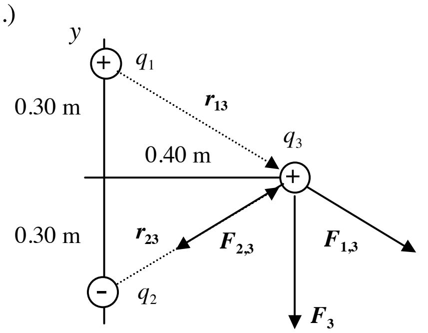
for the net force on q_3 the y-components cancel while the x-components are the same and add together

$$F_{2,3_x} = F_{1,3_x} = F_{1,3}\cos\theta = \left(0.288\text{ N}\right)\left(0.80\right) = 0.230\text{ N}$$

$$F_{3_x} = F_{1,3_x} + F_{2,3_x} = 2F_{1,3_x} = 2\left(0.230\text{ N}\right) = 0.460\text{ N}$$

$$\vec{F}_3 = \boxed{\left(0.460\text{ N}\right)\hat{i}}$$

3.)



$q_1 = 2.0 \text{ x } 10^{-6}\text{ C}$, $q_2 = -2.0 \text{ x } 10^{-6}\text{ C}$, and $q_3 = 4.0 \text{ x } 10^{-6}\text{ C}$

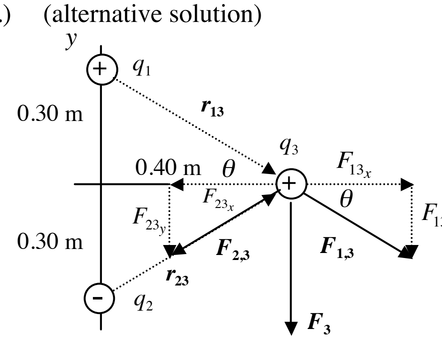
$r_{13} = r_{23} = 0.50\text{ m}$

$$\vec{F}_{1,3} = k\frac{q_1 q_3}{r_{13}^2}\hat{r}_{1,3} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(2.0 \text{ x } 10^{-6}\text{ C}\right)\left(4.0 \text{ x } 10^{-6}\text{ C}\right)}{\left(0.50\text{ m}\right)^2}\frac{\left(0.40\text{ m}\right)\hat{i} + \left(-0.30\text{ m}\right)\hat{j}}{0.50\text{ m}} = \left(0.230\text{ N}\right)\hat{i} + \left(-0.173\text{ N}\right)\hat{j}$$

$$\vec{F}_{2,3} = k\frac{q_1 q_3}{r_{23}^2}\hat{r}_{2,3} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-2.0 \text{ x } 10^{-6}\text{ C}\right)\left(4.0 \text{ x } 10^{-6}\text{ C}\right)}{\left(0.50\text{ m}\right)^2}\frac{\left(0.40\text{ m}\right)\hat{i} + \left(0.30\text{ m}\right)\hat{j}}{0.50\text{ m}} = \left(-0.230\text{ N}\right)\hat{i} + \left(-0.173\text{ N}\right)\hat{j}$$

$$\boxed{\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = \left(\text{-}0.346\text{ N}\right)\hat{j}}$$

3.) (alternative solution)



$q_1 = 2.0 \text{ x } 10^{-6}\text{ C}$, $q_2 = -2.0 \text{ x } 10^{-6}\text{ C}$, and $q_3 = 4.0 \text{ x } 10^{-6}\text{ C}$

$r_{13} = r_{23} = 0.50\text{ m}$

$$\sin\theta = \frac{0.3}{0.5} = 0.60$$

$$F_{2,3} = F_{1,3} = k\frac{q_1 q_3}{r_{13}^2} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(2.0 \text{ x } 10^{-6}\text{ C}\right)\left(4.0 \text{ x } 10^{-6}\text{ C}\right)}{\left(0.50\text{ m}\right)^2} = 0.288\text{ N}$$

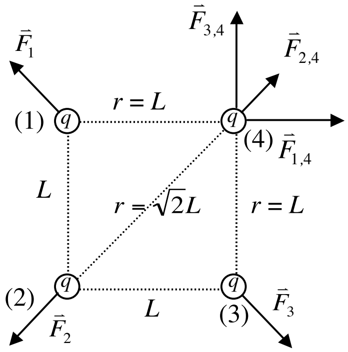
for the net force on q_3 the x-components cancel while the y-components are the same and add together

$$F_{23_y} = F_{13_y} = F_{13}\sin\theta = (0.288\text{ N})(0.60) = 0.173\text{ N}$$

$$F_{3_y} = F_{13_y} + F_{23_y} = 2F_{13_y} = 2(0.173\text{ N}) = 0.346\text{ N}$$

$$\vec{F}_3 = \boxed{(-0.346\text{ N})\hat{j}}$$

4.)



Considering the charge in the upper right-hand corner the magnitude of the forces from the adjacent charges (1) and (3) are:

$$\vec{F}_{1,4} = k\frac{qq}{r^2}\hat{r}_{1,4} = k\frac{q^2}{L^2}\hat{i} \quad \text{and} \quad \vec{F}_{1,3} = k\frac{qq}{r^2}\hat{r}_{1,3} = k\frac{q^2}{L^2}\hat{j}$$

For the diagonal charge (2) the force is

$$\vec{F}_{2,4} = k\frac{qq}{r^2}\hat{r}_{2,4} = k\frac{q^2}{(\sqrt{2}L)^2}\frac{L\hat{i}+L\hat{j}}{\sqrt{2}L} = k\frac{q^2}{(\sqrt{2}L)^2}\left(\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}\right) = k\frac{q^2}{2\sqrt{2}L^2}\hat{i} + k\frac{q^2}{2\sqrt{2}L^2}\hat{j}$$

The total force on charge (4) is

$$\vec{F}_4 = \vec{F}_{1,4} + \vec{F}_{2,4} + \vec{F}_{3,4} = \left(k\frac{q^2}{L^2} + \frac{q^2}{2\sqrt{2}L^2}\right)\hat{i} + \left(k\frac{q^2}{L^2} + \frac{q^2}{2\sqrt{2}L^2}\right)\hat{j} = \left(\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)\right)\hat{i} + \left(\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)\right)\hat{j}$$

The magnitude of the force on charge (4) is

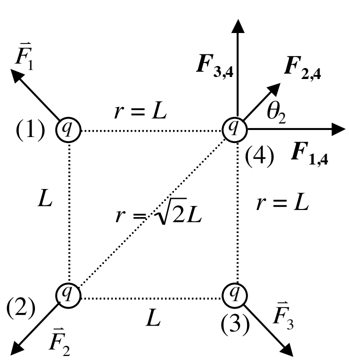
$$F_4 = \sqrt{\left(\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)\right)^2 + \left(\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)\right)^2} = \sqrt{2\left(\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)\right)^2} = \sqrt{2}\left(\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)\right) = \boxed{\frac{kq^2}{4L^2}\left(4\sqrt{2}+2\right)}$$

The direction of the force on charge (4) is

$$\theta_4 = \tan^{-1}\frac{F_{4_y}}{F_{4_x}} = \tan^{-1}\left(\frac{\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)}{\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)}\right) = \boxed{45^\circ}$$

The other charges experience the same force with directions $\boxed{\theta_1 = 135^\circ,\ \theta_2 = 225^\circ,\text{ and }\theta_3 = 315^\circ}$

4.) (alternative solution)



Considering the charge in the upper right-hand corner the magnitude of the forces from the adjacent charges (1) and (3) are both

$$F_{1,4} = F_{3,4} = k\frac{qq}{r^2} = k\frac{q^2}{L^2}$$

$F_{1,4}$ is in the positive x-direction and $F_{3,4}$ is in the positive y-direction

so $F_{1,4_x} = k\dfrac{q^2}{L^2}$ and $F_{3,4_y} = k\dfrac{q^2}{L^2}$

For the diagonal charge (2) the magnitude of the force is: $F_{2,4} = k\dfrac{qq}{r^2} = k\dfrac{q^2}{(\sqrt{2}L)^2} = k\dfrac{q^2}{2L^2}$

since $\theta_2 = 45°$ then $\cos\theta_2 = \frac{\sqrt{2}}{2}$ and $\sin\theta_2 = \frac{\sqrt{2}}{2}$

$$F_{2,4_x} = k\frac{q^2}{2L^2}\cos\theta_2 = k\frac{q^2}{2L^2}\left(\frac{\sqrt{2}}{2}\right) = k\frac{\sqrt{2}q^2}{4L^2} \text{ and } F_{2,4_y} = k\frac{q^2}{2L^2}\sin\theta_2 = k\frac{q^2}{2L^2}\left(\frac{\sqrt{2}}{2}\right) = k\frac{\sqrt{2}q^2}{4L^2}$$

The total x-component on charge (4) is

$$F_{4_x} = F_{1,4_x} + F_{2,4_x} = k\frac{q^2}{L^2} + k\frac{\sqrt{2}q^2}{4L^2} = kq^2\left(\frac{4+\sqrt{2}}{4L^2}\right) = \frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)$$

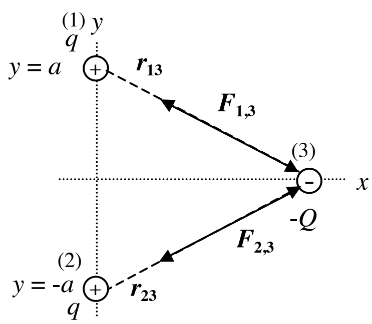
The total y-component on charge (4) is

$$F_{4_y} = F_{1,4_y} + F_{2,4_y} = k\frac{q^2}{L^2} + k\frac{\sqrt{2}q^2}{4L^2} = kq^2\left(\frac{4+\sqrt{2}}{4L^2}\right) = \frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)$$

The total force on charge (4) is

$$\vec{F}_4 = \left(\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)\right)\hat{i} + \left(\frac{kq^2}{4L^2}\left(4+\sqrt{2}\right)\right)\hat{j}$$

5.)

