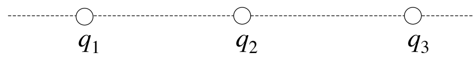
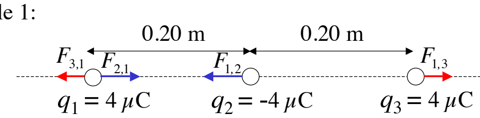


Example 1:

Three charges are fixed along a straight line as shown in the figure above with $q_1 = 4\ \mu$C, $q_2 = -4\ \mu$C, and $q_3 = 4\ \mu$C. The distance between q_1 and q_2 is 0.20 m and the distance between q_2 and q_3 is also 0.20 m. Find the net force on each charge due to the other charges.

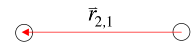
Example 1:



$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$$

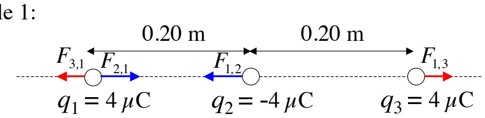
$$\vec{F}_{2,1} = \frac{1}{4\pi\varepsilon_o}\frac{q_2 q_1}{r^2}\hat{r}$$

$$k = \frac{1}{4\pi\varepsilon_o} = 9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$$

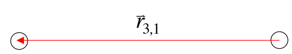


$$\vec{F}_{2,1} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-4 \text{ x } 10^{-6}\text{ C}\right)\left(4 \text{ x } 10^{-6}\text{ C}\right)}{(0.20\text{ m})^2}\left(-\hat{i}\right) = (3.6\text{ N})\hat{i}$$

Example 1:



$$\vec{F}_{3,1} = k\frac{q_3 q_1}{r^2}\hat{r}$$



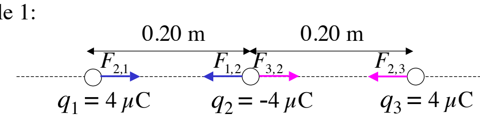
$$\vec{F}_{3,1} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(4 \text{ x } 10^{-6}\text{ C}\right)\left(4 \text{ x } 10^{-6}\text{ C}\right)}{(0.40\text{ m})^2}\left(-\hat{i}\right) = (-0.9\text{ N})\hat{i}$$

$$\vec{F}_{2,1} = (3.6\text{ N})\hat{i} \text{ and } \vec{F}_{3,1} = (-0.9\text{ N})\hat{i}$$

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1} = (3.6\text{ N})\hat{i} + (-0.9\text{ N})\hat{i}$$

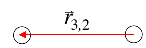
$$\boxed{\vec{F}_1 = (2.7\text{ N})\hat{i}}$$

Example 1:



$$\vec{F}_2 = \vec{F}_{1,2} + \vec{F}_{3,2} \qquad \vec{F}_{1,2} = -\vec{F}_{2,1} = (-3.6\text{ N})\hat{i}$$

$$\vec{F}_{3,2} = k\frac{q_3 q_2}{r^2}\hat{r}$$



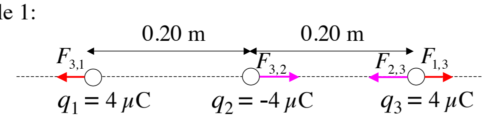
$$\vec{F}_{3,2} = k\frac{q_3 q_2}{r^2}\hat{r} = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(4 \text{ x } 10^{-6}\text{ C}\right)\left(-4 \text{ x } 10^{-6}\text{ C}\right)}{(0.20\text{ m})^2}\left(-\hat{i}\right)$$

$$\vec{F}_{3,2} = (3.6\text{ N})\hat{i}$$

$$\vec{F}_2 = \vec{F}_{1,2} + \vec{F}_{3,2} = (-3.6\text{ N})\hat{i} + (3.6\text{ N})\hat{i}$$

$$\boxed{\vec{F}_2 = 0}$$

Example 1:



$$\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3}$$

$$\vec{F}_{1,3} = -\vec{F}_{3,1} = (0.9\text{ N})\hat{i}$$

$$\vec{F}_{2,3} = -\vec{F}_{3,2} = (-3.6\text{ N})\hat{i}$$

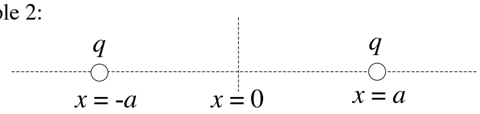
$$\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = (0.9\text{ N})\hat{i} + (-3.6\text{ N})\hat{i}$$

$$\boxed{\vec{F}_3 = (-2.7\text{ N})\hat{i}}$$

Example 2:

Two positive point charges q are placed on the x-axis at $x = a$ and $x = -a$. A negative point charge $-Q$ is located at some point on the x-axis. Find the net force that the two positive charges exert on $-Q$.

Example 2:



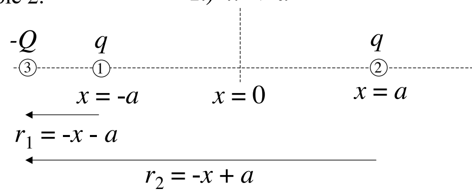
There are three cases that must be considered because the force is infinite at the location of each charge.

I.) $x < -a$

II.) $-a < x < a$

III.) $a < x$

Example 2: I.) $x < -a$



$$\bar{F}_{1,3} = k\frac{q_1 q_3}{r_1^2}\hat{r}_1 = k\frac{q(-Q)}{(-x-a)^2}\left(-\hat{i}\right) = k\frac{qQ}{(-x-a)^2}\hat{i}$$

$$\bar{F}_{2,3} = k\frac{q_2 q_3}{r_2^2}\hat{r}_2 = k\frac{q(-Q)}{(-x+a)^2}\left(-\hat{i}\right) = k\frac{qQ}{(-x+a)^2}\hat{i}$$

$$\bar{F}_3 = \bar{F}_{3,1} + \bar{F}_{3,2} = k\frac{qQ}{(-x-a)^2}\hat{i} + k\frac{qQ}{(-x+a)^2}\hat{i}$$

Example 2: I.) $x < -a$

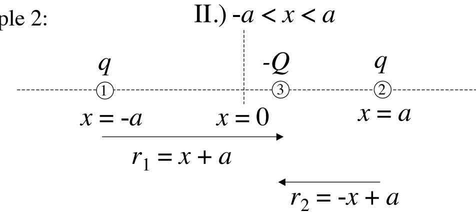
$$\bar{F}_3 = k\frac{qQ}{(-x-a)^2}\hat{i} + k\frac{qQ}{(-x+a)^2}\hat{i}$$

$$\bar{F}_3 = k\frac{qQ(-x+a)^2}{(-x-a)^2(-x+a)^2}\hat{i} + k\frac{qQ(-x-a)^2}{(-x-a)^2(-x+a)^2}\hat{i}$$

$$\bar{F}_3 = k\frac{qQ\left(x^2-2ax+a^2\right)+qQ\left(x^2+2ax+a^2\right)}{(-x-a)^2(-x+a)^2}\hat{i}$$

$$\boxed{\bar{F}_3 = \frac{2kqQ\left(x^2+a^2\right)}{(-x-a)^2(-x+a)^2}\hat{i} \quad (x < -a)}$$

Example 2: II.) $-a < x < a$



$$\bar{F}_{1,3} = k\frac{q_1 q_3}{r_1^2}\hat{r}_1 = k\frac{q(-Q)}{(x+a)^2}\hat{i} = -k\frac{qQ}{(x+a)^2}\hat{i}$$

$$\bar{F}_{2,3} = k\frac{q_2 q_3}{r_2^2}\hat{r}_2 = k\frac{q(-Q)}{(-x+a)^2}\left(-\hat{i}\right) = k\frac{qQ}{(-x+a)^2}\hat{i}$$

$$\bar{F}_3 = \bar{F}_{3,1} + \bar{F}_{3,2} = k\frac{qQ}{(-x+a)^2}\hat{i} - k\frac{qQ}{(x+a)^2}\hat{i}$$

