Example 1:

A block attached to a spring with unknown spring constant oscillates with a period of 2.00 s. What is the period if

a.) The mass is doubled?

b.) The mass is halved?

c.) The amplitude is doubled?

d.) The spring constant is doubled?

Example 1:

$$T_s = 2\pi\sqrt{\frac{m}{k}} = 2.00\text{ s}$$

a.) $m = 2m,\ T = ?$ $\quad T = 2\pi\sqrt{\frac{2m}{k}} = \sqrt{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \sqrt{2}(2.00\text{ s})$

$\boxed{T = 2.83\text{ s}}$

b.) $m = \frac{m}{2},\ T = ?$ $\quad T = 2\pi\sqrt{\frac{\frac{m}{2}}{k}} = \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{1}{\sqrt{2}}(2.00\text{ s})$

$\boxed{T = 1.41\text{ s}}$

c.) $A = 2A,\ T = ?$ $\quad T = 2\pi\sqrt{\frac{m}{k}}$ $\quad \boxed{T = 2.00\text{ s}}$

d.) $k = 2k,\ T = ?$ $\quad T = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{1}{\sqrt{2}}(2.00\text{ s})$

$\boxed{T = 1.41\text{ s}}$

Example 2:

A 2.00 kg object is attached to a horizontal spring of force constant 5000 N/m. The spring is stretched 10.0 cm from equilibrium and released. Find

a.) the period

b.) the frequency

c.) the angular frequency

d.) the amplitude of motion

e.) the maximum speed

f.) the maximum acceleration

Example 2: $\quad m = 2.00\text{ kg},\ k = 5000\ \frac{\text{N}}{\text{m}},\ \text{and } A = 10.0\text{ cm}$

a.) $T = ?$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(2.00\text{ kg})}{\left(5000\ \frac{\text{N}}{\text{m}}\right)}}$$

$\boxed{T = 0.1257\text{ s}}$

b.) $f = ?$

$$T = \frac{1}{f} \quad \text{so } f = \frac{1}{T} = \frac{1}{(0.1257\text{ s})}$$

$\boxed{f = 7.96\text{ Hz}}$

c.) $\omega = ?$

$$\frac{2\pi}{\omega} = \frac{1}{f} \quad \text{so } \omega = 2\pi f$$

$$\omega = 2\pi(7.96\text{ Hz})$$

$\boxed{\omega = 50.0\ \frac{\text{rad}}{\text{s}}}$

d.) $A = ?$ $\quad \boxed{A = 10.0\text{ cm}}$

e.) $v_{max} = ?$

$$v = -\omega A\sin(\omega t) = -v_{max}\sin(\omega t)$$

$$v_{max} = \omega A = \left(50\ \frac{\text{rad}}{\text{s}}\right)^2(10.0\text{ cm})$$

$\boxed{v_{max} = 500\ \frac{\text{cm}}{\text{s}}}$

f.) $a_{max} = ?$

$$a = -\omega^2 A\cos(\omega t) = -a_{max}\sin(\omega t)$$

$$a_{max} = \omega^2 A = \left(50\ \frac{\text{rad}}{\text{s}}\right)^2(10.0\text{ cm})$$

$\boxed{a_{max} = 2500\ \frac{\text{cm}^2}{\text{s}}}$

Example 3:

The position of a 50.0 g oscillating mass is given by the following equation

$$x = (2.0\text{ cm})\cos\left(10\frac{\text{rad}}{\text{s}}\cdot t\right)$$

Find:

a.) the amplitude

b.) the period

c.) the spring constant

d.) the maximum speed

e.) the total energy

Example 3: $\quad x = (2.0\text{ cm})\cos\left(10\frac{\text{rad}}{\text{s}}\cdot t\right)$ and $m = 50\text{ g}$

a.) $A = ?$ $\quad x = A\cos(\omega t)$

$\boxed{A = 2.0\text{ cm}}$

b.) $T = ?$ $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(10\ \frac{\text{rad}}{\text{s}}\right)}$

$\boxed{T = 0.63\text{ s}}$

c.) $k = ?$ $\quad T = 2\pi\sqrt{\frac{m}{k}}$

$$T^2 = 4\pi^2\frac{m}{k}$$

$$k = 4\pi^2\frac{m}{T^2} = 4\pi^2\frac{(0.050\text{ kg})}{(0.63\text{ s})^2}$$

$\boxed{k = 5.0\ \frac{\text{N}}{\text{m}}}$

d.) $v_{max} = ?$

$$v = -\omega A\sin(\omega t) = -v_{max}\sin(\omega t)$$

$$v_{max} = \omega A = \left(10\ \frac{\text{rad}}{\text{s}}\right)(2.0\text{ cm})$$

$\boxed{v_{max} = 20\ \frac{\text{cm}}{\text{s}}}$

e.) $E = ?$

$$E = U_s(A) = \frac{1}{2}kA^2$$

$$E = \frac{1}{2}kA^2$$

$$E = \frac{1}{2}\left(5.0\ \frac{\text{N}}{\text{m}}\right)(0.020\text{ m})^2$$

$\boxed{E = 0.0010\text{ J}}$

Example 4:

A 0.40 kg ball is suspended from a spring with spring constant 12 N/m. If the ball is pulled down 0.20 m from the equilibrium position and released, what is the maximum speed of the ball?