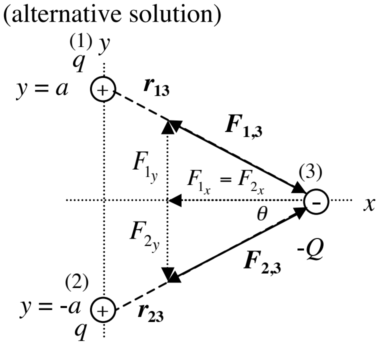


$$\vec{F}_{1,3} = k\frac{q_1q_3}{r_{13}^2}\hat{r}_{13} = k\frac{q(-Q)}{\left(\sqrt{x^2+a^2}\right)^2}\frac{x\hat{i} - a\hat{j}}{\sqrt{x^2+a^2}} = -\frac{kqQx}{\left(x^2+a^2\right)^{3/2}}\hat{i} + \frac{kqQa}{\left(x^2+a^2\right)^{3/2}}\hat{j}$$

$$\vec{F}_{2,3} = k\frac{q_2q_3}{r_{23}^2}\hat{r}_{23} = k\frac{q(-Q)}{\left(\sqrt{x^2+a^2}\right)^2}\frac{x\hat{i} + a\hat{j}}{\sqrt{x^2+a^2}} = -\frac{kqQx}{\left(x^2+a^2\right)^{3/2}}\hat{i} - \frac{kqQa}{\left(x^2+a^2\right)^{3/2}}\hat{j}$$

$$\boxed{\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = -\frac{2kqQx}{\left(x^2+a^2\right)^{3/2}}\hat{i}}$$

5.) (alternative solution)



for both charges $r = \sqrt{x^2+a^2}$ and $\cos\theta = \frac{x}{\sqrt{x^2+a^2}}$

the magnitude of the force on $-Q$ for both charges are

$$F_{1,3} = F_{2,3} = k\frac{qQ}{r^2} = k\frac{qQ}{\left(\sqrt{x^2+a^2}\right)^2} = k\frac{qQ}{x^2+a^2}$$

for the net force on Q the y-components cancel so $F_{3_y} = 0$ while the x-components are the same and add together

$$F_{1,3_x} = F_{2,3_x} = k\frac{qQ}{x^2+a^2}\cos\theta = k\frac{qQ}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{kqQx}{\left(x^2+a^2\right)^{\frac{3}{2}}}$$

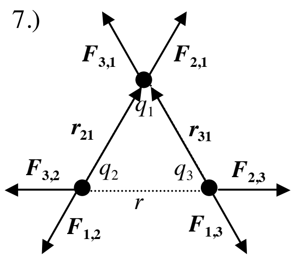
$F_{3_x} = F_{1,3_x} + F_{2,3_x} = 2F_{1,3_x} = \frac{2kqQx}{\left(x^2+a^2\right)^{\frac{3}{2}}}$ the direction is in the negative x-direction so $F_{3_x} = \frac{-2kqQx}{\left(x^2+a^2\right)^{\frac{3}{2}}}$

6.) total charge is $q = -30.0 \text{ x } 10^{-6}$ C

the charge of and electron is $q_e = -1.6 \text{ x } 10^{-19}$ C

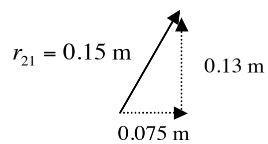
so the number of electrons is $\left(-30.0 \text{ x } 10^{-6} \text{ C}\right)\left(\dfrac{1 \text{ electron}}{-1.6 \text{ x } 10^{-19} \text{ C}}\right) = \boxed{1.875 \text{ x } 10^{14} \text{ electrons}}$

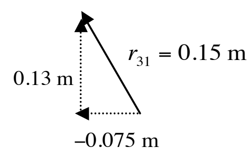
7.)



$q_1 = q_2 = q_3 = 11.0 \text{ x } 10^{-9}$ C

Looking at charge q_1: $\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$ and $r_{21} = r_{31} = r_{23} = 0.15$ m





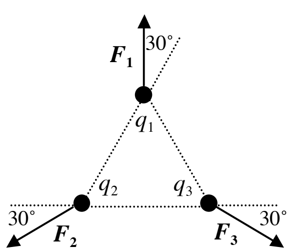
$\vec{r}_{21} = (0.075 \text{ m})\hat{i} + (0.13 \text{ m})\hat{j}$ $\qquad$ $\vec{r}_{31} = (-0.075 \text{ m})\hat{i} + (0.13 \text{ m})\hat{j}$

$$\vec{F}_{2,1} = k\frac{q_2 q_1}{r_{21}^{\;2}}\hat{r}_{21} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(11.0 \text{ x } 10^{-6} \text{ C}\right)\left(11.0 \text{ x } 10^{-6} \text{ C}\right)}{(0.15 \text{ m})^2}\frac{(0.075 \text{ m})\hat{i} + (0.13 \text{ m})\hat{j}}{(0.15 \text{ m})} = (24.2 \text{ N})\hat{i} + (41.9 \text{ N})\hat{j}$$

$$\vec{F}_{3,1} = k\frac{q_3 q_1}{r_{31}^{\;2}}\hat{r}_{31} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(11.0 \text{ x } 10^{-6} \text{ C}\right)\left(11.0 \text{ x } 10^{-6} \text{ C}\right)}{(0.15 \text{ m})^2}\frac{(-0.075 \text{ m})\hat{i} + (0.13 \text{ m})\hat{j}}{(0.15 \text{ m})} = (-24.2 \text{ N})\hat{i} + (41.9 \text{ N})\hat{j}$$

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} = (83.8 \text{ N})\hat{j}$$

$$\boxed{F_1 = \ 83.8 \text{ N} \ \text{ and } \ \theta_1 = \ 90°}$$



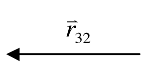
Since the charge distribution is symmetrical:

$$\boxed{F_3 = F_2 = F_1 = \ 83.8 \text{ N and } \theta_2 = \ 210°, \theta_3 = -30°}$$

If you don't see the symmetry:

Looking at charge q_2: $\vec{F}_2 = \vec{F}_{1,2} + \vec{F}_{3,2}$

$$\vec{F}_{1,2} = -\vec{F}_{2,1} = (-24.2 \text{ N})\hat{i} + (-41.9 \text{ N})\hat{j}$$



$$\vec{F}_{3,2} = k\frac{q_3 q_2}{r_{32}^{\;2}}\hat{r}_{32} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(11.0 \text{ x } 10^{-6} \text{ C}\right)\left(11.0 \text{ x } 10^{-6} \text{ C}\right)}{(0.15 \text{ m})^2}\left(-\hat{i}\right) = (-48.4 \text{ N})\hat{i}$$

$$\vec{F}_2 = \vec{F}_{1,2} + \vec{F}_{3,2} = (-72.6 \text{ N})\hat{i} + (-41.9 \text{ N})\hat{j}$$

$$F_2 = \sqrt{F_{2_x}^2 + F_{2_y}^2} = \sqrt{(-72.6 \text{ N})^2 + (-41.9 \text{ N})^2} = \boxed{83.8 \text{ N}}$$

$$\theta_2 = \tan^{-1}\left(\frac{F_{2_y}}{F_{2_x}}\right) = \tan^{-1}\left(\frac{-41.9 \text{ N}}{-72.6 \text{ N}}\right) = 30° + 180° = \boxed{210°}$$

Looking at charge q_3: $\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3}$

$$\vec{F}_{1,3} = -\vec{F}_{3,1} = (24.2\text{ N})\hat{i} + (-41.9\text{ N})\hat{j}$$

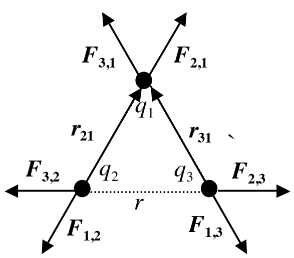
$$\vec{F}_{2,3} = -\vec{F}_{3,2} = (48.2\text{ N})\hat{i}$$

$$\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = (72.6\text{ N})\hat{i} + (-41.9\text{ N})\hat{j}$$

$$F_3 = \sqrt{F_{3_x}^2 + F_{3_y}^2} = \sqrt{(72.6\text{ N})^2 + (-41.9\text{ N})^2} = \boxed{83.8\text{ N}}$$

$$\theta_3 = \tan^{-1}\left(\frac{F_{3_y}}{F_{3_x}}\right) = \tan^{-1}\left(\frac{-41.9\text{ N}}{72.6\text{ N}}\right) = -30° + 360° = \boxed{330°}$$

7.) (alternative solution)



$q_1 = q_2 = q_3 = 11.0 \text{ x } 10^{-9}$ C and $r = 0.15$ m

since the charges and separations are all the same

$$F_{2,1} = F_{3,1} = F_{1,2} = F_{3,2} = F_{1,3} = F_{2,3} = k\frac{q_2 q_3}{r^2} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(11.0 \text{ x } 10^{-6}\text{ C})^2}{(0.15\text{ m})^2} = 48.4\text{ N}$$

looking at charge q_1, $\theta_{21} = 60°$ and $\theta_{31} = 120°$

$$\vec{F}_{2,1} = (F_{2,1}\cos\theta_{21})\hat{i} + (F_{2,1}\sin\theta_{21})\hat{j} = ((48.4\text{ N})\cos 60°)\hat{i} + ((48.4\text{ N})\sin 60°)\hat{j}$$

$$\vec{F}_{2,1} = (24.2\text{ N})\hat{i} + (41.9\text{ N})\hat{j}$$

$$\vec{F}_{3,1} = (F_{3,1}\cos\theta_{31})\hat{i} + (F_{3,1}\sin\theta_{31})\hat{j} = ((48.4\text{ N})\cos 120°)\hat{i} + ((48.4\text{ N})\sin 120°)\hat{j}$$

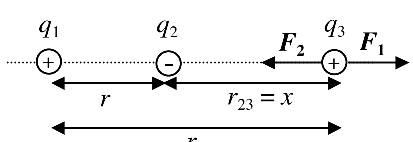
$$\vec{F}_{3,1} = (-24.2\text{ N})\hat{i} + (41.9\text{ N})\hat{j}$$

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} = (83.8\text{ N})\hat{j} \text{ so } F_1 = \boxed{83.8\text{ N}} \text{ and } \theta_1 = \boxed{90°}$$

Since the charge distribution is symmetrical:

$$\boxed{F_3 = F_2 = F_1 = 83.8\text{ N and } \theta_2 = 210°, \theta_3 = -30°}$$

8.)