Example 2: II.) $-a < x < a$

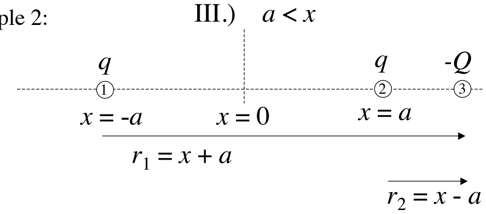
$$\bar{F}_3 = k\frac{qQ}{(-x+a)^2}\hat{i} - k\frac{qQ}{(x+a)^2}\hat{i}$$

$$\bar{F}_3 = k\frac{qQ(x+a)^2}{(-x+a)^2(x+a)^2}\hat{i} - k\frac{qQ(-x+a)^2}{(-x+a)^2(x+a)^2}\hat{i}$$

$$\bar{F}_3 = k\frac{qQ\left(x^2+2ax+a^2\right)-qQ\left(x^2-2ax+a^2\right)}{(-x+a)^2(x+a)^2}\hat{i}$$

$$\boxed{\bar{F}_3 = \frac{4kqQax}{(a-x)^2(a+x)^2}\hat{i} \quad (-a < x < a)}$$

Example 2: III.) $a < x$



$$\bar{F}_{1,3} = k\frac{q_1 q_3}{r_1^2}\hat{r}_1 = k\frac{q(-Q)}{(x+a)^2}\hat{i} = -k\frac{qQ}{(x+a)^2}\hat{i}$$

$$\bar{F}_{2,3} = k\frac{q_2 q_3}{r_2^2}\hat{r}_2 = k\frac{q(-Q)}{(x-a)^2}\hat{i} = -k\frac{qQ}{(x-a)^2}\hat{i}$$

$$\bar{F}_3 = \bar{F}_{3,1} + \bar{F}_{3,2} = -k\frac{qQ}{(x+a)^2}\hat{i} - k\frac{qQ}{(x-a)^2}\hat{i}$$

Example 2: III.) $a < x$

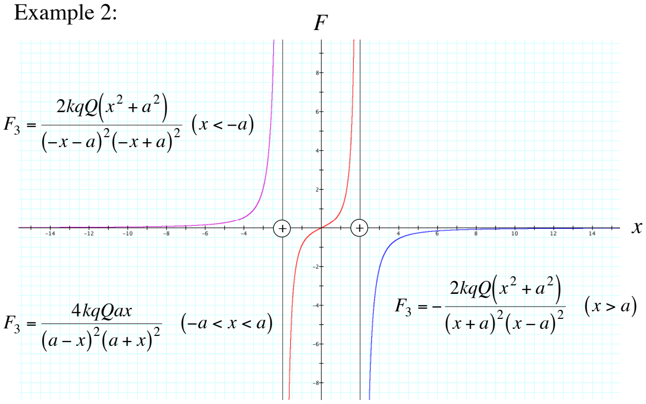
$$\vec{F}_3 = -k\frac{qQ}{(x+a)^2}\hat{i} - k\frac{qQ}{(x-a)^2}\hat{i}$$

$$\vec{F}_3 = -k\frac{qQ(x-a)^2}{(x+a)^2(x-a)^2}\hat{i} - k\frac{qQ(x+a)^2}{(x+a)^2(x-a)^2}\hat{i}$$

$$\vec{F}_3 = -k\frac{qQ\left(x^2-2ax+a^2\right)+qQ\left(x^2+2ax+a^2\right)}{(x+a)^2(x-a)^2}\hat{i}$$

$$\boxed{\vec{F}_3 = -\frac{2kqQ\left(x^2+a^2\right)}{(x+a)^2(x-a)^2}\hat{i} \quad (x > a)}$$

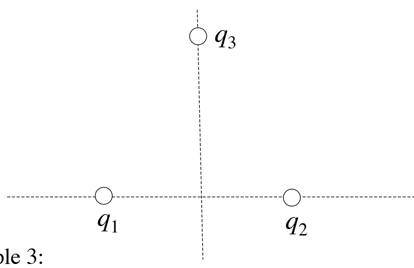
Example 2:



$$F_3 = \frac{2kqQ\left(x^2+a^2\right)}{(-x-a)^2(-x+a)^2} \quad (x < -a)$$

$$F_3 = \frac{4kqQax}{(a-x)^2(a+x)^2} \quad (-a < x < a)$$

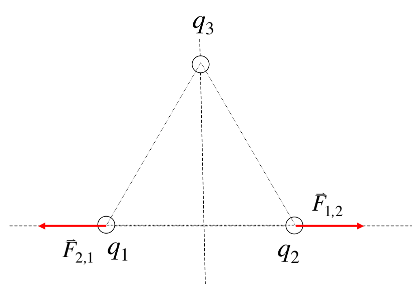
$$F_3 = -\frac{2kqQ\left(x^2+a^2\right)}{(x+a)^2(x-a)^2} \quad (x > a)$$



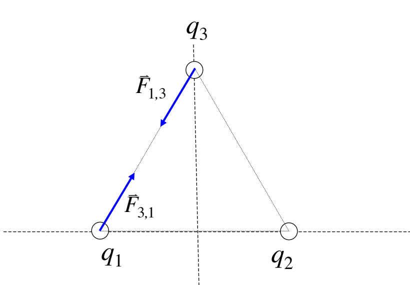
Example 3:

Three charges are fixed along the corners of an equilateral triangle with sides equal to 0.20 m as shown in the figure above with $q_1 = 6\ \mu$C, $q_2 = 6\ \mu$C, and $q_3 = -6\ \mu$C. Find the net force on each charge due to the other charges.

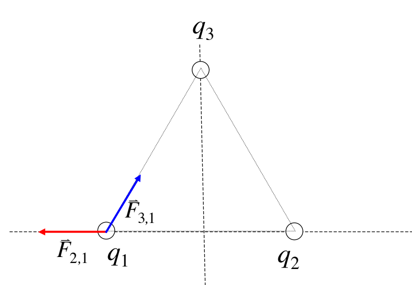
Example 3:



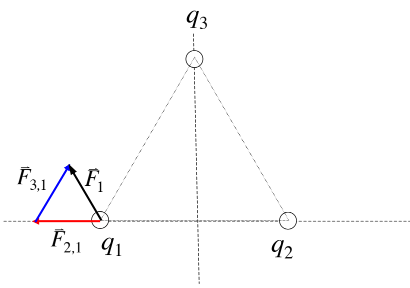
Example 3:



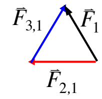
Example 3:



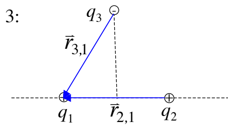
Example 3:



Example 3:



$$\vec F_{2,1} = k\frac{q_2 q_1}{r^2}\hat r$$



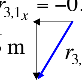
$$\vec F_{2,1} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6 \text{ x } 10^{-6}\text{ C}\right)\left(6 \text{ x } 10^{-6}\text{ C}\right)}{(0.20\text{ m})^2}\left(-\hat i\right)$$

$$\vec F_{2,1} = (-8.1\text{ N})\hat i$$

$$\vec F_{3,1} = k\frac{q_3 q_1}{r^2}\hat r$$

$r_{3,1_x} = -0.1$ m

$r_{3,1_y} = -\sqrt{0.03}$ m

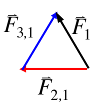


$r_{3,1} = 0.2$ m

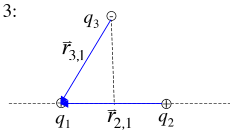
$$\vec F_{3,1} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-6 \text{ x } 10^{-6}\text{ C}\right)\left(6 \text{ x } 10^{-6}\text{ C}\right)}{(0.20\text{ m})^2}\left(\frac{(-0.1)\hat i + \left(-\sqrt{0.03}\right)\hat j}{0.2}\right)$$

$$\vec F_{3,1} = (4.05\text{ N})\hat i + (7.01\text{ N})\hat j$$

Example 3:



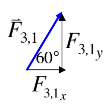
$$\vec F_{2,1} = k\frac{q_2 q_1}{r^2}\hat r$$



$$\vec F_{2,1} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6 \text{ x } 10^{-6}\text{ C}\right)\left(6 \text{ x } 10^{-6}\text{ C}\right)}{(0.20\text{ m})^2}\left(-\hat i\right)$$

$$\vec F_{2,1} = (-8.1\text{ N})\hat i$$

$F_{3,1} = 8.1$ N



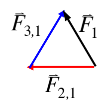
$$F_{3,1_x} = F_{3,1}\cos\theta_{3,1} = (8.1\text{ N})\cos 60°$$

$$F_{3,1_x} = 4.05\text{ N}$$

$$F_{3,1_y} = F_{3,1}\sin\theta_{3,1} = (8.1\text{ N})\sin 60°$$

$$F_{3,1_y} = 7.01\text{ N}$$

Example 3:



$$\vec F_{2,1} = (-8.1\text{ N})\hat i$$

$$\vec F_{3,1} = (4.05\text{ N})\hat i + (7.01\text{ N})\hat j$$

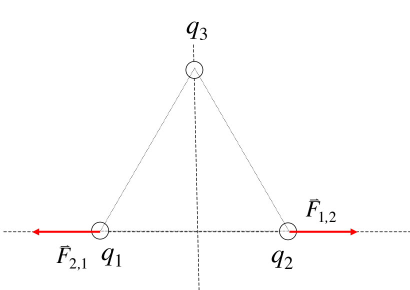
$$\vec F_1 = \vec F_{3,1} + \vec F_{2,1} = (-4.05\text{ N})\hat i + (7.01\text{ N})\hat j$$

$$F_1 = \sqrt{F_{1_x}^2 + F_{1_y}^2} = \sqrt{(-4.05\text{ N})^2 + (7.01\text{ N})^2} = 8.1\text{ N}$$

