Example 1:

A closed pipe resonator has a length of 0.60 m.

a.) Find the fundamental frequency.

b.) What is the wavelength of the seventh harmonic?

c.) How many antinodes are in the third harmonic?

d.) How many nodes are in the fifth harmonic?

e.) Repeat a-d for an open-pipe resonator.

Example 1:

closed pipe $L = 0.60\text{ m},\ v = 340\ \frac{\text{m}}{\text{s}}$

a.) $f_1 = \ ?$

$v = \lambda_1 f_1$ and $\lambda_n = \frac{4L}{n}$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{\left(\frac{4L}{1}\right)} = \frac{v}{4L} = \frac{\left(340\ \frac{\text{m}}{\text{s}}\right)}{4(0.60\text{ m})}$$

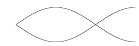
$\boxed{f_1 = 142\text{ Hz}}$

b.) $n = 7,\ \lambda_n = \ ?$

$$\lambda_7 = \frac{4L}{7} = \frac{4(0.60\text{ m})}{7}$$

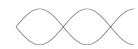
$\boxed{\lambda_7 = 0.343\text{ m}}$

c.) antinodes in third harmonic = ?



$\boxed{\text{2 antinodes}}$

d.) nodes in fifth harmonic = ?



$\boxed{\text{3 nodes}}$

Example 1e:

open pipe $L = 0.60\text{ m},\ v = 340\ \frac{\text{m}}{\text{s}}$

a.) $f_1 = \ ?$

$v = \lambda_1 f_1$ and $\lambda_n = \frac{2L}{n}$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{\left(\frac{2L}{1}\right)} = \frac{v}{2L} = \frac{\left(340\ \frac{\text{m}}{\text{s}}\right)}{2(0.60\text{ m})}$$

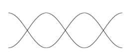
$\boxed{f_1 = 283\text{ Hz}}$

b.) $n = 7,\ \lambda_n = \ ?$

$$\lambda_7 = \frac{2L}{7} = \frac{2(0.60\text{ m})}{7}$$

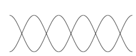
$\boxed{\lambda_7 = 0.171\text{ m}}$

c.) antinodes in third harmonic = ?



$\boxed{\text{4 antinodes}}$

d.) nodes in fifth harmonic = ?



$\boxed{\text{5 nodes}}$

Example 2:

The frequency of the fifth harmonic of a closed-pipe is 1500 Hz.

a.) Find the length of the pipe.

b.) Find the first 3 resonant frequencies.

c.) What is the longest wavelength that resonates?

d.) Repeat a-c for an open-pipe resonator.

Example 2:

closed pipe, $f_5 = 1500\text{ Hz},\ v = 340\ \frac{\text{m}}{\text{s}}$

a.) $L = \ ?$

$v = \lambda_5 f_5$ and $\lambda_n = \frac{4L}{n}$

$$\lambda_5 = \frac{v}{f_5} = \frac{4L}{5} \text{ so } L = \frac{5v}{4f_5} = \frac{5\left(340\ \frac{\text{m}}{\text{s}}\right)}{4(1500\text{ Hz})}$$

$\boxed{L = 0.283\text{ m}}$

b.) $f_1, f_3, f_5 = \ ?$

$f_n = nf_1$

$f_5 = 5f_1$ so $f_1 = \frac{f_5}{5} = \frac{1500\text{ Hz}}{5}$ $\quad f_3 = 3f_1 = 3(300\text{ Hz})$

$\boxed{f_1 = 300\text{ Hz}}$ $\boxed{f_3 = 900\text{ Hz}}$ $\boxed{f_5 = 1500\text{ Hz}}$

c.) $\lambda_1 = \ ?$

$$\lambda_1 = \frac{4L}{1} = \frac{4(0.283\text{ m})}{1}$$

$\boxed{\lambda_1 = 1.13\text{ m}}$

Example 2d:

open pipe, $f_5 = 1500\text{ Hz},\ v = 340\ \frac{\text{m}}{\text{s}}$

a.) $L = \ ?$

$v = \lambda_5 f_5$ and $\lambda_n = \frac{2L}{n}$

$$\lambda_5 = \frac{v}{f_5} = \frac{2L}{5} \text{ so } L = \frac{5v}{2f_5} = \frac{5\left(340\ \frac{\text{m}}{\text{s}}\right)}{2(1500\text{ Hz})}$$

$\boxed{L = 0.567\text{ m}}$

b.) $f_1, f_2, f_3 = \ ?$

$f_n = nf_1$

$f_5 = 5f_1$ so $f_1 = \frac{f_5}{5} = \frac{1500\text{ Hz}}{5}$ $\quad f_2 = 2f_1 = 2(300\text{ Hz})$ $\quad f_3 = 3f_1 = 3(300\text{ Hz})$

$\boxed{f_1 = 300\text{ Hz}}$ $\boxed{f_2 = 600\text{ Hz}}$ $\boxed{f_3 = 900\text{ Hz}}$

c.) $\lambda_1 = \ ?$

$$\lambda_1 = \frac{2L}{1} = \frac{2(0.567\text{ m})}{1}$$

$\boxed{\lambda_1 = 1.13\text{ m}}$

Example 3:

A tuning fork has a frequency of 512 Hz.