$q_1 = 5.7 \text{ x } 10^{-6}$ C, $q_1 = -3.5 \text{ x } 10^{-6}$ C, and $r_1 = 0.25$ m

The third charge must be placed to the right of q_2 where the force due to each charge are in opposite directions. Between the charges the forces are both directed to the right and on the left side of q_1 the forces are in opposite directions but F_1 is always larger than F_2.

$$F_1 = k\frac{q_1 q_3}{r_{13}^2} = k\frac{q_1 q_3}{(r+x)^2} \text{ and } F_2 = k\frac{q_2 q_3}{r_{23}^2} = k\frac{q_2 q_3}{x^2} \text{ so } F_1 = F_2 \text{ means } k\frac{q_1 q_3}{(r+x)^2} = k\frac{q_2 q_3}{x^2} \text{ or } \frac{q_1}{(r+x)^2} = \frac{q_2}{x^2}$$

$$q_1 x^2 = q_2 (r+x)^2 = q_2 (r^2 + 2rx + x^2)$$

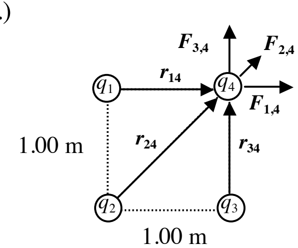
$$\frac{q_1}{q_2}x^2 = r^2 + 2rx + x^2 \text{ so } 0 = \left(1 - \frac{q_1}{q_2}\right)x^2 + 2rx + r^2$$

$$0 = \left(1 - \frac{(5.7 \text{ x } 10^{-6} \text{ C})}{(3.5 \text{ x } 10^{-6} \text{ C})}\right)x^2 + 2(0.25 \text{ m})x + (0.25 \text{ m})^2$$

$$0 = (-0.629)x^2 + (0.50 \text{ m})x + (0.0625 \text{ m}^2) \text{ using quadratic formula } x = \boxed{0.905 \text{ m}} \text{ or } x = -0.110 \text{ m}$$

so the charge should be replaced 0.905 m to the right of q_2

9.)



$$q_1 = q_2 = q_3 = q_4 = 6.00 \text{ x } 10^{-3} \text{ C}$$

looking at the charge q_4: $\vec{F}_4 = \vec{F}_{1,4} + \vec{F}_{2,4} + \vec{F}_{3,4}$ and $r_{14} = r_{34} = 1.00 \text{ m}$, $r_{24} = \sqrt{2} \text{ m}$

$$\vec{F}_{1,4} = k\frac{q_1 q_4}{r_{14}^{\ 2}}\hat{r}_{14} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.00 \text{ x } 10^{-3} \text{ C})(6.00 \text{ x } 10^{-3} \text{ C})}{(1.00 \text{ m})^2}\hat{i} = (3.24 \text{ x } 10^5 \text{ N})\hat{i}$$

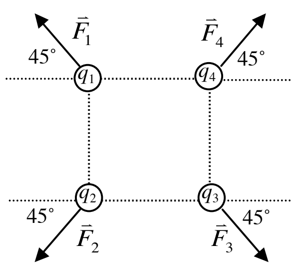
$$\vec{F}_{3,4} = k\frac{q_3 q_4}{r_{34}^{\ 2}}\hat{r}_{34} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.00 \text{ x } 10^{-3} \text{ C})(6.00 \text{ x } 10^{-3} \text{ C})}{(1.00 \text{ m})^2}\hat{j} = (3.24 \text{ x } 10^5 \text{ N})\hat{j}$$

$$\vec{F}_{2,4} = k\frac{q_2 q_4}{r_{24}^{\ 2}}\hat{r}_{24} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.00 \text{ x } 10^{-3} \text{ C})(6.00 \text{ x } 10^{-3} \text{ C})}{(\sqrt{2} \text{ m})^2}\frac{(1.00 \text{ m})\hat{i} + (1.00 \text{ m})\hat{j}}{(\sqrt{2} \text{ m})} = (1.15 \text{ x } 10^5 \text{ N})\hat{i} + (1.15 \text{ x } 10^5 \text{ N})\hat{j}$$

$$\vec{F}_4 = \vec{F}_{1,4} + \vec{F}_{2,4} + \vec{F}_{3,4} = (4.39 \text{ x } 10^5 \text{ N})\hat{i} + (4.39 \text{ x } 10^5 \text{ N})\hat{j}$$

$$F_4 = \sqrt{F_{4_x}^2 + F_{4_y}^2} = \sqrt{(4.39 \text{ x } 10^5 \text{ N})^2 + (4.39 \text{ x } 10^5 \text{ N})^2} = \boxed{6.20 \text{ x } 10^5 \text{ N}}$$

$$\theta_4 = \tan^{-1}\left(\frac{F_{4_y}}{F_{4_x}}\right) = \tan^{-1}\left(\frac{4.39 \text{ x } 10^5 \text{ N}}{4.39 \text{ x } 10^5 \text{ N}}\right) = \boxed{45°}$$



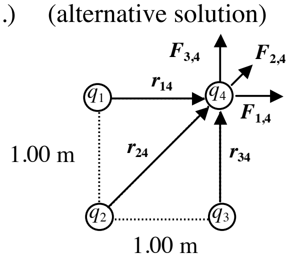
because of symmetry the magnitude of the force on the other charges are the same

$$\boxed{F_1 = F_2 = F_3 = F_4 = 6.20 \text{ x } 10^5 \text{ N}}$$

the directions of these forces are:

$$\boxed{\theta_1 = 135°,\ \theta_2 = 225°, \text{ and } \theta_3 = 315°}$$

9.) (alternative solution)



$q_1 = q_2 = q_3 = q_4 = 6.00 \text{ x } 10^{-3} \text{ C}$ and $r_1 = r_3 = 1.00 \text{ m}$, $r_2 = \sqrt{2} \text{ m}$

looking at the charge in the upper right-hand corner

$$F_{1,4} = F_{3,4} = k\frac{q_3 q_4}{r_{34}^2} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.00 \text{ x } 10^{-3} \text{ C})^2}{(1.00 \text{ m})^2} = 3.24 \text{ x } 10^5 \text{ N (repulsive)}$$

$F_{1,4}$ only has an x-component so $F_{1,4_x} = 3.24 \text{ x } 10^5 \text{ N}$ and $F_{3,4}$ only has an y-component so $F_{3,4_y} = 3.24 \text{ x } 10^5 \text{ N}$

$$F_{2,4}=k\frac{q_2q_4}{r_{24}^2}=\left(9.0\text{ x }10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00\text{ x }10^{-3}\text{ C}\right)^2}{\left(\sqrt{2}\text{ m}\right)^2}=1.62\text{ x }10^5\text{ N (repulsive)}$$

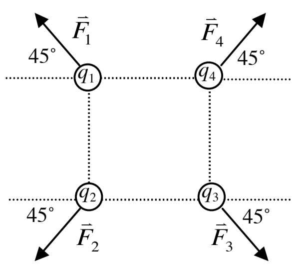
$$F_{2,4_x}=F_{2,4}\cos\theta=\left(1.62\text{ x }10^5\text{ N}\right)\cos45°=1.15\text{ x }10^5\text{ N and }F_{2,4_y}=F_{2,4}\sin\theta=\left(1.62\text{ x }10^5\text{ N}\right)\sin45°=1.15\text{ x }10^5\text{ N}$$

the *x*-component of the total force on q_4 is $F_{4_x}=F_{1,4_x}+F_{2,4_x}=3.24\text{ x }10^5\text{ N}+1.15\text{ x }10^5\text{ N}=4.39\text{ x }10^5\text{ N}$

the *y*-component of the total force on q_4 is $F_{4_y}=F_{3,4_y}+F_{2,4_y}=3.24\text{ x }10^5\text{ N}+1.15\text{ x }10^5\text{ N}=4.39\text{ x }10^5\text{ N}$

$$F_4=\sqrt{F_{4_x}^2+F_{4_y}^2}=\sqrt{\left(4.39\text{ x }10^5\text{ N}\right)^2+\left(4.39\text{ x }10^5\text{ N}\right)^2}=\boxed{6.20\text{ x }10^5\text{ N}}$$

$$\theta_4=\tan^{-1}\left(\frac{F_{4_y}}{F_{4_x}}\right)=\tan^{-1}\left(\frac{4.39\text{ x }10^5\text{ N}}{4.39\text{ x }10^5\text{ N}}\right)=\boxed{45°}$$



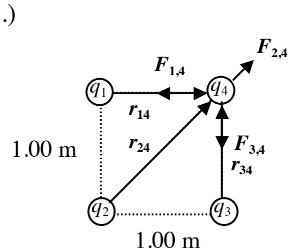
because of symmetry the magnitude of the force on the other charges are the same

$$\boxed{F_1=F_2=F_3=F_4=\ 6.20\text{ x }10^5\text{ N}}$$

the directions of these forces are:

$$\boxed{\theta_1=135°,\ \theta_2=225°,\text{ and }\theta_3=315°}$$

10.)



$q_1=q_3=-6.00\text{ x }10^{-3}\text{ C}$ and $q_2=q_4=6.00\text{ x }10^{-3}\text{ C}$

looking at the charge q_4: $\vec{F}_4=\vec{F}_{1,4}+\vec{F}_{2,4}+\vec{F}_{3,4}$ and $r_{14}=r_{34}=1.00\text{ m}$, $r_{24}=\sqrt{2}\text{ m}$

$$\vec{F}_{1,4}=k\frac{q_1q_4}{r_{14}^2}\hat{r}_{14}=\left(9.0\text{ x }10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-6.00\text{ x }10^{-3}\text{ C}\right)\left(6.00\text{ x }10^{-3}\text{ C}\right)}{\left(1.00\text{ m}\right)^2}\hat{i}=\left(-3.24\text{ x }10^5\text{ N}\right)\hat{i}$$

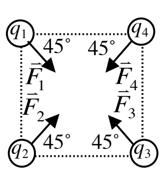
$$\vec{F}_{3,4}=k\frac{q_3q_4}{r_{34}^2}\hat{r}_{34}=\left(9.0\text{ x }10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-6.00\text{ x }10^{-3}\text{ C}\right)\left(6.00\text{ x }10^{-3}\text{ C}\right)}{\left(1.00\text{ m}\right)^2}\hat{j}=\left(-3.24\text{ x }10^5\text{ N}\right)\hat{j}$$

$$\vec{F}_{2,4}=k\frac{q_2q_4}{r_{24}^2}\hat{r}_{24}=\left(9.0\text{ x }10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00\text{ x }10^{-3}\text{ C}\right)\left(6.00\text{ x }10^{-3}\text{ C}\right)}{\left(\sqrt{2}\text{ m}\right)^2}\frac{\left(1.00\text{ m}\right)\hat{i}+\left(1.00\text{ m}\right)\hat{j}}{\left(\sqrt{2}\text{ m}\right)}=\left(1.15\text{ x }10^5\text{ N}\right)\hat{i}+\left(1.15\text{ x }10^5\text{ N}\right)\hat{j}$$

$$\vec{F}_4=\vec{F}_{1,4}+\vec{F}_{2,4}+\vec{F}_{3,4}=\left(-2.09\text{ x }10^5\text{ N}\right)\hat{i}+\left(-2.09\text{ x }10^5\text{ N}\right)\hat{j}$$

$$F_4=\sqrt{F_{4_x}^2+F_{4_y}^2}=\sqrt{\left(-2.09\text{ x }10^5\text{ N}\right)^2+\left(-2.09\text{ x }10^5\text{ N}\right)^2}=\boxed{2.96\text{ x }10^5\text{ N}}$$

$$\theta_4=\tan^{-1}\left(\frac{F_{4_y}}{F_{4_x}}\right)=\tan^{-1}\left(\frac{-2.09\text{ x }10^5\text{ N}}{-2.09\text{ x }10^5\text{ N}}\right)=\ 45°+180°=\boxed{225°}$$