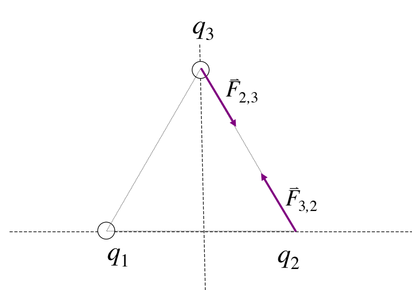
$$\theta_1 = \tan^{-1}\left(\frac{F_{1_y}}{F_{1_x}}\right) = \tan^{-1}\left(\frac{7.01}{-4.05}\right) = -60° + 180° = 120°$$

$$\boxed{\vec F_1 = 8.1\text{ N}\angle 120°}$$

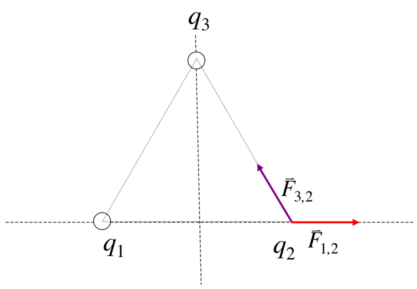
Example 3:



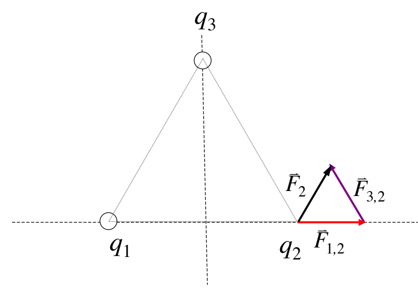
Example 3:



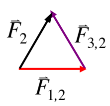
Example 3:

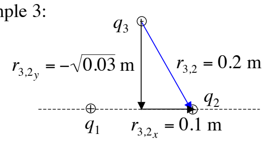


Example 3:



Example 3:





$$\bar{F}_{1,2} = -\bar{F}_{2,1} = (8.1\text{ N})\hat{i}$$

$$\bar{F}_{3,2} = k\frac{q_3 q_1}{r^2}\hat{r}$$

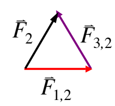
$$\bar{F}_{3,2} = \left(9.0 \text{ x } 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(-6\text{ x }10^{-6}\text{ C})(6\text{ x }10^{-6}\text{ C})}{(0.20\text{ m})^2}\left(\frac{(0.1)\hat{i} + (-\sqrt{0.03})\hat{j}}{0.2}\right)$$

$$\bar{F}_{3,2} = (-4.05\text{ N})\hat{i} + (7.01\text{ N})\hat{j}$$

$$\bar{F}_2 = \bar{F}_{1,2} + \bar{F}_{3,2} = (4.05\text{ N})\hat{i} + (7.01\text{ N})\hat{j}$$

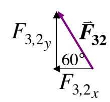
$$\boxed{\bar{F}_2 = 8.1\text{ N}\angle 60°}$$

Example 3:



$$\bar{F}_{1,2} = -\bar{F}_{2,1} = (8.1\text{ N})\hat{i}$$

$$F_{3,2} = 8.1\text{ N}$$



$$F_{3,2_x} = F_{3,2}\cos\theta_{3,2} = (8.1\text{ N})\cos 120°$$

$$F_{3,2_x} = -4.05\text{ N}$$

$$F_{3,2_y} = F_{3,2}\sin\theta_{3,2} = (8.1\text{ N})\sin 120°$$

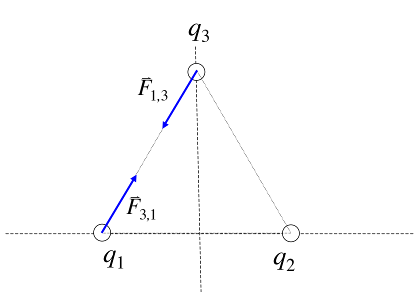
$$F_{3,2_y} = 7.01\text{ N}$$

$$\bar{F}_{3,2} = (-4.05\text{ N})\hat{i} + (7.01\text{ N})\hat{j}$$

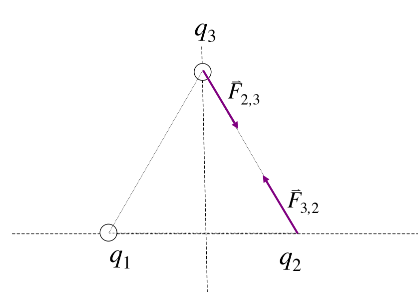
$$\bar{F}_2 = \bar{F}_{1,2} + \bar{F}_{3,2} = (4.05\text{ N})\hat{i} + (7.01\text{ N})\hat{j}$$

$$\boxed{\bar{F}_2 = 8.10\text{ N}\angle 60°}$$

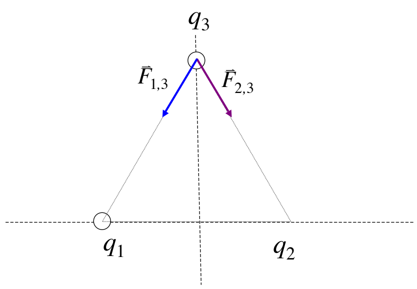
Example 3:



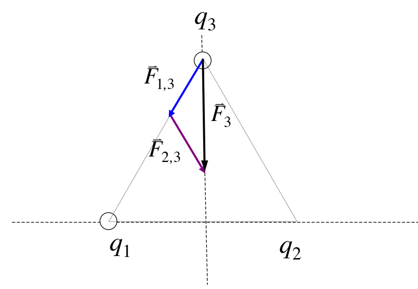
Example 3:



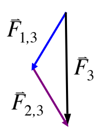
Example 3:



Example 3:



Example 3:



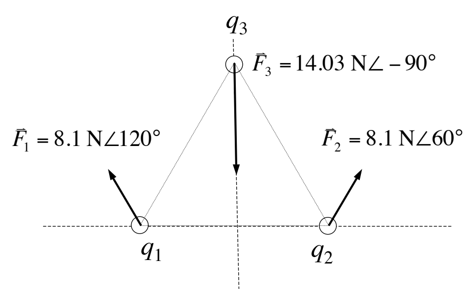
$$\vec{F}_{1,3} = -\vec{F}_{3,1} = (-4.05\text{ N})\hat{i} + (-7.01\text{ N})\hat{j}$$

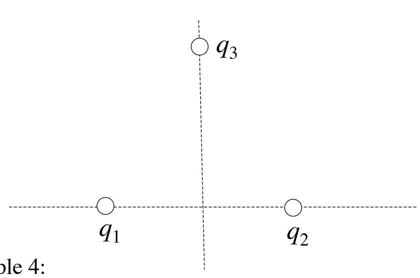
$$\vec{F}_{2,3} = -\vec{F}_{3,2} = (4.05\text{ N})\hat{i} + (-7.01\text{ N})\hat{j}$$

$$\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = (\text{-}14.03\text{ N})\hat{j}$$

$$\boxed{\vec{F}_3 = 14.03\text{ N}\angle -90^\circ}$$

Example 3:

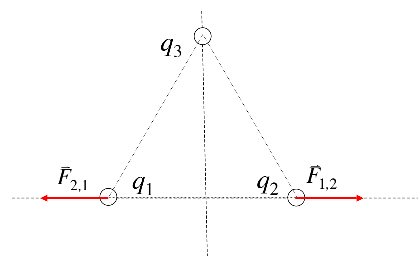




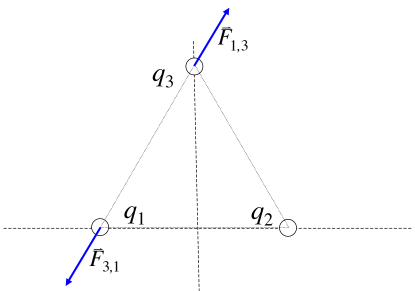
Example 4:

Three charges are fixed along the corners of an equilateral triangle with sides equal to 0.20 m as shown in the figure above with $q_1 = 6\,\mu\text{C}$, $q_2 = 6\,\mu\text{C}$, and $q_3 = 6\,\mu\text{C}$. Find the net force on each charge due to the other charges.