Example 4:

$$m = 0.40\text{ kg},\ k = 12\ \frac{\text{N}}{\text{m}},\ A = 0.20\text{ m},\ v_{max} = ?$$

$$v = -\omega A\sin(\omega t) = -v_{max}\sin(\omega t)$$

$$v_{max} = \omega A$$

$$T = \frac{2\pi}{\omega} \text{ so } \omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{2\pi\sqrt{\frac{m}{k}}} = \sqrt{\frac{k}{m}}$$

$$v_{max} = \sqrt{\frac{k}{m}}A$$

$$v_{max} = \sqrt{\frac{12\ \frac{\text{N}}{\text{m}}}{0.40\text{ kg}}}(0.20\text{ m})$$

$$\boxed{v_{max} = 1.1\ \frac{\text{m}}{\text{s}}}$$

Example 5:

A 3.00 kg object oscillating on a spring of force constant 2000 N/m has a total energy of 0.900 J. (*a*) What is the amplitude of the motion? (*b*) What is the maximum speed?

Example 5:

$$m = 3\text{ kg},\ k = 2000\ \frac{\text{N}}{\text{m}},\text{ and } E = 0.900\text{ J}$$

a.) $A = ?$

$$E = K + U$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

when $x = A,\ v = 0$ and $E = 0 + \frac{1}{2}kA^2 = \frac{1}{2}kA^2$

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.900\text{ J})}{\left(2000\ \frac{\text{N}}{\text{m}}\right)}}$$

$$\boxed{A = 0.0300\text{ m}}$$

b.) $v_{max} = ?$

$v = -\omega A\sin(\omega t)$

$v = -v_{max}\sin(\omega t)$

$$v_{max} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{\left(2000\ \frac{\text{N}}{\text{m}}\right)}{3\text{ kg}}}(0.03\text{ m})$$

$$\boxed{v_{max} = 0.775\ \frac{\text{m}}{\text{s}}}$$

Example 6:

A 200 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At one instant, the mass is at x = 5.0 cm and and has v_x = -30 cm/s. Determine:

a.) the period

b.) the amplitude

Example 6:

$$m = 200\text{ g},\ f = 2.0\text{ Hz, at } x = 5.0\text{ cm},\ v_x = -30\ \frac{\text{cm}}{\text{s}}$$

a.) $T = ?$ $\quad T = \frac{1}{f} = \frac{1}{(2.0\text{ Hz})}$

$$\boxed{T = 0.50\text{ s}}$$

b.) $A = ?$

$$x = A\cos(\omega t) \text{ and } v_x = -\omega A\sin(\omega t)$$

$$\frac{v_x}{x} = \frac{-\omega A\sin(\omega t)}{A\cos(\omega t)} = -\omega\tan(\omega t)$$

$$\tan(\omega t) = \frac{-v_x}{x\omega} \text{ so } \omega t = \tan^{-1}\left(\frac{-v_x}{x\omega}\right) = \tan^{-1}\left(\frac{-v_x}{x2\pi f}\right)$$

$$\omega t = \tan^{-1}\left(\frac{30\ \frac{\text{cm}}{\text{s}}}{(5.0\text{ cm})2\pi(2.0\text{ Hz})}\right) = 0.4455\text{ rad}$$

$$A = \frac{x}{\cos(\omega t)} = \frac{5.0\text{ cm}}{\cos(0.4455\text{ rad})} = \boxed{5.5\text{ cm}}$$

Example 7:

A mass on a string of unknown length oscillates as a pendulum with a period of 4.00 s. What is the period if

a.) The mass is doubled?

b.) The string length is doubled?

c.) The string length is halved?

d.) The amplitude is halved?

Example 7:

$$T_p = 2\pi\sqrt{\frac{\ell}{g}} = 4.00\ \text{s}$$

a.) $m = 2m,\ T = \ ?$ $\boxed{T = 4.00\ \text{s}}$

b.) $\ell = 2\ell,\ T = \ ?$ $T = 2\pi\sqrt{\frac{2\ell}{g}} = \sqrt{2}\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \sqrt{2}(4.00\ \text{s})$

$$\boxed{T = 5.66\ \text{s}}$$

c.) $\ell = \frac{\ell}{2},\ T = \ ?$ $T = 2\pi\sqrt{\frac{\frac{\ell}{2}}{g}} = \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \frac{1}{\sqrt{2}}(4.00\ \text{s})$

$$\boxed{T = 2.83\ \text{s}}$$

d.) $A = \frac{A}{2},\ T = \ ?$ $\boxed{T = 4.00\ \text{s}}$

Example 8:

a.) Find the length of a simple pendulum if the period is 5.0 s at a point where $g = 9.8\ \text{m/s}^2$.

b.) What would be the period of the pendulum in (a) on the moon, where the acceleration due to gravity is one-sixth that on earth.

Example 8:

a.) $T = 5.0\ \text{s},\ g = 9.8\ \frac{\text{m}}{\text{s}^2},\ \ell = \ ?$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = 4\pi^2\frac{\ell}{g}$$

$$\ell = \frac{gT^2}{4\pi^2} = \frac{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(5.0\ \text{s})^2}{4\pi^2}$$

$$\boxed{\ell = 6.2\ \text{m}}$$

b.) $g = \frac{g}{6},\ T = \ ?$ $T = 2\pi\sqrt{\frac{\ell}{\frac{g}{6}}} = \sqrt{6}\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \sqrt{6}(5.0\ \text{s})$

$$\boxed{T = 12.2\ \text{s}}$$