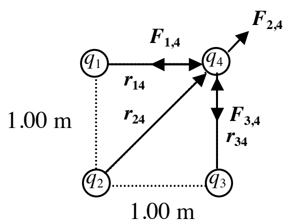
because of symmetry the magnitude of the force on the other charges are the same

$$\boxed{F_1=F_2=F_3=F_4=\ 2.96\text{ x }10^5\text{ N}}$$

the directions of these forces are:

$$\boxed{\theta_1=315°,\ \theta_2=45°,\text{ and }\theta_3=135°}$$

10.) (alternative solution)



$q_1 = q_3 = -6.00 \text{ x } 10^{-3} \text{ C}$ and $q_2 = q_4 = 6.00 \text{ x } 10^{-3} \text{ C}$

$r_1 = r_3 = 1.00 \text{ m}$, $r_2 = \sqrt{2} \text{ m}$

looking at the charge q_4 in the upper right-hand corner

$$F_{1,4} = F_{3,4} = k\frac{q_3 q_4}{r_{34}^2} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00 \text{ x } 10^{-3} \text{ C}\right)\left(6.00 \text{ x } 10^{-3} \text{ C}\right)}{\left(1.00 \text{ m}\right)^2} = 3.24 \text{ x } 10^5 \text{ N}$$

$F_{1,4}$ only has a negative x-component and $F_{1,4_x} = -3.24 \text{ x } 10^5 \text{ N}$

$F_{3,4}$ only has a negative y-component and $F_{3,4_y} = -3.24 \text{ x } 10^5 \text{ N}$

$$F_{2,4} = k\frac{q_2 q_4}{r_{24}^2} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00 \text{ x } 10^{-3} \text{ C}\right)\left(6.00 \text{ x } 10^{-3} \text{ C}\right)}{\left(\sqrt{2} \text{ m}\right)^2} = 1.62 \text{ x } 10^5 \text{ N}$$

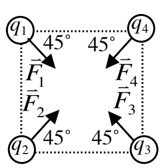
$F_{2,4_x} = F_{2,4}\cos\theta = \left(1.62 \text{ x } 10^5 \text{ N}\right)\cos 45° = 1.15 \text{ x } 10^5 \text{ N}$ and $F_{2,4_y} = F_{2,4}\sin\theta = \left(1.62 \text{ x } 10^5 \text{ N}\right)\sin 45° = 1.15 \text{ x } 10^5 \text{ N}$

the x-component of the total force on q_4 is $F_{4_x} = F_{1,4_x} + F_{2,4_x} = -3.24 \text{ x } 10^5 \text{ N} + 1.15 \text{ x } 10^5 \text{ N} = -2.09 \text{ x } 10^5 \text{ N}$

the y-component of the total force on q_4 is $F_4 = F_{3,4_y} + F_{2,4_y} = -3.24 \text{ x } 10^5 \text{ N} + 1.15 \text{ x } 10^5 \text{ N} = -2.09 \text{ x } 10^5 \text{ N}$

$$F_4 = \sqrt{F_{4_x}^2 + F_{4_y}^2} = \sqrt{\left(-2.09 \text{ x } 10^5 \text{ N}\right)^2 + \left(-2.09 \text{ x } 10^5 \text{ N}\right)^2} = \boxed{2.96 \text{ x } 10^5 \text{ N}}$$

$$\theta_4 = \tan^{-1}\left(\frac{F_{4_y}}{F_{4_x}}\right) = \tan^{-1}\left(\frac{-2.09 \text{ x } 10^5 \text{ N}}{-2.09 \text{ x } 10^5 \text{ N}}\right) = 45° + 180° = \boxed{225°}$$



because of symmetry the magnitude of the force on the other charges are the same

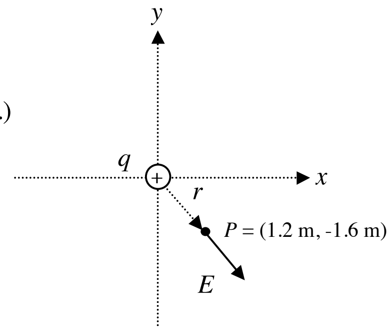
$$\boxed{F_1 = F_2 = F_3 = F_4 = 2.96 \text{ x } 10^5 \text{ N}}$$

the directions of these forces are:

$$\boxed{\theta_1 = 315°,\ \theta_2 = 45°,\text{ and } \theta_3 = 135°}$$

1.)

a.)



$$q = 6.0 \text{ x } 10^{-6}\text{ C} \text{ and } \vec{r} = (1.2\text{ m})\hat{i} + (-1.6\text{ m})\hat{j}$$

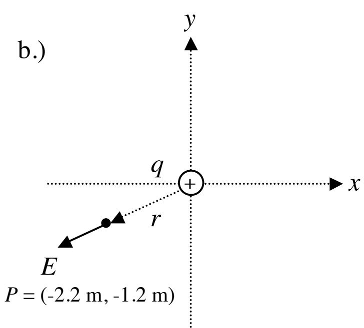
$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2\text{ m})^2 + (-1.6\text{ m})^2} = 2.0\text{ m}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(1.2\text{ m})\hat{i} + (-1.6\text{ m})\hat{j}}{2.0\text{ m}} = 0.6\hat{i} - 0.8\hat{j}$$

$$\vec{E} = k\frac{q}{r^2}\hat{r} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.0 \text{ x } 10^{-6}\text{ C})}{(2.0\text{ m})^2}\left(0.6\hat{i} - 0.8\hat{j}\right) = \left(1.35 \text{ x } 10^4\ \frac{\text{N}}{\text{C}}\right)\left(0.6\hat{i} - 0.8\hat{j}\right)$$

$$\vec{E} = \boxed{\left(8.10 \text{ x } 10^3\ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(-1.08 \text{ x } 10^4\ \frac{\text{N}}{\text{C}}\right)\hat{j}}$$

b.)



$$q = 6.0 \text{ x } 10^{-6}\text{ C} \text{ and } \vec{r} = (-2.2\text{ m})\hat{i} + (-1.2\text{ m})\hat{j}$$

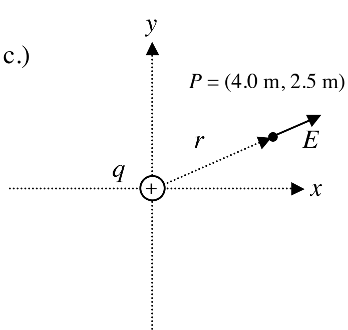
$$r = \sqrt{x^2 + y^2} = \sqrt{(-2.2\text{ m})^2 + (-1.2\text{ m})^2} = 2.51\text{ m}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(-2.2\text{ m})\hat{i} + (-1.2\text{ m})\hat{j}}{2.51\text{ m}} = -0.877\hat{i} - 0.478\hat{j}$$

$$\vec{E} = k\frac{q}{r^2}\hat{r} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.0 \text{ x } 10^{-6}\text{ C})}{(2.51\text{ m})^2}\left(-0.877\hat{i} - 0.478\hat{j}\right) = \left(8.57 \text{ x } 10^3\ \frac{\text{N}}{\text{C}}\right)\left(-0.877\hat{i} - 0.478\hat{j}\right)$$

$$\vec{E} = \boxed{\left(-7.52 \text{ x } 10^3\ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(-4.10 \text{ x } 10^3\ \frac{\text{N}}{\text{C}}\right)\hat{j}}$$

c.)



$$q = 6.0 \text{ x } 10^{-6}\text{ C} \text{ and } \vec{r} = (4.0\text{ m})\hat{i} + (2.5\text{ m})\hat{j}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(4.0\text{ m})^2 + (2.5\text{ m})^2} = 4.72\text{ m}$$

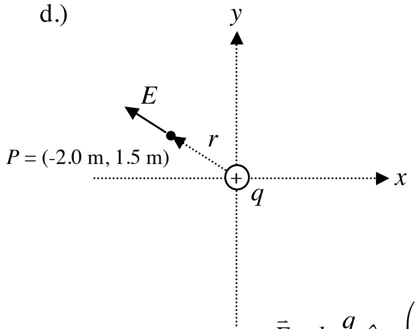
$$\hat{r} = \frac{\vec{r}}{r} = \frac{(4.0\text{ m})\hat{i} + (2.5\text{ m})\hat{j}}{4.72\text{ m}} = 0.847\hat{i} + 0.530\hat{j}$$

$$\vec{E} = k\frac{q}{r^2}\hat{r} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.0 \text{ x } 10^{-6}\text{ C})}{(4.72\text{ m})^2}\left(0.847\hat{i} + 0.530\hat{j}\right) = \left(2.42 \text{ x } 10^3\ \frac{\text{N}}{\text{C}}\right)\left(0.847\hat{i} + 0.530\hat{j}\right)$$

$$\vec{E} = \boxed{\left(2.05 \text{ x } 10^3\ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(1.28 \text{ x } 10^3\ \frac{\text{N}}{\text{C}}\right)\hat{j}}$$

1.) (continued)

d.)



$$q = 6.0 \text{ x } 10^{-6} \text{ C and } \vec{r} = (-2.0 \text{ m})\hat{i} + (1.5 \text{ m})\hat{j}$$

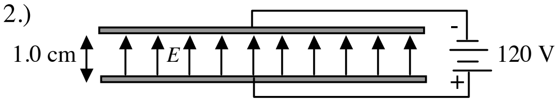
$$r = \sqrt{x^2 + y^2} = \sqrt{(-2.0 \text{ m})^2 + (1.5 \text{ m})^2} = 2.50 \text{ m}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(-2.0 \text{ m})\hat{i} + (1.5 \text{ m})\hat{j}}{2.50 \text{ m}} = -0.80\hat{i} + 0.60\hat{j}$$

$$\vec{E} = k\frac{q}{r^2}\hat{r} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6.0 \text{ x } 10^{-6} \text{ C})}{(2.50 \text{ m})^2}\left(-0.80\hat{i} + 0.60\hat{j}\right) = \left(8.64 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\left(-0.80\hat{i} + 0.60\hat{j}\right)$$

$$\vec{E} = \boxed{\left(-6.91 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(5.18 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{j}}$$

2.)



$$E = 1.20 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}, \ m_e = 9.11 \text{ x } 10^{-31} \text{ kg}, \ e = 1.60 \text{ x } 10^{-19} \text{ C}$$

a.)

$$\vec{F}_E = q\vec{E} = \left(-1.60 \text{ x } 10^{-19} \text{ C}\right)\left(1.20 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\hat{j} = \boxed{\left(-1.92 \text{ x } 10^{-15} \text{ N}\right)\hat{j}} \quad \text{(downward)}$$

b.)