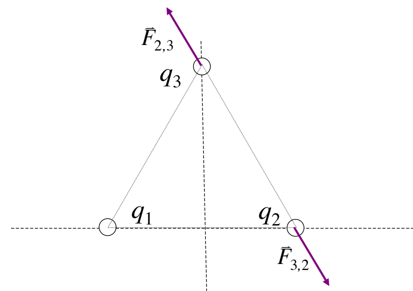
Example 4:



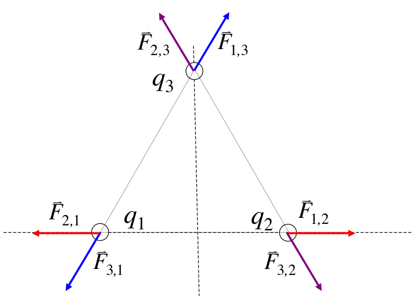
Example 4:



Example 4:

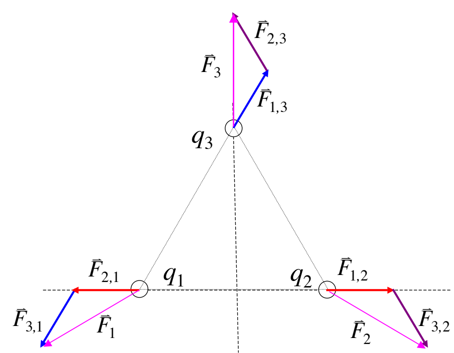


Example 4:

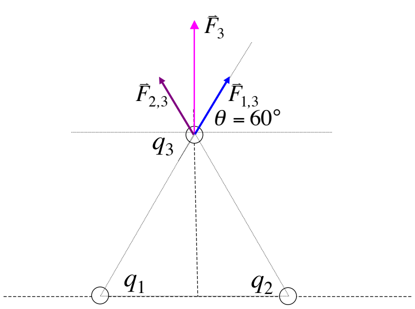


$$F_{1,2} = F_{1,3} = F_{2,3} = \left(9 \text{ x } 10^{9}\,\frac{\text{N}\cdot\text{ m}^2}{\text{C}^2}\right)\frac{\left(6 \text{ x } 10^{-6}\text{C}\right)\left(6 \text{ x } 10^{-6}\text{C}\right)}{\left(0.20 \text{ m}\right)^2} = 8.10 \text{ N}$$

Example 4:



Example 4:

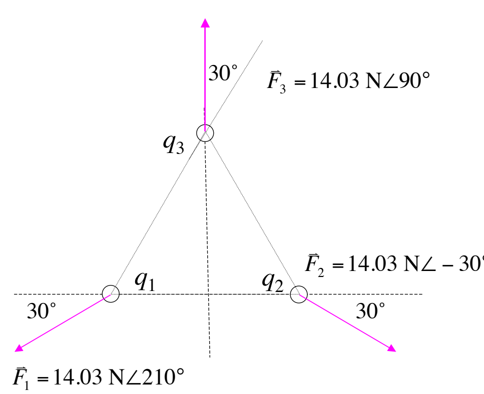


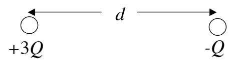
$$F_3 = 2F_{1,3_y} = 2F_{1,3}\sin\theta$$

$$F_3 = 2\left(8.10 \text{ N}\right)\sin 60° = 14.03 \text{ N}$$

$$\vec{F}_3 = 14.03 \text{ N}\angle 90°$$

Example 4:

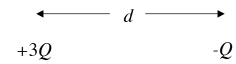




Example 5:

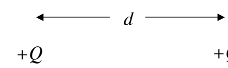
Two identical conducting spheres are charged to $+3Q$ and $-Q$, respectively, and are separated by a distance d (much greater than the radii of the spheres) as shown above. The magnitude of the force of attraction on the left sphere is F_1. After the spheres are made to touch and then are reseparated by distance d, the magnitude of the force on the left sphere is F_2. What is the relationship between F_1 and F_2?

Example 5:

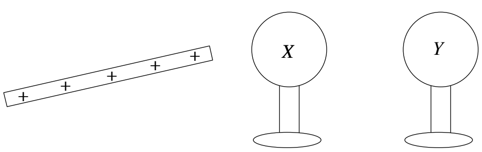


$$F_1 = k\frac{(3Q)(Q)}{d^2} = k\frac{3Q^2}{d^2}$$

$$Q_{Total} = 3Q + (-Q) = 2Q$$



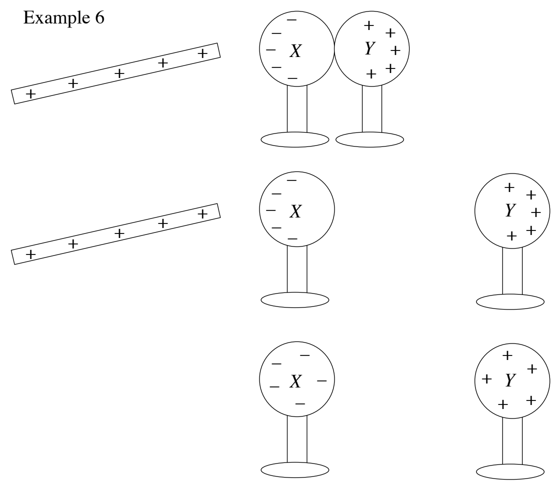
$$F_2 = k\frac{(Q)(Q)}{d^2} = k\frac{Q^2}{d^2} = \frac{1}{3}F_1$$

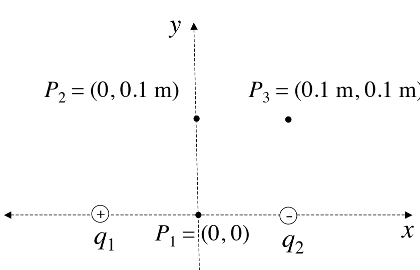


Example 6:

Two metal spheres that are initially uncharged are mounted on insulating stands, as shown above. A positively charged rubber rod is brought close to, but does not make contact with sphere X. Sphere Y is then brought close to X on the side opposite to the rubber rod. Y is allowed to touch X and then is removed some distance away. The rubber rod is then moved far away from X and Y. What are the final charges on the spheres?

Example 6

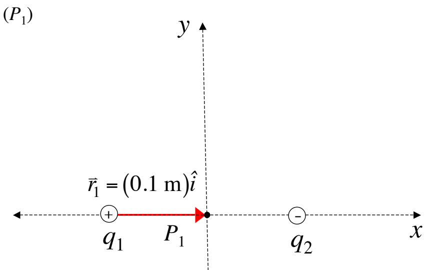




Example 7:

Two charges are fixed along the x-axis at $x = -0.1$ m and $x = 0.1$ m as shown in the figure above with $q_1 = 6\ \mu$C and $q_2 = -6\ \mu$C. Find the electric field due to both charges at P_1, P_2, and P_3.

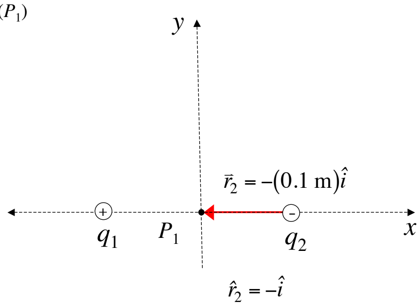
Example 7: (P_1)



$\hat{r}_1 = \hat{i}$

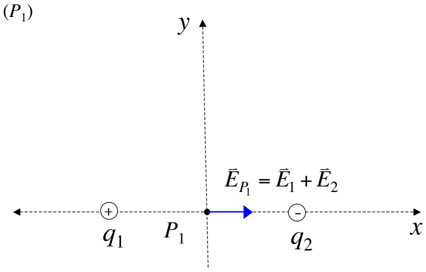
$$\vec{E}_1 = k\frac{q_1}{r_1^2}\hat{r}_1 = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6 \text{ x } 10^{-6}\text{ C}\right)}{(0.10\text{ m})^2}\hat{i} = \left(5.4 \text{ x } 10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i}$$

Example 7: (P_1)



$$\vec{E}_2 = k\frac{q_2}{r_2^{\,2}}\hat{r}_2 = \left(9.0\text{ x }10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-6\text{ x }10^{-6}\text{ C}\right)}{(0.10\text{ m})^2}\left(-\hat{i}\right) = \left(5.4\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i}$$

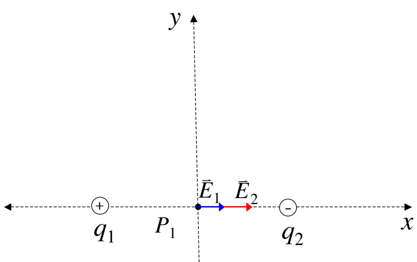
Example 7: (P_1)



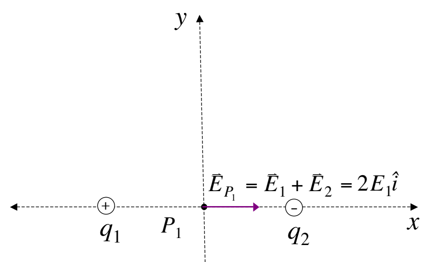
$$\vec{E}_{P_1} = \vec{E}_1 + \vec{E}_2 = \left(5.4\text{ x }10^6\ \frac{\text{N}}{\text{C}} + 5.4\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i}$$

$$\boxed{\vec{E}_{P_1} = \left(10.8\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i}}$$