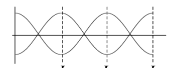
a.) What are the three shortest lengths that will resonate this frequency if both ends of the pipe are open?

b.) What are the three shortest lengths that will resonate this frequency if one of the ends of the pipe is closed?

Example 3: open pipe, $f = 512\text{ Hz},\ v = 340\ \dfrac{\text{m}}{\text{s}}$

a.) L_1, L_2, and L_3,= ?

$$\lambda = \frac{v}{f} = \frac{\left(340\ \frac{\text{m}}{\text{s}}\right)}{(512\text{ Hz})} = 0.664\text{ m}$$



$$\lambda_n = \frac{2L}{n}$$

for a fixed wavelength $\lambda = \dfrac{2L_n}{n}$

For a fixed wavelength, the lengths that will resonate a given wavelength are:

$$L_n = \frac{n\lambda}{2}$$

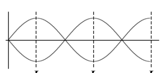
$$L_1 = \frac{\lambda}{2} = \frac{0.664\text{ m}}{2} \qquad L_2 = \frac{2\lambda}{2} = \frac{3(0.664\text{ m})}{4} \qquad L_3 = \frac{3\lambda}{2} = \frac{3(0.664\text{ m})}{2}$$

$\boxed{L_1 = 0.332\text{ m}}$ $\boxed{L_2 = 0.664\text{ m}}$ $\boxed{L_3 = 0.996\text{ m}}$

Example 3: closed pipe, $f = 512\text{ Hz},\ v = 340\ \dfrac{\text{m}}{\text{s}}$

b.) L_1, L_2, and L_3,= ?

$$\lambda = \frac{v}{f} = \frac{\left(340\ \frac{\text{m}}{\text{s}}\right)}{(512\text{ Hz})} = 0.664\text{ m}$$



$$\lambda_n = \frac{4L}{n}\ (n = 1,\ 3,\ 5,\ \ldots)$$

for a fixed wavelength $\lambda = \dfrac{4L_n}{n}$

For a fixed wavelength, the lengths that will resonate a given wavelength are:

$$L_n = \frac{n\lambda}{4}$$

$$L_1 = \frac{\lambda}{4} = \frac{0.664\text{ m}}{4} \qquad L_2 = \frac{3\lambda}{4} = \frac{3(0.664\text{ m})}{4} \qquad L_3 = \frac{5\lambda}{4} = \frac{5(0.664\text{ m})}{4}$$

$\boxed{L_1 = 0.166\text{ m}}$ $\boxed{L_2 = 0.498\text{ m}}$ $\boxed{L_3 = 0.830\text{ m}}$

Example 4:

A beat frequency of 4 Hz is detected when two tones are played together. If the frequency of one of the tones is 520 Hz, what are the possible frequencies for the second tone?

$$f_b = |f_1 - f_2| = 4\text{ Hz}$$

$$f_1 = 520\text{ Hz}$$

$$\boxed{f_2 = 516\text{ Hz or } 524\text{ Hz}}$$

Example 5:

Rat is being pursued by the police while traveling on MoPac. Rat's speed is 30 m/s and the cop's speed is 50 m/s. The frequency of the police siren (at rest) is 500 Hz.

a.) What is the frequency of the siren heard by the policeman?

b.) What is the frequency of the siren heard by Rat?

c.) Suppose Larry is traveling in the opposite direction at a speed of 40 m/s towards the policeman. What frequency does Larry hear?

Example 5:

$f_s = 500\text{ Hz},\ v_d = -30\ \dfrac{\text{m}}{\text{s}},\ v_s = 50\ \dfrac{\text{m}}{\text{s}},\ v = 340\ \dfrac{\text{m}}{\text{s}}$

a.) $f_d =$? for the policeman

policeman hears wave at $f_d =$ 500 Hz

b.) $f_d =$? for Rat

$$f_d = \frac{(v + v_d)}{(v - v_s)} f_s = \frac{\left(340\ \frac{\text{m}}{\text{s}} - 30\ \frac{\text{m}}{\text{s}}\right)}{\left(340\ \frac{\text{m}}{\text{s}} - 50\ \frac{\text{m}}{\text{s}}\right)}(500\text{ Hz})$$

$$\boxed{f_d = 534\text{ Hz}}$$

c.) $f_d =$? for Larry $\left(v_d = 40\ \dfrac{\text{m}}{\text{s}}\right)$

$$f_d = \frac{(v + v_d)}{(v - v_s)} f_s = \frac{\left(340\ \frac{\text{m}}{\text{s}} + 40\ \frac{\text{m}}{\text{s}}\right)}{\left(340\ \frac{\text{m}}{\text{s}} - 50\ \frac{\text{m}}{\text{s}}\right)}(500\text{ Hz})$$

$$\boxed{f_d = 655\text{ Hz}}$$