$$\vec{F}_g = m_e\vec{g} = \left(9.11 \text{ x } 10^{-31} \text{ kg}\right)\left(-9.8 \ \frac{\text{m}}{\text{s}^2}\right)\hat{j} = \boxed{\left(-8.93 \text{ x } 10^{-30} \text{ N}\right)\hat{j}} \quad \text{(downward)}$$

c.) using Newton's 2nd Law $F_{net} = \sum F = ma$

$$\text{so } \vec{a} = \frac{\vec{F}_E + \vec{F}_g}{m_e} = \frac{\left(-1.92 \text{ x } 10^{-15} \text{ N}\right)\hat{j} + \left(-8.93 \text{ x } 10^{-30} \text{ N}\right)\hat{j}}{9.11 \text{ x } 10^{-31} \text{ kg}} = \boxed{-2.11 \text{ x } 10^{15} \ \frac{\text{m}}{\text{s}^2}} \quad \text{(downward)}$$

d.)

$$v^2 = v_o^2 + 2a\Delta y$$

$$v = \sqrt{2a\Delta y} \quad (v_o = 0) \quad \text{so} \quad v = \sqrt{2\left(-2.11 \text{ x } 10^{15} \ \frac{\text{m}}{\text{s}^2}\right)(-0.01 \text{ m})} = \boxed{6.49 \text{ x } 10^6 \ \frac{\text{m}}{\text{s}}}$$

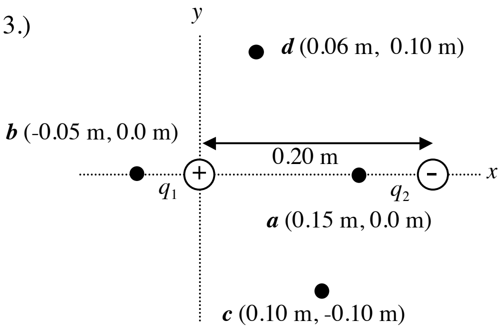
$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(9.11 \text{ x } 10^{-31} \text{ kg}\right)\left(6.49 \text{ x } 10^6 \ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{1.92 \text{ x } 10^{-17} \text{ J}}$$

e.) electron launched with initial horizontal velocity v_{x_o} will be accelerated downward and follow the path of a horizontally launched projectile "falling" with the acceleration found in part (c).

$$y = \frac{1}{2}at^2 + v_{y_o}t \quad \text{or } y = \frac{1}{2}\left(-2.11 \text{ x } 10^{15} \ \frac{\text{m}}{\text{s}^2}\right)t^2 + 0 = \left(-1.06 \text{ x } 10^{15} \ \frac{\text{m}}{\text{s}^2}\right)t^2$$

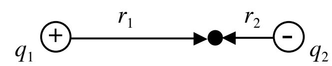
$$x = v_{x_o}t \text{ so } t = \frac{x}{v_{x_o}} \text{ and } y = \left(-1.06 \text{ x } 10^{15} \ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{x}{v_{x_o}}\right)^2 = \boxed{\frac{\left(-1.06 \text{ x } 10^{15} \ \frac{\text{m}}{\text{s}^2}\right)}{v_{x_o}^2}x^2}$$

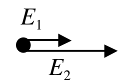
3.)



$q_1 = 22 \text{ x } 10^{-9}$ C and $q_2 = -22 \text{ x } 10^{-9}$ C

at point ***a***





$\vec{r}_1 = (0.15 \text{ m})\hat{i}$ and $\vec{r}_2 = (-0.05 \text{ m})\hat{i}$

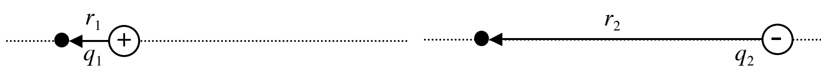
$r_1 = 0.15$ m, $\hat{r}_1 = \hat{i}$ and $r_2 = 0.05$ m, $\hat{r}_2 = -\hat{i}$

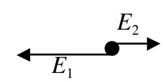
$$\vec{E}_1 = k\frac{q_1}{r_1^2}\hat{r}_1 = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(22 \text{ x } 10^{-9} \text{ C})}{(0.15 \text{ m})^2}\hat{i} = \left(8.80 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{i}$$

$$\vec{E}_2 = k\frac{q_2}{r_2^2}\hat{r}_2 = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(-22 \text{ x } 10^{-9} \text{ C})}{(0.05 \text{ m})^2}(-\hat{i}) = \left(7.92 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\hat{i}$$

$$\vec{E}_a = \vec{E}_1 + \vec{E}_2 = \boxed{\left(8.80 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\hat{i}}$$

at point ***b***





$\vec{r}_1 = (-0.05 \text{ m})\hat{i}$ and $\vec{r}_2 = (-0.25 \text{ m})\hat{i}$

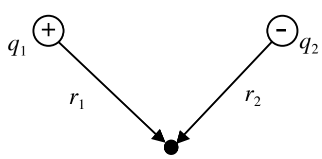
$r_1 = 0.05$ m, $\hat{r}_1 = -\hat{i}$ and $r_2 = 0.25$ m, $\hat{r}_2 = -\hat{i}$

$$\vec{E}_1 = k\frac{q_1}{r_1^2}\hat{r}_1 = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(22 \text{ x } 10^{-9} \text{ C})}{(0.05 \text{ m})^2}(-\hat{i}) = \left(-7.92 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\hat{i}$$

$$\vec{E}_2 = k\frac{q_2}{r_2^2}\hat{r}_2 = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(-22 \text{ x } 10^{-9} \text{ C})}{(0.25 \text{ m})^2}(-\hat{i}) = \left(3.17 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{i}$$

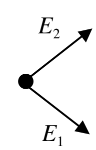
$$\vec{E}_b = \vec{E}_1 + \vec{E}_2 = \boxed{\left(-7.60 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\hat{i}}$$

at point ***c***



$\vec{r}_1 = (0.10 \text{ m})\hat{i} + (-0.10 \text{ m})\hat{j}$

$\vec{r}_2 = (-0.10 \text{ m})\hat{i} + (-0.10 \text{ m})\hat{j}$



$$r_1 = \sqrt{(0.10 \text{ m})^2 + (-0.10 \text{ m})^2} = 0.1414 \text{ m}$$

$$\hat{r}_1 = \frac{\vec{r}_1}{r_1} = \frac{(0.10 \text{ m})\hat{i} + (-0.10 \text{ m})\hat{j}}{0.1414 \text{ m}} = 0.7072\hat{i} - 0.7072\hat{j}$$

$$r_2 = \sqrt{(-0.10 \text{ m})^2 + (-0.10 \text{ m})^2} = 0.1414 \text{ m}$$

$$\hat{r}_2 = \frac{\vec{r}_2}{r_2} = \frac{(-0.10 \text{ m})\hat{i} + (-0.10 \text{ m})\hat{j}}{0.1414 \text{ m}} = -0.7072\hat{i} - 0.7072\hat{j}$$

3.) (continued)

$$\vec{E}_1 = k\frac{q_1}{r_1^2}\hat{r}_1 = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(22 \text{ x } 10^{-9} \text{ C}\right)}{\left(0.1414 \text{ m}\right)^2}\left(0.7072\hat{i} - 0.7072\hat{j}\right) = \left(9.91 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\left(0.7072\hat{i} - 0.7072\hat{j}\right)$$

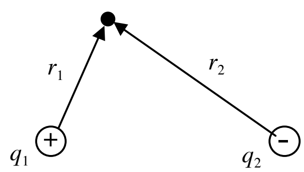
$$\vec{E}_1 = \left(7.01 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{i} - \left(7.01 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

$$\vec{E}_2 = k\frac{q_2}{r_2^2}\hat{r}_2 = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-22 \text{ x } 10^{-9} \text{ C}\right)}{\left(0.1414 \text{ m}\right)^2}\left(-0.7072\hat{i} - 0.7072\hat{j}\right) = \left(9.91 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\left(0.7072\hat{i} + 0.7072\hat{j}\right)$$

$$\vec{E}_2 = \left(7.01 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(7.01 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

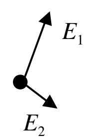
$$\vec{E}_c = \vec{E}_1 + \vec{E}_2 = \boxed{\left(1.40 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\hat{i}}$$

at point ***d***



$$\vec{r}_1 = \left(0.06 \text{ m}\right)\hat{i} + \left(0.10 \text{ m}\right)\hat{j}$$

$$\vec{r}_2 = \left(-0.14 \text{ m}\right)\hat{i} + \left(0.10 \text{ m}\right)\hat{j}$$



$$r_1 = \sqrt{\left(0.06 \text{ m}\right)^2 + \left(0.10 \text{ m}\right)^2} = 0.1166 \text{ m}$$

$$\hat{r}_1 = \frac{\vec{r}_1}{r_1} = \frac{\left(0.06 \text{ m}\right)\hat{i} + \left(0.10 \text{ m}\right)\hat{j}}{0.1166 \text{ m}} = 0.5146\hat{i} + 0.8576\hat{j}$$

$$r_2 = \sqrt{\left(-0.14 \text{ m}\right)^2 + \left(0.10 \text{ m}\right)^2} = 0.1720 \text{ m}$$

$$\hat{r}_2 = \frac{\vec{r}_2}{r_2} = \frac{\left(-0.14 \text{ m}\right)\hat{i} + \left(0.10 \text{ m}\right)\hat{j}}{0.1720 \text{ m}} = -0.8140\hat{i} + 0.5814\hat{j}$$

$$\vec{E}_1 = k\frac{q_1}{r_1^2}\hat{r}_1 = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(22 \text{ x } 10^{-9} \text{ C}\right)}{\left(0.1166 \text{ m}\right)^2}\left(0.5146\hat{i} + 0.8576\hat{j}\right) = \left(1.456 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\left(0.5146\hat{i} + 0.8576\hat{j}\right)$$

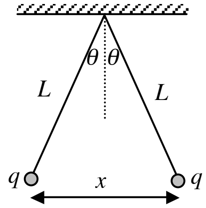
$$\vec{E}_1 = \left(7.49 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(1.25 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

$$\vec{E}_2 = k\frac{q_2}{r_2^2}\hat{r}_2 = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-22 \text{ x } 10^{-9} \text{ C}\right)}{\left(0.1720 \text{ m}\right)^2}\left(-0.8140\hat{i} + 0.5814\hat{j}\right) = \left(6.693 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\left(0.8140\hat{i} - 0.5814\hat{j}\right)$$

$$\vec{E}_2 = \left(5.45 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(-3.89 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

$$\vec{E}_d = \vec{E}_1 + \vec{E}_2 = \boxed{\left(1.29 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(8.61 \text{ x } 10^3 \ \frac{\text{N}}{\text{C}}\right)\hat{j}}$$