Example 7: (P_1)



Example 7: (P_1)



Example 7: (P_1)

$$\vec{E}_{P_1} = \vec{E}_1 + \vec{E}_2 = 2E_1\hat{i}$$

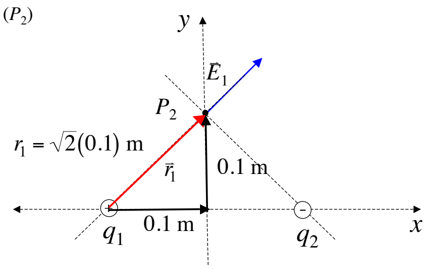
$$E_1 = k\frac{q_1}{r_1^{\,2}}$$

$$E_1 = \left(9.0\text{ x }10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6\text{ x }10^{-6}\text{ C}\right)}{(0.10\text{ m})^2} = 5.4\text{ x }10^6\ \frac{\text{N}}{\text{C}}$$

$$\vec{E}_{P_1} = 2E_1\hat{i} = 2\left(5.4\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i}$$

$$\boxed{\vec{E}_{P_1} = \left(10.8\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i}}$$

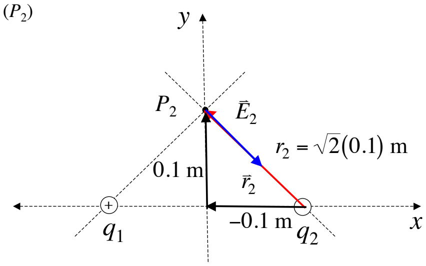
Example 7: (P_2)



$$\vec{E}_1 = k\frac{q_1}{r_1^{\,2}}\hat{r}_1 \qquad \vec{r}_1 = (0.1\text{ m})\hat{i} + (0.1\text{ m})\hat{j}$$

$$\hat{r}_1 = \frac{\vec{r}_1}{r_1} = \frac{(0.1\text{ m})\hat{i} + (0.1\text{ m})\hat{j}}{\sqrt{(0.1\text{ m})^2 + (0.1\text{ m})^2}} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

Example 7: (P_2)



$$\vec{E}_2 = k\frac{q_2}{r_2^2}\hat{r}_2 \qquad \vec{r}_2 = (-0.1\text{ m})\hat{i} + (0.1\text{ m})\hat{j}$$

$$\hat{r}_2 = \frac{\vec{r}_2}{r_2} = \frac{(-0.1\text{ m})\hat{i} + (0.1\text{ m})\hat{j}}{\sqrt{(0.1\text{ m})^2 + (0.1\text{ m})^2}} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

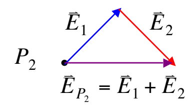
Example 7: (P_2)

$$\vec{E}_1 = k\frac{q_1}{r_1^2}\hat{r}_1 = \left(9.0\text{ x }10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6\text{ x }10^{-6}\text{ C})}{(\sqrt{2}(0.1)\text{ m})^2}\hat{r}_1 = \left(2.7\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{r}_1$$

$$\vec{E}_1 = 2.7\text{ x }10^6\ \frac{\text{N}}{\text{C}}\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) = \left(1.91\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(1.91\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

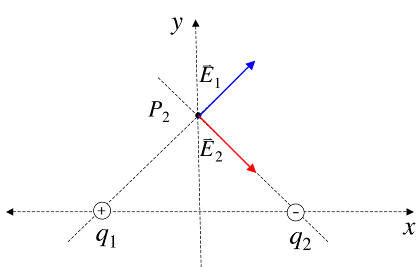
$$\vec{E}_2 = k\frac{q_2}{r_2^2}\hat{r}_2 = \left(9.0\text{ x }10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(-6\text{ x }10^{-6}\text{ C})}{(\sqrt{2}\,(0.1)\text{ m})^2}\hat{r}_2 = \left(-2.7\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{r}_2$$

$$\vec{E}_2 = -2.7\text{ x }10^6\ \frac{\text{N}}{\text{C}}\left(-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) = \left(1.91\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(-1.91\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

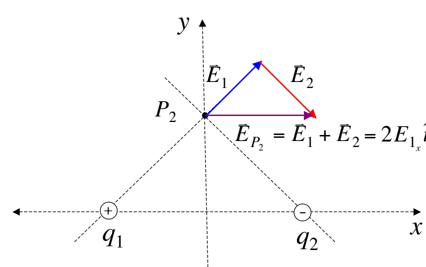


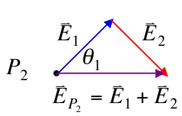
$$\boxed{\vec{E}_{P_2} = \left(3.82\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i} = 3.82\text{ x }10^6\ \frac{\text{N}}{\text{C}}\ \angle 0}$$

Example 7: (P_2)



Example 7: (P_2)



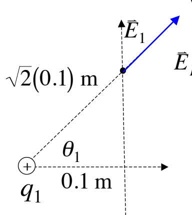


Example 7: (P_2)

$$\vec{E}_{P_2} = \vec{E}_1 + \vec{E}_2 = 2E_{1_x}\hat{i} = (2E_1\cos\theta_1)\hat{i}$$

$$E_1 = k\frac{q_1}{r_1^2}$$

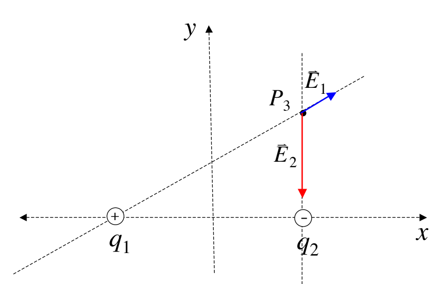
$$E_1 = \left(9.0\text{ x }10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6\text{ x }10^{-6}\text{ C})}{(\sqrt{2}(0.10\text{ m}))^2} = 2.7\text{ x }10^6\ \frac{\text{N}}{\text{C}}$$



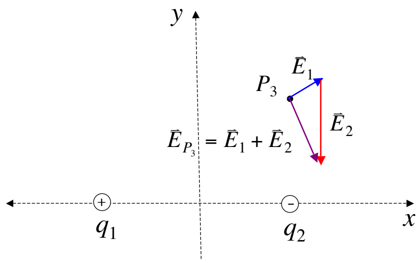
$$\vec{E}_{P_2} = (2E_1\cos\theta_1)\hat{i} = 2\left(2.7\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\left(\frac{0.1\text{ m}}{\sqrt{2}(\,0.1)\text{ m}}\right)\hat{i}$$

$$\boxed{\vec{E}_{P_2} = \left(3.82\text{ x }10^6\ \frac{\text{N}}{\text{C}}\right)\hat{i}}$$

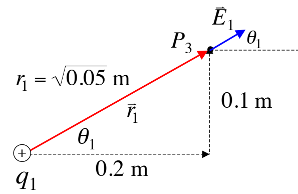
Example 7: (P_3)



Example 7: (P_3)



Example 7: (P_3)

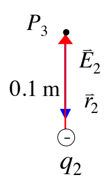


$$\vec{E}_1 = k\frac{q_1}{r_1^2}\hat{r}_1 \qquad \vec{r}_1 = (0.2\text{ m})\hat{i} + (0.1\text{ m})\hat{j}$$

$$\hat{r}_1 = \frac{\vec{r}_1}{r_1} = \frac{(0.2\text{ m})\hat{i} + (0.1\text{ m})\hat{j}}{\sqrt{(0.2\text{ m})^2 + (0.1\text{ m})^2}} = \left(\frac{0.2\text{ m}}{\sqrt{0.05}\text{ m}}\right)\hat{i} + \left(\frac{0.1\text{ m}}{\sqrt{0.05}\text{ m}}\right)\hat{j}$$

$$\left(\hat{r}_1 = (\cos\theta_1)\hat{i} + (\sin\theta_1)\hat{j}\right)$$

Example 7: (P_3)



$$\vec{E}_2 = k\frac{q_2}{r_2^2}\hat{r}_2 \qquad \vec{r}_2 = (0.1\text{ m})\hat{j} \qquad \hat{r}_2 = \hat{j}$$

Example 7: (P_3)

$$\vec{E}_1 = k\frac{q_1}{r^2}\hat{r}_1 = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(6 \text{ x } 10^{-6}\text{ C})}{(\sqrt{0.05}\text{ m})^2}\hat{r}_1 = \left(1.08 \text{ x } 10^6\ \frac{\text{N}}{\text{C}}\right)\hat{r}_1$$

