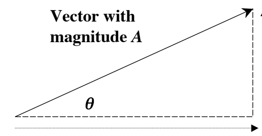
Vector Basics

Vector Components



$$A_y = A\sin\theta$$

$$A_x = A\cos\theta$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

Unit Vector Notation

Unit Vectors are vectors having unit length.

If $\bar{A}$ is any vector with length $A > 0$, then $\bar{A}/A$ is a unit vector, denoted by $\hat{a}$, having the same direction as $\bar{A}$. Then $\bar{A} = A\hat{a}$.

The rectangular unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors having the direction of the positive x, y, and z axes of a rectangular coordinate system.

Unit Vector Notation

If a two - dimensinal vector $\bar{A}$ has components A_x and A_y, then the vector $\bar{A}$ can be written in unit vector notation as:

$$\bar{A} = A_x\hat{i} + A_y\hat{j}$$

The unit vector pointing in the same direction as $\bar{A}$ is:

$$\hat{a} = \frac{\bar{A}}{A} = \frac{A_x\hat{i} + A_y\hat{j}}{\sqrt{A_x^2 + A_y^2}}$$

Dot or Scalar Product

The dot or scalar product of two vectors $\bar{A}$ and $\bar{B}$, denoted by $\bar{A}\cdot\bar{B}$ is defined as the product of the magnitudes of $\bar{A}$ and $\bar{B}$ and the cosine of the angle between them (when placed tail - to - tail).

$$\bar{A}\cdot\bar{B} = AB\cos\theta \qquad 0 \le \theta \le \pi$$

Note that $\bar{A}\cdot\bar{B}$ is a scalar and not a vector.

Dot or Scalar Product

The following laws are valid:

1.) $\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$

2.) $\bar{A}\cdot\left(\bar{B}+\bar{C}\right) = \bar{A}\cdot\bar{B} + \bar{A}\cdot\bar{C}$

3.) $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

4.) $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

5.) **If $\bar{A} = A_x\hat{i} + A_y\hat{j}$ and $\bar{B} = B_x\hat{i} + B_y\hat{j}$, then**

$$\bar{A}\cdot\bar{B} = A_xB_x + A_yB_y$$

$$\bar{A}\cdot\bar{A} = A^2 = A_x^2 + A_y^2$$

6.) **If $\bar{A}\cdot\bar{B} = 0$ and $\bar{A}$ and $\bar{B}$ are not null vectors, then $\bar{A}$ and $\bar{B}$ are perpendicular.**

Cross or Vector Product

The cross or vector product of $\vec{A}$ and $\vec{B}$ is a vector $\vec{C} = \vec{A} \times \vec{B}$. The magnitude of $\vec{A} \times \vec{B}$ is defined as the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and the sine of angle between them. The direction of the vector $\vec{C} = \vec{A} \times \vec{B}$ is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and such that $\vec{A}$, $\vec{B}$, and $\vec{C}$ form a right - handed system.

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{u} \qquad 0 \le \theta \le \pi$$

where $\hat{u}$ is a unit vector indicating the direction of $\vec{A} \times \vec{B}$.

Cross or Vector Product

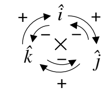
The following laws are valid :

1.) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

2.) $\vec{A} \times \left(\vec{B} + \vec{C}\right) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

3.) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

4.) $\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}$



5.) **If** $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ **and** $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, **then**

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

6.) **If** $\vec{A} \times \vec{B} = 0$ **and** $\vec{A}$ **and** $\vec{B}$ **are not null vectors, then** $\vec{A}$ **and** $\vec{B}$ **are parallel.**

Example 1

If $\vec{A} = 10\angle 36.87°$ and $\vec{B} = 14.142\angle 135°$ find :

a.) $\vec{A}$ **and** $\vec{B}$ **in unit vector notation**

b.) $\hat{a}$ **and** $\hat{b}$ **(unit vectors)**

c.) $\vec{A} \cdot \vec{B}$

d.) **the angle between** $\vec{A}$ **and** $\vec{B}$

e.) $\vec{A} \times \vec{B}$ **and** $\vec{B} \times \vec{A}$

Example 1

If $\vec{A} = 10\angle 36.87°$ and $\vec{B} = 14.142\angle 135°$ find :

a.) $\vec{A}$ **and** $\vec{B}$ **in unit vector notation**

$$\vec{A} = A_x\hat{i} + A_y\hat{j} = A\cos\theta_A\hat{i} + A\sin\theta_A\hat{j}$$

$$= \left(10\cos 36.87°\right)\hat{i} + \left(10\sin 36.87°\right)\hat{j}$$

$$\boxed{\vec{A} = 8\hat{i} + 6\hat{j}}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} = B\cos\theta_B\hat{i} + B\sin\theta_B\hat{j}$$

$$= \left(14.142\cos 135°\right)\hat{i} + \left(14.142\sin 135°\right)\hat{j}$$

$$\boxed{\vec{B} = -10\hat{i} + 10\hat{j}}$$

Example 1

If $\vec{A} = 10\angle 36.87°$ and $\vec{B} = 14.142\angle 135°$ find :

b.) $\hat{a}$ **and** $\hat{b}$ **(unit vectors)**

$$\hat{a} = \frac{\vec{A}}{A} = \frac{8\hat{i} + 6\hat{j}}{10} \qquad \hat{b} = \frac{\vec{B}}{B} = \frac{-10\hat{i} + 10\hat{j}}{14.142}$$

$$\boxed{\hat{a} = 0.8\hat{i} + 0.6\hat{j}} \qquad \boxed{\hat{b} = -0.7071\hat{i} + 0.7071\hat{j}}$$