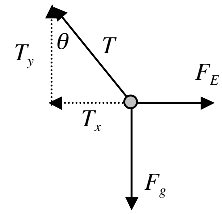
4.)



looking at the forces on q

$T_x = T\sin\theta$ and $T_y = T\cos\theta$

$F_E = k\dfrac{q^2}{x^2}$ and $F_g = mg$



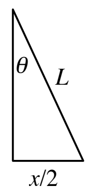
for static equilibrium $\sum F_x = 0$ and $\sum F_y = 0$

if $\sum F_x = 0$ then $F_E - T_x = 0$ and $F_E = T_x$ and $k\dfrac{q^2}{x^2} = T\sin\theta$

if $\sum F_y = 0$ then $T_y - F_g = 0$ and $T_y = F_g$ and $mg = T\cos\theta$

therefore $\dfrac{k\dfrac{q^2}{x^2}}{mg} = \dfrac{T\sin\theta}{T\cos\theta} = \tan\theta$ and for small angles $\dfrac{k}{mg}\dfrac{q^2}{x^2} = \sin\theta$

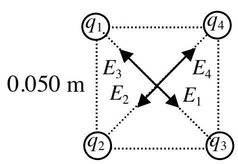
since charges are the same the separation is symmetric and



$$\sin\theta = \frac{\frac{x}{2}}{L} = \frac{x}{2L} \text{ so } \frac{k}{mg}\frac{q^2}{x^2} = \frac{x}{2L} \text{ and } x^3 = \frac{2Lkq^2}{mg}$$

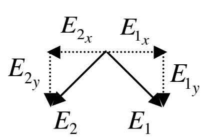
$$x = \left(\frac{2Lkq^2}{mg}\right)^{\frac{1}{3}} = \left(\frac{2Lq^2}{4\pi\varepsilon_o mg}\right)^{\frac{1}{3}} = \left(\frac{q^2 L}{2\pi\varepsilon_o mg}\right)^{\frac{1}{3}}$$

5.)



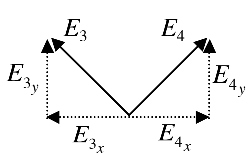
$q_1 = 2.00 \text{ x } 10^{-9}$ C , $q_2 = -2.00 \text{ x } 10^{-9}$ C , $q_3 = 4.00 \text{ x } 10^{-9}$ C , $q_4 = -4.00 \text{ x } 10^{-9}$ C

$r_1 = r_2 = r_3 = r_4 = 0.025\sqrt{2}$ m



$$E_2 = E_1 = k\frac{q_1}{r_1^2} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(2.00 \text{ x } 10^{-9} \text{ C}\right)}{\left(0.025\sqrt{2} \text{ m}\right)^2} = 1.44 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}$$

$E_{2_x} = -E_{1_x}$ and $E_{2_y} = E_{1_y} = \left(-1.44 \text{ x } 10^4 \ \dfrac{\text{N}}{\text{C}}\right)\sin 45° = -1.018 \text{ x } 10^4 \ \dfrac{\text{N}}{\text{C}}$



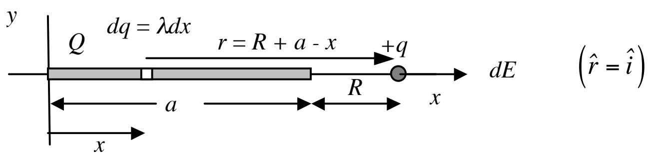
$$E_4 = E_3 = k\frac{q_3}{r_3^2} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(4.00 \text{ x } 10^{-9} \text{ C}\right)}{\left(0.025\sqrt{2} \text{ m}\right)^2} = 2.88 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}$$

$E_{4_x} = -E_{3_x}$ and $E_{4_y} = E_{3_y} = \left(2.88 \text{ x } 10^4 \ \dfrac{\text{N}}{\text{C}}\right)\sin 45° = 2.036 \text{ x } 10^4 \ \dfrac{\text{N}}{\text{C}}$

the x-components cancel so $E_x = 0$ and $E_y = 2\left(-1.018 \text{ x } 10^4 \ \dfrac{\text{N}}{\text{C}}\right) + 2\left(2.036 \text{ x } 10^4 \ \dfrac{\text{N}}{\text{C}}\right) = 2.036 \text{ x } 10^4 \ \dfrac{\text{N}}{\text{C}}$

$$\vec{E} = \boxed{\left(2.036 \text{ x } 10^4 \ \frac{\text{N}}{\text{C}}\right)\hat{j}}$$

1.)



$$dq = \lambda dx = \frac{Q}{a}dx$$

a.)

$$d\vec{E} = \frac{k\lambda dx}{r^2}\hat{r} = \frac{k\lambda dx}{(R+a-x)^2}\hat{i}$$

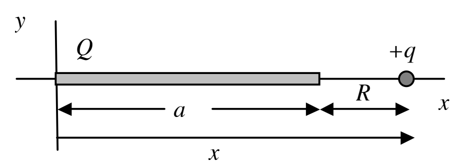
$$E_x = \int_0^a \frac{k\lambda dx}{(R+a-x)^2} = k\lambda\int_0^a (-1)(R+a-x)^{-2}(-1)dx$$

$$E_x = \int_0^a \frac{k\lambda dx}{(R+a-x)^2} = k\lambda \frac{(-1)(R+a-x)^{-1}}{(-1)}\Bigg]_0^a = k\lambda\left(\frac{1}{R} - \frac{1}{(R+a)}\right)$$

$$E_x = k\lambda\left(\frac{(R+a)-R}{R(R+a)}\right) = k\lambda\,\frac{a}{R(R+a)}$$

$$x = R + a$$
$$R = x - a$$

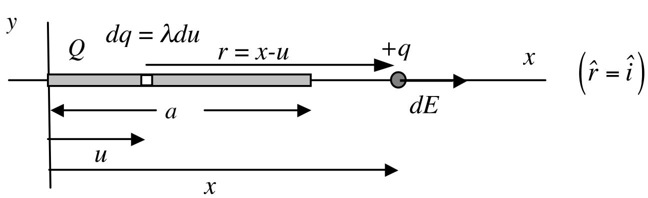
$$E_x = \frac{k\lambda a}{(x-a)x} = \frac{kQ}{(x-a)x}$$



$$E_y = 0$$

$$\boxed{\vec{E} = \frac{kQ}{(x-a)x}\hat{i} \quad (x > a)}$$

a.) (alternative solution)



$$dq = \lambda du = \frac{Q}{a}du$$

$$d\vec{E} = \frac{k\lambda du}{r^2}\hat{r} = \frac{k\lambda du}{(x-u)^2}\hat{i}$$

$$E_x = \int_0^a \frac{k\lambda du}{(x-u)^2} = k\lambda\int_0^a (-1)(x-u)^{-2}(-1)du$$

$$E_x = k\lambda\frac{(-1)(x-u)^{-1}}{(-1)}\Bigg]_0^a = k\lambda\frac{1}{x-u}\Bigg]_0^a = k\lambda\left(\frac{1}{x-a} - \frac{1}{x}\right)$$

$$E_x = k\lambda\left(\frac{x-(x-a)}{(x-a)x}\right) = k\lambda\,\frac{a}{(x-a)x} = k\,\frac{Q}{a}\,\frac{a}{(x-a)x}$$

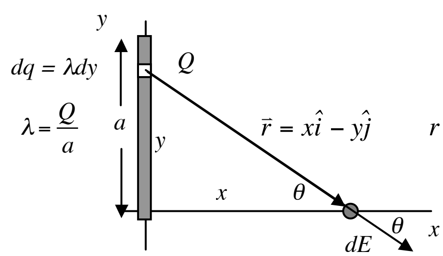
$$E_x = \frac{kQ}{(x-a)x}$$

$$E_y = 0 \qquad \boxed{\vec{E} = \frac{kQ}{(x-a)x}\hat{i} \quad (x > a)}$$

b.) $$\boxed{\vec{F} = q_o\vec{E} = \frac{kqQ}{(x-a)x}\hat{i}}$$

c.) $$x >> a \Rightarrow x - a \approx x \qquad \boxed{F(x >> a) = \frac{kqQ}{x^2}}$$ (same as for a point charge)

2.)



$$\vec{r} = x\hat{i} - y\hat{j} \qquad r = \sqrt{x^2+y^2} \qquad \hat{r} = \frac{x\hat{i} - y\hat{j}}{\sqrt{x^2+y^2}}$$

a.) $$d\vec{E} = \frac{kdq}{r^2}\hat{r} = \frac{k\lambda dy}{(x^2+y^2)}\frac{(x\hat{i} - y\hat{j})}{\sqrt{x^2+y^2}} = \frac{k\lambda xdy}{(x^2+y^2)^{3/2}}\hat{i} - \frac{k\lambda ydy}{(x^2+y^2)^{3/2}}\hat{j}$$

$$dE_x = \frac{k\lambda xdy}{(x^2+y^2)^{3/2}} \quad \text{and} \quad dE_y = -\frac{k\lambda ydy}{(x^2+y^2)^{3/2}}$$

$$E_x = \int_0^a \frac{k\lambda xdy}{(x^2+y^2)^{3/2}} = k\lambda x\int_0^a \frac{dy}{(x^2+y^2)^{3/2}}$$

$$E_x = k\lambda x\int_0^a \frac{dy}{\left(x^2\left(1+\left(\frac{y}{x}\right)^2\right)\right)^{3/2}}$$

Let $\frac{y}{x} = \tan\theta$

$y = x\tan\theta$ and $dy = x\sec^2\theta d\theta$

$$E_x = k\lambda x\int \frac{x\sec^2\theta d\theta}{(x^2(1+\tan^2\theta))^{3/2}} = k\lambda\int \frac{x^2\sec^2\theta d\theta}{(x^2\sec^2\theta)^{3/2}} = k\lambda\int \frac{x^2\sec^2\theta d\theta}{x^3\sec^3\theta} = k\lambda\int \frac{d\theta}{x\sec\theta}$$

$$E_x = \frac{k\lambda}{x}\int \cos\theta d\theta = \frac{k\lambda}{x}\sin\theta = \frac{k\lambda}{x}\frac{y}{\sqrt{x^2+y^2}}\Bigg]_0^a \qquad \left(\sin\theta = \frac{y}{\sqrt{x^2+y^2}}\right)$$

$$E_x = \frac{k\lambda}{x}\left(\frac{a}{\sqrt{x^2+a^2}} - \frac{0}{\sqrt{x^2+0^2}}\right) = \frac{k\lambda a}{x\sqrt{x^2+a^2}}$$

$$E_x = \frac{kQ}{x\sqrt{x^2+a^2}} \qquad \left(\lambda = \frac{Q}{a}\right)$$

$$E_y = -\int_0^a \frac{k\lambda y dy}{\left(x^2+y^2\right)^{3/2}} = -k\lambda \int_0^a \frac{y dy}{\left(x^2+y^2\right)^{3/2}} = -k\lambda \int_0^a \frac{\left(x^2+y^2\right)^{-3/2}}{2} 2y dy$$

$$E_y = -k\lambda \left. \frac{\left(x^2+y^2\right)^{-1/2}}{2(-1/2)} \right]_0^a = k\lambda \left. \frac{\left(x^2+y^2\right)^{-1/2}}{2(1/2)} \right]_0^a = k\lambda\left(\left(x^2+a^2\right)^{-1/2} - \left(x^2\right)^{-1/2}\right)$$

$$E_y = k\lambda\left(\frac{1}{\sqrt{x^2+a^2}} - \frac{1}{x}\right) = k\lambda \frac{x-\sqrt{x^2+a^2}}{x\sqrt{x^2+a^2}}$$

$$E_y = -\frac{kQ}{a}\frac{\sqrt{x^2+a^2}-x}{x\sqrt{x^2+a^2}} \qquad \left(\lambda = \frac{Q}{a}\right)$$