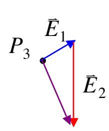
$$\vec{E}_1 = \left(1.08 \text{ x } 10^6\ \frac{\text{N}}{\text{C}}\right)\left(\left(\frac{0.2\text{ m}}{\sqrt{0.05}\text{ m}}\right)\hat{i} + \left(\frac{0.1\text{ m}}{\sqrt{0.05}\text{ m}}\right)\hat{j}\right)$$

$$\vec{E}_1 = \left(9.66 \text{ x } 10^5\ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(4.83 \text{ x } 10^5\ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

$$\vec{E}_2 = k\frac{q_2}{r_2^2}\hat{r}_2 = \left(9.0 \text{ x } 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(-6 \text{ x } 10^{-6}\text{ C})}{(0.10\text{ m})^2}\hat{j}$$

$$\vec{E}_2 = -\left(5.4 \text{ x } 10^6\ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

Example 7: (P_3)



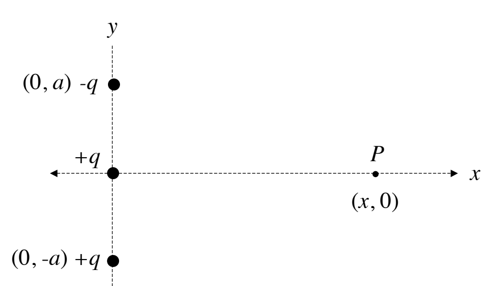
$$\vec{E}_{P_3} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_1 = \left(9.66 \text{ x } 10^5\ \frac{\text{N}}{\text{C}}\right)\hat{i} + \left(4.83 \text{ x } 10^5\ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

$$\vec{E}_2 = -\left(5.4 \text{ x } 10^6\ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

$$\vec{E}_{P_3} = \vec{E}_1 + \vec{E}_2 = \left(9.66 \text{ x } 10^5\ \frac{\text{N}}{\text{C}}\right)\hat{i} - \left(4.92 \text{ x } 10^6\ \frac{\text{N}}{\text{C}}\right)\hat{j}$$

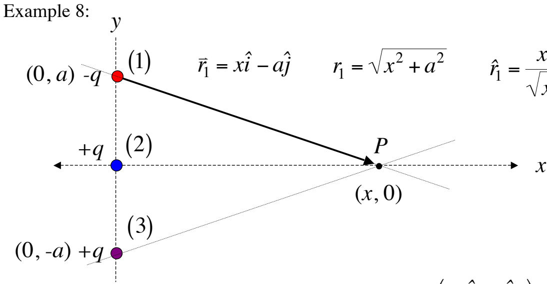
$$\boxed{\vec{E}_{P_3} = 5.01 \text{ x } 10^6\ \frac{\text{N}}{\text{C}}\angle\text{-}79°}$$



Example 8:

Three charges are fixed along the y-axis as shown in the figure above. Find the electric field at point P on the x-axis at a distance of x from the origin.

Example 8:

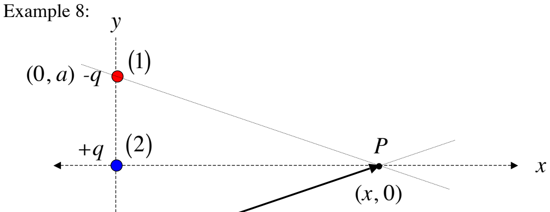


$$\vec{r}_1 = x\hat{i} - a\hat{j} \qquad r_1 = \sqrt{x^2 + a^2} \qquad \hat{r}_1 = \frac{x\hat{i} - a\hat{j}}{\sqrt{x^2 + a^2}}$$

$$\vec{E}_1 = k\frac{q}{r_1^2}\hat{r}_1 = k\frac{-q}{\left(\sqrt{x^2 + a^2}\right)^2}\left(\frac{x\hat{i} - a\hat{j}}{\sqrt{x^2 + a^2}}\right)$$

$$\vec{E}_1 = k\frac{q}{\left(x^2 + a^2\right)^{3/2}}\left(-x\hat{i} + a\hat{j}\right)$$

Example 8:

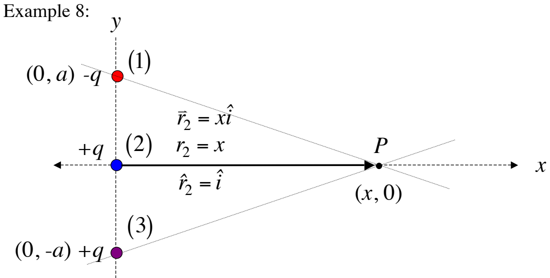


$$\vec{r}_3 = x\hat{i} + a\hat{j} \qquad r_3 = \sqrt{x^2 + a^2} \qquad \hat{r}_3 = \frac{x\hat{i} + a\hat{j}}{\sqrt{x^2 + a^2}}$$

$$\vec{E}_3 = k\frac{q}{r_3^2}\hat{r}_3 = k\frac{q}{\left(\sqrt{x^2 + a^2}\right)^2}\left(\frac{x\hat{i} + a\hat{j}}{\sqrt{x^2 + a^2}}\right)$$

$$\vec{E}_3 = k\frac{q}{\left(x^2 + a^2\right)^{3/2}}\left(x\hat{i} + a\hat{j}\right)$$

Example 8:

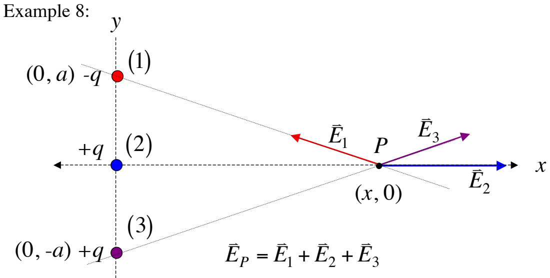


$$\vec{r}_2 = x\hat{i} \qquad r_2 = x \qquad \hat{r}_2 = \hat{i}$$

$$\vec{E}_2 = k\frac{q}{r_2^2}\hat{r}_2 = k\frac{q}{x^2}\hat{i}$$

$$\vec{E}_2 = k\frac{q}{x^2}\hat{i}$$

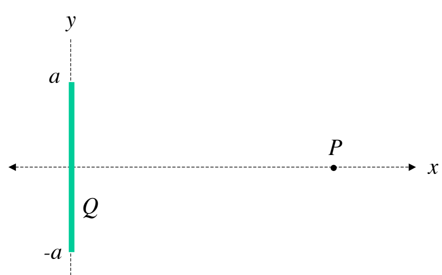
Example 8:



$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$\vec{E}_P = k\frac{q}{\left(x^2 + a^2\right)^{3/2}}\left(-x\hat{i} + a\hat{j}\right) + k\frac{q}{x^2}\hat{i} + k\frac{q}{\left(x^2 + a^2\right)^{3/2}}\left(x\hat{i} + a\hat{j}\right)$$

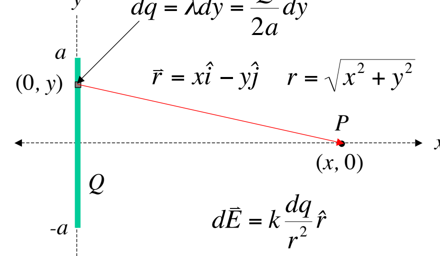
$$\boxed{\vec{E}_P = \frac{kq}{x^2}\hat{i} + \frac{2akq}{\left(x^2 + a^2\right)^{3/2}}\hat{j}}$$



Example 9:

Positive electric charge Q is distributed uniformly along a line with length $2a$, lying along the y-axis between $y = -a$ and $y = a$. Find the electric field at point P on the x-axis at a distance of x from the origin.