Example 1

If $\vec{A} = 10\angle 36.87°$ and $\vec{B} = 14.142\angle 135°$ find :

c.) $\vec{A} \cdot \vec{B}$

$$\vec{A} \cdot \vec{B} = \left(A_x\hat{i} + A_y\hat{j}\right) \cdot \left(B_x\hat{i} + B_y\hat{j}\right)$$

$$= \left(A_x\hat{i}\right) \cdot \left(B_x\hat{i}\right) + \left(A_x\hat{i}\right) \cdot \left(B_y\hat{j}\right) + \left(A_y\hat{j}\right) \cdot \left(B_x\hat{i}\right) + \left(A_y\hat{j}\right) \cdot \left(B_y\hat{j}\right)$$

$$= A_xB_x\left(\hat{i} \cdot \hat{i}\right) + A_xB_y\left(\hat{i} \cdot \hat{j}\right) + A_yB_x\left(\hat{j} \cdot \hat{i}\right) + A_yB_y\left(\hat{j} \cdot \hat{j}\right)$$

$$= A_xB_x(1) + A_xB_y(0) + A_yB_x(0) + A_yB_y(1)$$

$$= A_xB_x + A_yB_y = (8)(-10) + (6)(10) \quad \boxed{= -20}$$

Example 1

If $\vec{A} = 10\angle 36.87°$ and $\vec{B} = 14.142\angle 135°$ find :

d.) the angle between $\vec{A}$ and $\vec{B}$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

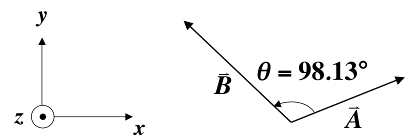
$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \cos^{-1}\left(\frac{-20}{(10)(14.142)}\right)$$

$$\boxed{\theta = 98.13°}$$

Example 1

If $\vec{A} = 10\angle 36.87°$ and $\vec{B} = 14.142\angle 135°$ find :

c.) $\vec{A}\cdot\vec{B}$



$$\vec{A}\cdot\vec{B} = AB\cos\theta = (10)(14.142)\cos(98.13°)$$

$$\boxed{\vec{A}\cdot\vec{B} = -20}$$

Example 1

If $\vec{A} = 10\angle 36.87°$ and $\vec{B} = 14.142\angle 135°$ find :

e.) $\vec{A}\times\vec{B}$

$$\vec{A}\times\vec{B} = \left(A_x\hat{i} + A_y\hat{j}\right)\times\left(B_x\hat{i} + B_y\hat{j}\right)$$

$$= \left(A_x\hat{i}\right)\times\left(B_x\hat{i}\right) + \left(A_x\hat{i}\right)\times\left(B_y\hat{j}\right) + \left(A_y\hat{j}\right)\times\left(B_x\hat{i}\right) + \left(A_y\hat{j}\right)\times\left(B_y\hat{j}\right)$$

$$= A_xB_x\left(\hat{i}\times\hat{i}\right) + A_xB_y\left(\hat{i}\times\hat{j}\right) + A_yB_x\left(\hat{j}\times\hat{i}\right) + A_yB_y\left(\hat{j}\times\hat{j}\right)$$

$$= A_xB_x(0) + A_xB_y\left(\hat{k}\right) + A_yB_x\left(-\hat{k}\right) + A_yB_y(0)$$

$$= \left(A_xB_y - A_yB_x\right)\hat{k} = ((8)(10) - (6)(-10))\hat{k} \quad \boxed{= 140\hat{k}}$$

Example 1

If $\vec{A} = 10\angle 36.87°$ and $\vec{B} = 14.142\angle 135°$ find :

e.) $\vec{B}\times\vec{A}$

$$\vec{B}\times\vec{A} = \left(B_x\hat{i} + B_y\hat{j}\right)\times\left(A_x\hat{i} + A_y\hat{j}\right)$$

$$= \left(B_x\hat{i}\right)\times\left(A_x\hat{i}\right) + \left(B_x\hat{i}\right)\times\left(A_y\hat{j}\right) + \left(B_y\hat{j}\right)\times\left(A_x\hat{i}\right) + \left(B_y\hat{j}\right)\times\left(A_y\hat{j}\right)$$

$$= B_xA_x\left(\hat{i}\times\hat{i}\right) + B_xA_y\left(\hat{i}\times\hat{j}\right) + B_yA_x\left(\hat{j}\times\hat{i}\right) + B_yA_y\left(\hat{j}\times\hat{j}\right)$$

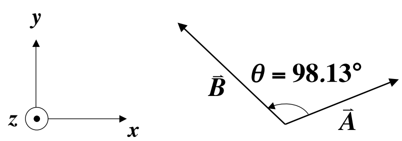
$$= B_xA_x(0) + B_xA_y\left(\hat{k}\right) + B_yA_x\left(-\hat{k}\right) + B_yA_y(0)$$

$$= \left(B_xA_y - B_yA_x\right)\hat{k} = ((-10)(6) - (10)(8))\hat{k} \quad \boxed{= -140\hat{k}}$$

Example 1

If $\vec{A} = 10\angle 36.87°$ and $\vec{B} = 14.142\angle 135°$ find :

e.) $\vec{A}\times\vec{B}$



$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta = (10)(14.142)\sin(98.1°)$$

$$\left|\vec{A}\times\vec{B}\right| = 140$$