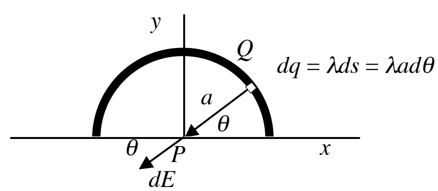
$$\boxed{\vec{E} = \left(\frac{kQ}{x\sqrt{x^2+a^2}}\right)\hat{i} - \left(\frac{kQ}{a}\frac{\sqrt{x^2+a^2}-x}{x\sqrt{x^2+a^2}}\right)\hat{j}}$$

b.) $\vec{F} = q_o\vec{E} = -q\vec{E}$

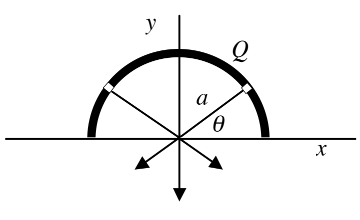
$$\boxed{\vec{F} = -\left(\frac{kqQ}{x\sqrt{x^2+a^2}}\right)\hat{i} + \left(\frac{kqQ}{a}\frac{\sqrt{x^2+a^2}-x}{x\sqrt{x^2+a^2}}\right)\hat{j}}$$

3.)

$$\left(\lambda = \frac{Q}{\pi a}\right)$$



$$\vec{r} = -a\cos\theta\hat{i} - a\sin\theta\hat{j}$$
$$r = a$$
$$\hat{r} = -\cos\theta\hat{i} - \sin\theta\hat{j}$$



$$d\vec{E} = \frac{kdq}{r^2}\hat{r} = \frac{k\lambda a d\theta}{a^2}\left(-\cos\theta\hat{i} - \sin\theta\hat{j}\right) = \frac{k\lambda d\theta}{a}\left(-\cos\theta\hat{i} - \sin\theta\hat{j}\right)$$

$E_x = 0$ due to symmetry.

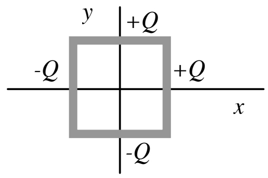
$$dE_y = -\frac{k\lambda d\theta}{a}\sin\theta$$

$$E_y = -\int_0^\pi \frac{k\lambda d\theta}{a}\sin\theta = -\frac{k\lambda}{a}\int_0^\pi \sin\theta d\theta = \left.\frac{k\lambda}{a}\cos\theta\right]_0^\pi$$

$$E_y = \frac{k\lambda}{a}\left(\cos\pi - \cos 0\right) = \frac{k\lambda}{a}\left((-1)-1\right) = \frac{-2k\lambda}{a}$$

$$\boxed{\vec{E} = -\frac{2k\lambda}{a}\hat{j} = -\frac{2kQ}{\pi a^2}\hat{j}} \qquad \left(\lambda = \frac{Q}{\pi a}\right)$$

4.)



a,)

Using result from Example 8 for points on the *x*-axis the field from a uniform line charge on the *y*-axis ($-a < y < a$):

$$E = \frac{kQ}{x\sqrt{x^2 + \left(\frac{a}{2}\right)^2}}$$

Note that in Example 8 the segment was 2*a* long and in this problem each segment is *a* long.

Square is composed of 4 such segments. The left segment attracts a positive charge placed the center to the left and the right segment repels a positive charge to the left. The upper segment repels a positive charge downward and the lower segment attracts a positive charge downward. The center of the square is a distance $x = a/2$ from each line segment.

$$E = \frac{kQ}{\frac{a}{2}\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}} = \frac{kQ}{\frac{a}{2}\sqrt{2\frac{a^2}{4}}} = \frac{kQ}{\frac{a}{2}\sqrt{\frac{a^2}{2}}} = \frac{kQ}{\frac{a^2}{2\sqrt{2}}} = \frac{2\sqrt{2}kQ}{a^2}$$

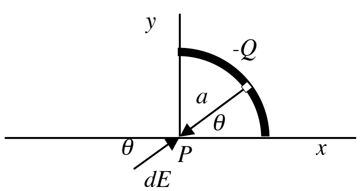
$$\vec{E}_{right} = \vec{E}_{left} = -\frac{2\sqrt{2}kQ}{a^2}\hat{i} \text{ and } \vec{E}_{top} = \vec{E}_{bottom} = -\frac{2\sqrt{2}kQ}{a^2}\hat{j}$$

$$\boxed{\vec{E} = -\frac{4\sqrt{2}kQ}{a^2}\hat{i} - \frac{4\sqrt{2}kQ}{a^2}\hat{j} = \frac{8kQ}{a^2}\angle 225^\circ}$$

b.)

Square is composed of 4 segments. The left segment repels a positive charge placed the center to the right which is canceled by the right segment which repels a positive charge to the left. The upper segment repels a positive charge downward which is canceled by the lower segment repels a positive charge upward. Therefore the electric field at the center of the square is **zero.**

5.)



$$dq = \lambda ds = \lambda a d\theta \quad \left(\lambda = \frac{-Q}{\frac{\pi}{2}a} = \frac{-2Q}{\pi a}\right)$$

$$r = a$$

$$\vec{r} = -a\cos\theta\hat{i} - a\sin\theta\hat{j}$$

$$\hat{r} = -\cos\theta\hat{i} - \sin\theta\hat{j}$$

$$d\vec{E} = \frac{kdq}{R^2}\hat{r} = \frac{k\lambda a d\theta}{a^2}\left(-\cos\theta\hat{i} - \sin\theta\hat{j}\right) = -\frac{k\lambda d\theta}{a}\cos\theta\hat{i} - \frac{k\lambda d\theta}{a}\sin\theta\hat{j}$$

$$dE_x = -\frac{k\lambda d\theta}{a}\cos\theta$$

$$E_x = \int_0^{\frac{\pi}{2}} -\frac{k\lambda d\theta}{a}\cos\theta = -\frac{k\lambda}{a}\int_0^{\frac{\pi}{2}}\cos\theta d\theta = -\frac{k\lambda}{a}\sin\theta\Big]_0^{\frac{\pi}{2}}$$

$$E_x = -\frac{k\lambda}{a}\left(\sin\frac{\pi}{2} - \sin 0\right) = -\frac{k\lambda}{a}(1 - 0) = -\frac{k\lambda}{a}$$

$$dE_y = -\frac{k\lambda d\theta}{a}\sin\theta$$

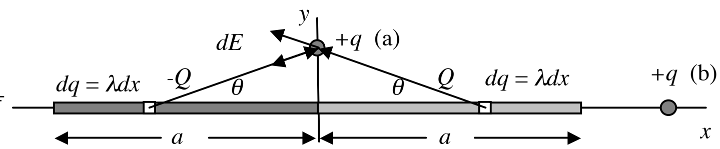
$$E_y = \int_0^{\frac{\pi}{2}} -\frac{k\lambda d\theta}{a}\sin\theta = -\frac{k\lambda}{a}\int_0^{\frac{\pi}{2}}\sin\theta d\theta = \frac{k\lambda}{a}\cos\theta\Big]_0^{\frac{\pi}{2}}$$

$$E_y = \frac{k\lambda}{a}\left(\cos\frac{\pi}{2} - \cos 0\right) = \frac{k\lambda}{a}(0-1) = -\frac{k\lambda}{a}$$

$$\boxed{\vec{E} = -\frac{k\lambda}{a}\hat{i} - \frac{k\lambda}{a}\hat{j} = \frac{2kQ}{\pi a^2}\hat{i} + \frac{2kQ}{\pi a^2}\hat{j}}$$

6.)

$$\vec{r}_- = -x\hat{i} + y\hat{j} \qquad \hat{r}_- = \frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$$



$$\vec{r}_+ = -x\hat{i} + y\hat{j} \qquad \hat{r}_+ = \frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$$

a.)

$$dE = \frac{kdq}{r^2} = \frac{k\lambda dx}{x^2 + y^2}$$

for the $-Q$ charge on $-x$-axis

$$d\vec{E} = \frac{k\lambda dx}{\left(x^2 + y^2\right)}\hat{r}_- = \frac{k\lambda dx}{\left(x^2 + y^2\right)}\frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}} = -\frac{k\lambda x dx}{\left(x^2 + y^2\right)^{3/2}}\hat{i} + \frac{k\lambda y dx}{\left(x^2 + y^2\right)^{3/2}}\hat{j}$$

$$E_x = \int_{-a}^{0} -\frac{k\lambda x dx}{\left(x^2 + y^2\right)^{3/2}} = -k\lambda\int_{-a}^{0}\frac{x dx}{\left(x^2 + y^2\right)^{3/2}} = -k\lambda\int_{-a}^{0}\frac{\left(x^2 + y^2\right)^{3/2}}{2}2x dx$$

$$E_x = -k\lambda\frac{\left(x^2 + y^2\right)^{-1/2}}{2(-1/2)}\Bigg]_{-a}^{0} = k\lambda\frac{\left(x^2 + y^2\right)^{-1/2}}{2(1/2)}\Bigg]_{-a}^{0} = k\lambda\left(\left(y^2\right)^{-1/2} - \left((-a)^2 + y^2\right)^{-1/2}\right)$$

$$E_x = k\lambda\left(\frac{1}{y} - \frac{1}{\sqrt{a^2 + y^2}}\right) = k\lambda\frac{\sqrt{a^2 + y^2} - y}{y\sqrt{a^2 + y^2}}$$