Example 9:



$$dq = \lambda dy = \frac{Q}{2a}dy$$

$$\vec{r} = x\hat{i} - y\hat{j} \qquad r = \sqrt{x^2 + y^2} \qquad \hat{r} = \frac{x\hat{i} - y\hat{j}}{\sqrt{x^2 + y^2}}$$

$$d\vec{E} = k\frac{dq}{r^2}\hat{r}$$

$$d\vec{E} = k\frac{\lambda dy}{x^2 + y^2}\frac{x\hat{i} - y\hat{j}}{\sqrt{x^2 + y^2}}$$

$$d\vec{E} = k\frac{\lambda dy}{\left(x^2 + y^2\right)^{3/2}}\left(x\hat{i} - y\hat{j}\right)$$

Example 9 :

$$d\vec{E} = k\frac{\lambda dy}{\left(x^2+y^2\right)^{3/2}}\left(x\hat{i} - y\hat{j}\right)$$

$$dE_x = k\frac{\lambda x dy}{\left(x^2+y^2\right)^{3/2}}$$

$$dE_y = k\frac{-\lambda y dy}{\left(x^2+y^2\right)^{3/2}}$$

Example 9 :

$$\int dE_x = \int_{-a}^{a} k\frac{\lambda x dy}{\left(x^2+y^2\right)^{3/2}}$$

$$E_x = \int_{-a}^{a} k\frac{\lambda x dy}{\left(x^2+y^2\right)^{3/2}} = k\lambda x\int_{-a}^{a}\frac{dy}{\left(x^2+y^2\right)^{3/2}}$$

From integral tables: $\int\frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{y}{x^2\left(x^2+y^2\right)^{1/2}} + C$

$$E_x = \left.\frac{kx\lambda y}{x^2\left(x^2+y^2\right)^{1/2}}\right]_{-a}^{a} = \frac{kx\lambda a}{x^2\left(x^2+a^2\right)^{1/2}} - \frac{kx\lambda(-a)}{x^2\left(x^2+a^2\right)^{1/2}}$$

Example 9 :

$$E_x = \left.\frac{kx\lambda y}{x^2\left(x^2+y^2\right)^{1/2}}\right]_{-a}^{a} = \frac{kx\lambda a}{x^2\left(x^2+a^2\right)^{1/2}} - \frac{kx\lambda(-a)}{x^2\left(x^2+a^2\right)^{1/2}}$$

$$E_x = \frac{kx\lambda a}{x^2\left(x^2+a^2\right)^{1/2}} + \frac{kx\lambda a}{x^2\left(x^2+a^2\right)^{1/2}}$$

$$E_x = \frac{2kx\lambda a}{x^2\left(x^2+a^2\right)^{1/2}} = \frac{\lambda a}{2\pi\varepsilon_o x\left(x^2+a^2\right)^{1/2}} = \frac{Q}{4\pi\varepsilon_o x\left(x^2+a^2\right)^{1/2}}$$

$$\left(k = \frac{1}{4\pi\varepsilon_o}\right) \qquad \left(\lambda = \frac{Q}{2a}\right)$$

Example 9 :

$$\int dE_y = \int_{-a}^{a} k\frac{-\lambda y dy}{\left(x^2+y^2\right)^{3/2}}$$

$$E_y = -k\lambda\int_{-a}^{a}\frac{y dy}{\left(x^2+y^2\right)^{3/2}} \qquad \text{(Easy integral)}$$

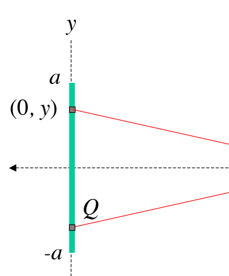
$$E_y = \frac{-k\lambda}{2}\int_{-a}^{a}\left(x^2+y^2\right)^{-3/2} 2y dy$$

$$E_y = \left.\frac{-k\lambda}{2}\frac{\left(x^2+y^2\right)^{-1/2}}{\left(-1/2\right)}\right]_{-a}^{a} = \frac{-k\lambda}{2}\frac{\left(x^2+a^2\right)^{-1/2}}{\left(-1/2\right)} - \frac{-k\lambda}{2}\frac{\left(x^2+a^2\right)^{-1/2}}{\left(-1/2\right)}$$

$$E_y = 0$$

Example 9 :

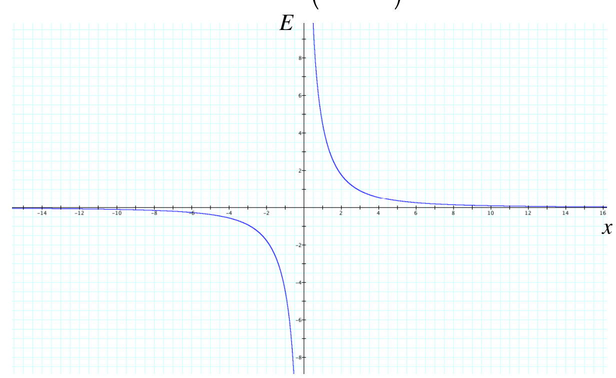
$$\vec{E} = \frac{Q}{4\pi\varepsilon_o x\left(x^2+a^2\right)^{1/2}}\hat{i}$$



Charge distribution is symmetrical with respect to the y-axis.

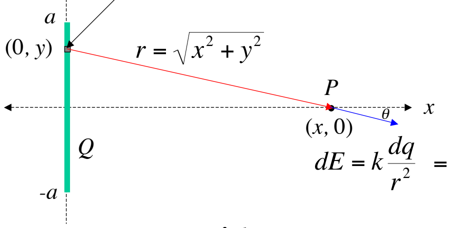
Example 9 :

$$\vec{E} = \frac{Q}{4\pi\varepsilon_o}\frac{1}{x\left(x^2+a^2\right)^{1/2}}\hat{i}$$



Example 9 :

$$dq = \lambda dy = \frac{Q}{2a}dy$$



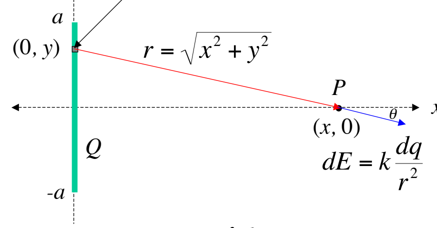
$$dE = k\frac{dq}{r^2} = k\frac{\lambda dy}{\left(x^2 + y^2\right)}$$

$$dE_x = dE\cos\theta = k\frac{\lambda dy}{\left(x^2 + y^2\right)}\frac{x}{\sqrt{x^2 + y^2}} = k\frac{\lambda x dy}{\left(x^2 + y^2\right)^{3/2}}$$

$$E_x = \int_{-a}^{a} k\frac{\lambda x dy}{\left(x^2 + y^2\right)^{3/2}}$$

Example 9 :

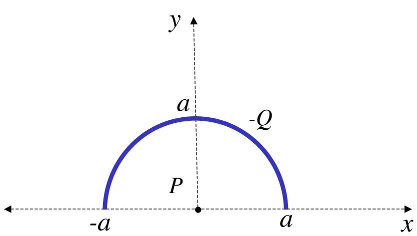
$$dq = \lambda dy = \frac{Q}{2a}dy$$



$$dE = k\frac{dq}{r^2} = k\frac{\lambda dy}{\left(x^2 + y^2\right)}$$

$$dE_y = dE\sin\theta = k\frac{\lambda dy}{\left(x^2 + y^2\right)}\frac{-y}{\sqrt{x^2 + y^2}} = k\frac{-\lambda y dy}{\left(x^2 + y^2\right)^{3/2}}$$

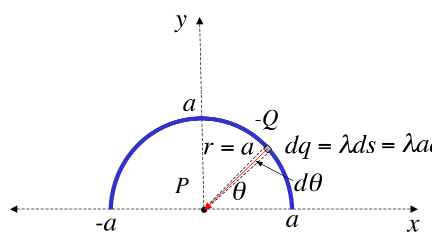
$$E_y = \int_{-a}^{a} k\frac{-\lambda y dy}{\left(x^2 + y^2\right)^{3/2}} = -k\lambda\int_{-a}^{a}\frac{y dy}{\left(x^2 + y^2\right)^{3/2}}$$



Example 10:

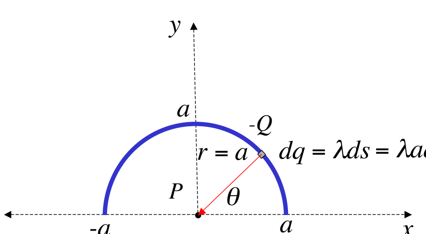
A negative charge -Q is uniformly distributed around a semicircle of radius a. Find the electric field at the center of curvature P.

Example 10 :



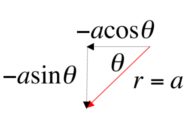
$$\left(\lambda = \frac{-Q}{\pi a}\right)$$

Example 10 :



$$\left(\lambda = \frac{-Q}{\pi a}\right)$$

$$d\vec{E} = k\frac{dq}{r^2}\hat{r} = k\frac{\lambda a d\theta}{a^2}\hat{r}$$



$$\hat{r} = \frac{-a\cos\theta\hat{i} - a\sin\theta\hat{j}}{a} = -\cos\theta\hat{i} - \sin\theta\hat{j}$$

$$d\vec{E} = k\frac{\lambda a d\theta}{a^2}\left(-\cos\theta\hat{i} - \sin\theta\hat{j}\right)$$

Example 10 :

$$d\vec{E} = k\frac{\lambda a d\theta}{a^2}\left(-\cos\theta\hat{i} - \sin\theta\hat{j}\right)$$

$$\vec{E} = \int_0^{\pi} k\frac{\lambda a d\theta}{a^2}\left(-\cos\theta\hat{i} - \sin\theta\hat{j}\right)$$

$$E_x = \int_0^{\pi} k\frac{\lambda a d\theta}{a^2}\left(-\cos\theta\right)$$

$$E_y = \int_0^{\pi} k\frac{\lambda a d\theta}{a^2}\left(-\sin\theta\right)$$

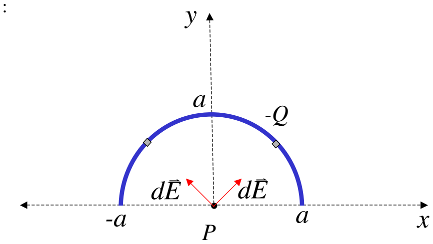
Example 10 :

$$E_x = \int_0^{\pi} k\frac{\lambda a d\theta}{a^2}(\text{-}\cos\theta) = -\frac{k\lambda}{a}\int_0^{\pi}\cos\theta d\theta \quad \text{(Easy integral)}$$

$$E_x = -\frac{k\lambda}{a}\sin\theta\Big]_0^{\pi} = -\frac{k\lambda}{a}(\sin\pi - \sin 0) = -\frac{k\lambda}{a}(0-0)$$

$$E_x = 0$$

Example 10 :



Once again we have symmetry.