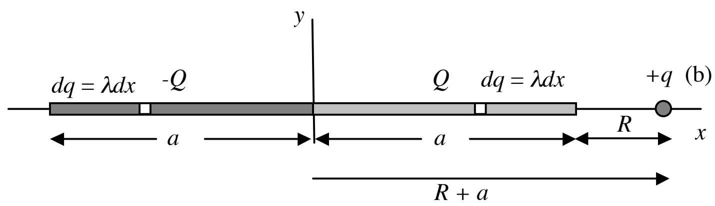
$$E_x = -\frac{kQ}{a}\frac{\sqrt{a^2 + y^2} - y}{y\sqrt{a^2 + y^2}} \qquad \left(\lambda = \frac{-Q}{a}\right)$$

The y-component is nonzero, but is cancelled out by the y-component from the $+Q$ distribution on the $+x$-axis. The $+Q$ charge on the $+x$-axis also contributes the same amount to E_x and both create a field in the $-x$ direction and total field is:

$$\vec{E} = -\frac{2kQ}{a}\frac{\left(\sqrt{a^2 + y^2} - y\right)}{y\sqrt{a^2 + y^2}}\hat{i}$$ the field at a point on the y-axis

$$\boxed{\vec{F} = q_o\vec{E} = -\frac{2kqQ}{a}\frac{\left(\sqrt{a^2 + y^2} - y\right)}{y\sqrt{a^2 + y^2}}\hat{i}}$$ the force on charge q

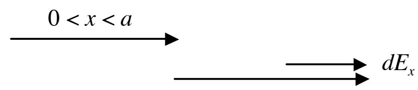
b.)



for $-Q$ charge

$r_- = a + R - x \qquad \hat{r}_- = \hat{i}$

for $+Q$ charge



$r_+ = a + R - x \qquad \hat{r}_+ = \hat{i}$

for $-Q$ charge

$$d\vec{E} = \frac{k\lambda dx}{r^2}\hat{r} = \frac{k\lambda dx}{(R+a-x)^2}\hat{i} \quad \text{so} \quad dE_x = \frac{k\lambda dx}{(R+a-x)^2}$$

$$E_x = \int_{-a}^{0} \frac{k\lambda dx}{(R+a-x)^2} = k\lambda \int_{-a}^{0} (-1)(R+a-x)^{-2}(-1)dx$$

$$E_x = k\lambda \frac{(-1)(R+a-x)^{-1}}{(-1)}\Bigg]_{-a}^{0} = k\lambda\left(\frac{1}{(R+a-0)} - \frac{1}{(R+a+a)}\right)$$

$$E_x = k\lambda\left(\frac{1}{(R+a)} - \frac{1}{(R+2a)}\right) = k\lambda\,\frac{(R+2a)-(R+a)}{(R+a)(R+2a)} = k\lambda\,\frac{a}{(R+a)(R+2a)}$$

$$E_x = -\frac{kQ}{(R+a)(R+2a)} \qquad \left(\lambda = \frac{-Q}{a}\right)$$

for $+Q$ charge

$$d\vec{E} = \frac{k\lambda dx}{r^2}\hat{r} = \frac{k\lambda dx}{(R+a-x)^2}\hat{i} \quad \text{so} \quad dE_x = \frac{k\lambda dx}{(R+a-x)^2}$$

$$E_x = \int_{0}^{a} \frac{k\lambda dx}{(R+a-x)^2} = k\lambda \int_{0}^{a} (-1)(R+a-x)^{-2}(-1)dx$$

$$E_x = \int_{0}^{a} \frac{k\lambda dx}{(R+a-x)^2} = k\lambda \frac{(-1)(R+a-x)^{-1}}{(-1)}\Bigg]_{0}^{a} = k\lambda\left(\frac{1}{R} - \frac{1}{(R+a)}\right)$$

$$E_x = k\lambda\left(\frac{(R+a)-R}{R(R+a)}\right) = k\lambda\,\frac{a}{R(R+a)}$$

$$E_x = \frac{kQ}{R(R+a)} \qquad \left(\lambda = \frac{Q}{a}\right)$$

For points on the x-axis that are greater than a and a distance R from a: $x = R + a$ so $R = x - a$.

for $-Q$ charge

$$E_x = -\frac{kQ}{(R+a)(R+2a)} = -\frac{kQ}{x(x+a)} \quad \text{and points in the } -x \text{ direction}$$

for $+Q$ charge

$$E_x = \frac{kQ}{R(R+a)} = \frac{kQ}{(x-a)x} \quad \text{and points in the } +x \text{ direction}$$

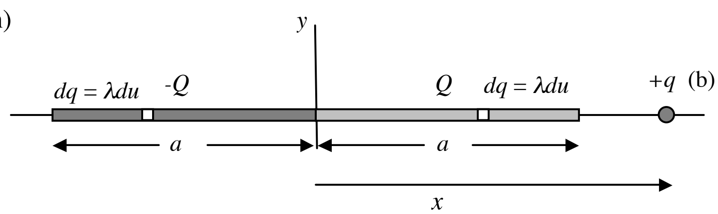
The total field is the superposition of these:

$$\vec{E} = \frac{kQ}{(x-a)x}\hat{i} - \frac{kQ}{x(x+a)}\hat{i} = kQ\frac{(x+a)-(x-a)}{x(x+a)(x-a)}\hat{i}$$

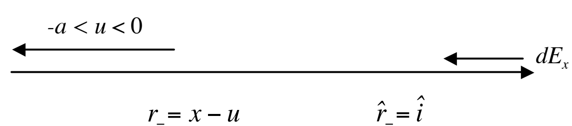
$$\vec{E} = \frac{2kQa}{x(x^2-a^2)}\hat{i} \qquad \text{the field at point } b\ (x > a)$$

$$\boxed{\vec{F} = q_o\vec{E} = \frac{2kqQa}{x(x^2-a^2)}\hat{i} \qquad \text{the force on charge } q}$$

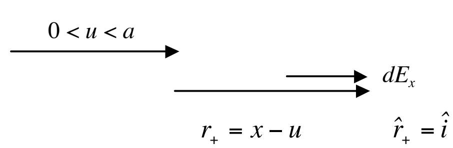
b.) (alternative solution)



for $-Q$ charge



for $+Q$ charge



for $-Q$ charge

$$d\vec{E} = \frac{k\lambda du}{r^2}\hat{r} = \frac{k\lambda du}{(x-u)^2}\hat{i} \qquad \text{so} \qquad dE_x = \frac{k\lambda du}{(x-u)^2}$$

$$E_x = \int_{-a}^{0}\frac{k\lambda du}{(x-u)^2} = k\lambda\int_{-a}^{0}(-1)(x-u)^{-2}(-1)du$$

$$E_x = k\lambda\frac{(-1)(x-u)^{-1}}{(-1)}\Bigg]_{-a}^{0} = k\lambda\left(\frac{1}{(x-0)} - \frac{1}{(x+a)}\right)$$

$$E_x = k\lambda\left(\frac{1}{x} - \frac{1}{(x+a)}\right) = k\lambda\frac{(x+a)-x}{x(x+a)} = k\lambda\frac{a}{x(x+a)}$$

$$E_x = -\frac{kQ}{x(x+a)} \qquad \left(\lambda = \frac{-Q}{a}\right)$$

for $+Q$ charge $\qquad d\vec{E} = \frac{k\lambda du}{r^2}\hat{r} = \frac{k\lambda du}{(x-u)^2}\hat{i}$ so $dE_x = \frac{k\lambda du}{(x-u)^2}$

$$E_x = \int_0^a \frac{k\lambda du}{(x-u)^2} = k\lambda\int_0^a (-1)(x-u)^{-2}(-1)du$$

$$E_x = k\lambda\frac{(-1)(x-u)^{-1}}{(-1)}\Bigg]_0^a = k\lambda\left(\frac{1}{(x-a)} - \frac{1}{x}\right)$$

$$E_x = k\lambda\left(\frac{x-(x-a)}{x(x-a)}\right) = k\lambda\frac{a}{x(x-a)}$$

$$E_x = \frac{kQ}{x(x-a)} \qquad \left(\lambda = \frac{Q}{a}\right)$$

for $-Q$ charge

$$E_x = -\frac{kQ}{x(x+a)}$$

for $+Q$ charge

$$E_x = \frac{kQ}{x(x-a)}$$

The total field is the superposition of these:

$$\vec{E} = \frac{kQ}{(x-a)x}\hat{i} - \frac{kQ}{x(x+a)}\hat{i} = kQ\frac{(x+a)-(x-a)}{x(x+a)(x-a)}\hat{i}$$

$$\vec{E} = \frac{2kQa}{x(x^2-a^2)}\hat{i}$$ the field at point b ($x > a$)

$$\boxed{\vec{F} = q_o\vec{E} = \frac{2kqQa}{x(x^2-a^2)}\hat{i}}$$ the force on charge q

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Answers in terms of ε_o. and Q. $\left(k = \frac{1}{4\pi\varepsilon_o}\right)$

1.) a.) $\vec{E} = \frac{Q}{4\pi\varepsilon_o(x-a)x}\hat{i}$ b.) $\vec{F} = \frac{qQ}{4\pi\varepsilon_o(x-a)x}\hat{i}$ c.) $F(x >> a) = \frac{qQ}{4\pi\varepsilon_o x^2}$

2.) a.) $\vec{E} = \left(\frac{Q}{4\pi\varepsilon_o x\sqrt{x^2+a^2}}\right)\hat{i} - \left(\frac{Q}{4\pi\varepsilon_o a}\frac{\sqrt{x^2+a^2}-x}{x\sqrt{x^2+a^2}}\right)\hat{j}$ b.) $\vec{F} = -\left(\frac{qQ}{4\pi\varepsilon_o x\sqrt{x^2+a^2}}\right)\hat{i} + \left(\frac{qQ}{4\pi\varepsilon_o a}\frac{\sqrt{x^2+a^2}-x}{x\sqrt{x^2+a^2}}\right)\hat{j}$

3.) $\vec{E} = -\frac{Q}{2\pi^2\varepsilon_o a^2}\hat{j}$ 4.) a.) $\vec{E} = -\frac{\sqrt{2}Q}{\pi\varepsilon_o a^2}\hat{i} - \frac{\sqrt{2}Q}{\pi\varepsilon_o a^2}\hat{j} = \frac{2Q}{\pi\varepsilon_o a^2}\angle 225°$ b.) $\vec{E} = 0$

5.) $\vec{E} = \frac{Q}{2\pi^2\varepsilon_o a^2}\hat{i} + \frac{Q}{2\pi^2\varepsilon_o a^2}\hat{j} = \frac{Q}{\sqrt{2}\pi^2\varepsilon_o a^2}\angle 45°$ 6.) a.) $\vec{F} = -\frac{qQ}{2\pi\varepsilon_o a}\frac{\left(y-\sqrt{a^2+y^2}\right)}{y\sqrt{a^2+y^2}}\hat{i}$ b.)

$\vec{F} = \frac{qQa}{2\pi\varepsilon_o x(x^2-a^2)}\hat{i}$