Example 10 :

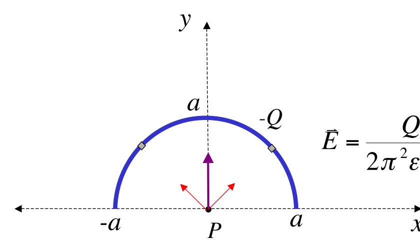
$$E_y = \int_0^{\pi} k\frac{\lambda a d\theta}{a^2}(\text{-}\sin\theta) = -\frac{k\lambda}{a}\int_0^{\pi}\sin\theta d\theta \quad \text{(Easy integral)}$$

$$E_y = \frac{k\lambda}{a}\cos\theta\Big]_0^{\pi} = \frac{k\lambda}{a}(\cos\pi - \cos 0) = \frac{k\lambda}{a}((-1)-1)$$

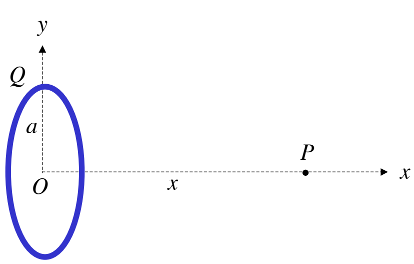
$$E_y = \frac{-2k\lambda}{a} = \frac{-\lambda}{2\pi\varepsilon_o a} \qquad \left(k = \frac{1}{4\pi\varepsilon_o}\right)$$

$$E_y = \frac{Q}{2\pi^2\varepsilon_o a^2} \qquad \left(\lambda = \frac{-Q}{\pi a}\right)$$

Example 10 :



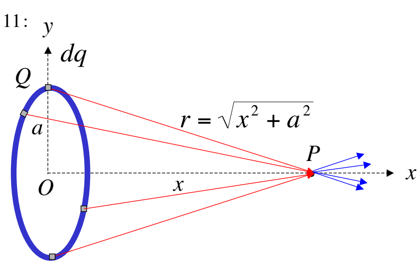
$$\vec{E} = \frac{Q}{2\pi^2\varepsilon_o a^2}\hat{j}$$



Example 11:

A ring-shaped conductor with radius a in the y-z plane carries a total charge Q uniformly distributed around it. Find the electric field at a point P that lies on the axis of the ring at a distance x from the origin.

Example 11 :

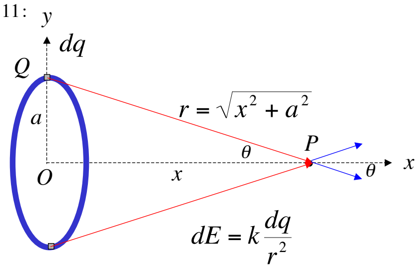


$E_y = 0$ because of symmetry.

$E_z = 0$ because of symmetry.

E_x is only component that is nonzero

Example 11:



$$dE_x = dE\cos\theta = k\frac{dq}{\left(x^2 + a^2\right)}\frac{x}{\sqrt{x^2 + a^2}} = k\frac{xdq}{\left(x^2 + a^2\right)^{3/2}}$$

Example 11:

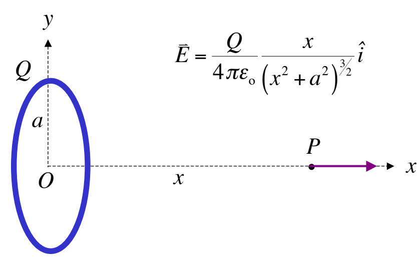
$$dE_x = k\frac{xdq}{\left(x^2 + a^2\right)^{3/2}}$$

$$\int dE_x = \int k\frac{xdq}{\left(x^2 + a^2\right)^{3/2}}$$

$$E_x = k\frac{x}{\left(x^2 + a^2\right)^{3/2}}\int dq = k\frac{x}{\left(x^2 + a^2\right)^{3/2}}Q$$

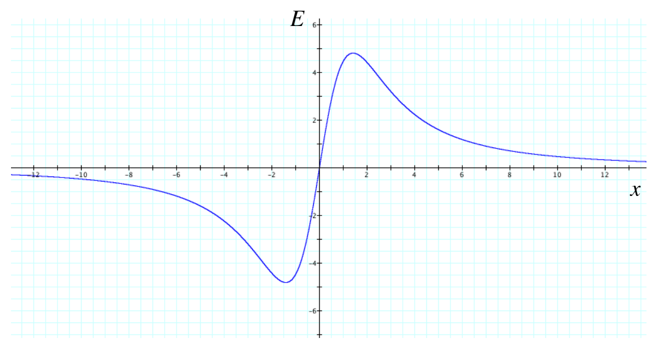
$$E_x = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}} = \frac{Qx}{4\pi\varepsilon_o\left(x^2 + a^2\right)^{3/2}}$$

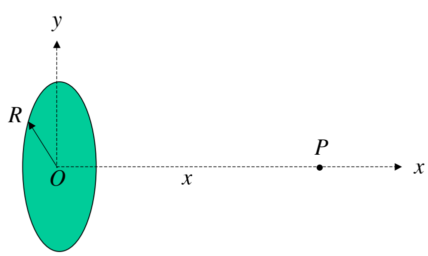
Example 11:



Example 11:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_o}\frac{x}{\left(x^2 + a^2\right)^{3/2}}\hat{i}$$

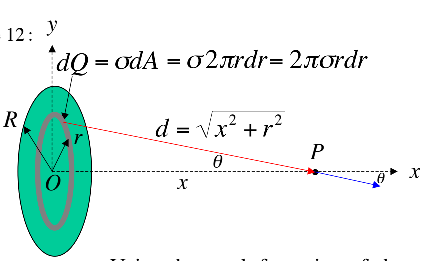




Example 12:

A disk of radius R in the y-z plane has a positive charge Q uniformly distributed on its surface. Find the electric field at a point along the axis of the disk a distance x from its center.

Example 12:



Using the result for a ring of charge

$$E_x = k\frac{Qx}{\left(x^2 + a^2\right)^{3/2}}$$

$$dE_x = k\frac{(2\pi\sigma r dr)x}{\left(x^2 + r^2\right)^{3/2}}$$

Example 12 :

$$dE_x = k\frac{(2\pi\sigma r dr)x}{\left(x^2 + r^2\right)^{3/2}}$$

$$E_x = \int_0^R k\frac{(2\pi\sigma r dr)x}{\left(x^2 + r^2\right)^{3/2}} = 2\pi\sigma kx\int_0^R \frac{rdr}{\left(x^2 + r^2\right)^{3/2}}$$

$$E_x = \pi\sigma kx\int_0^R \left(x^2 + r^2\right)^{-3/2} 2rdr$$

$$E_x = \pi\sigma kx\left.\frac{\left(x^2 + r^2\right)^{-1/2}}{\left(-1/2\right)}\right]_0^R = -2\pi\sigma kx\left(\left(x^2 + R^2\right)^{-1/2} - \left(x^2 + 0^2\right)^{-1/2}\right)$$

Example 12 :

$$E_x = -2\pi\sigma kx\left(\left(x^2 + R^2\right)^{-1/2} - \left(x^2 + 0^2\right)^{-1/2}\right)$$

$$E_x = 2\pi\sigma kx\left(\frac{1}{x} - \frac{1}{\left(x^2 + R^2\right)^{1/2}}\right)$$

$$E_x = 2\pi\sigma k\left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right)$$

$$E_x = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right) = \frac{Q}{2\pi\varepsilon_o R^2}\left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right)$$

$$\left(k = \frac{1}{4\pi\varepsilon_o}\right) \qquad \left(\sigma = \frac{Q}{\pi R^2}\right)$$

Example 12 :

$$\vec{E} = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right)\hat{i}$$