Example 1:

a.) What is the force of gravity between a gazelle with a mass of 100 kg and a lion with a mass that is 250 kg if the lion is lying in wait 100 meters from the gazelle?

b.) What would happen to the force of gravity between the gazelle and the lion if the distance increased to 200 meters?

Example 1: $m_1 = 100$ kg and $m_2 = 250$ kg

a.) $r_1 = 100 \text{ m}, \ F_{g_1} = \ ?$

$$F_{g_1} = G\frac{m_1 m_2}{r_1^2} = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(100\text{ kg})(250\text{ kg})}{(100\text{ m})^2}$$

$$\boxed{F_g = 1.67\times10^{-10}\ \text{N}}$$

b.) $r_2 = 200 \text{ m}, \ F_{g_2} = \ ?$

$$F_{g_2} = G\frac{m_1 m_2}{r_2^2} = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(100\text{ kg})(250\text{ kg})}{(200\text{ m})^2}$$

$$\boxed{F_{g_2} = 4.17\times10^{-11}\ \text{N} = \frac{1}{4}F_{g_1}}$$

Example 2:

Two masses m_1 and m_2 are separated by a distance r. The force of gravitational attraction between the two masses is F_1. If m_1 is doubled, m_2 tripled, and r is halved, what is the new gravitational attraction F_2 in terms of the original force F_1 ?

Example 2:

$$F_1 = G\frac{m_1 m_2}{r^2}$$

$$F_2 = G\frac{2m_1\, 3m_2}{\left(\frac{r}{2}\right)^2}$$

$$F_2 = (2)(3)G\frac{m_1 m_2}{\left(\frac{1}{4}\right)r^2}$$

$$F_2 = (2)(3)(4)G\frac{m_1 m_2}{r^2}$$

$$F_2 = 24G\frac{m_1 m_2}{r^2}$$

$$\boxed{F_2 = 24F_1}$$

Example 3:

What is the gravitational force between the Earth and the Sun?

(M_S = 1.99 x 10^{30} kg, M_E = 5.98 x 10^{24} kg and r = 1.5 x 10^{11} m)

Example 3:

$$M_E = 5.98\times10^{24}\ \text{kg},\ M_S = 1.99\times10^{30}\ \text{kg},\ r = 1.5\times10^{11}\ \text{m},\ F_g = ?$$

$$F_g = G\frac{M_E M_S}{r^2}$$

$$F_g = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(1.99\times10^{30}\ \text{kg}\right)}{\left(1.5\times10^{11}\ \text{m}\right)^2}$$

$$\boxed{F_g = 3.53\times10^{22}\ \text{N}}$$

Example 4:

a.) What is the weight of 85 kg person on the surface of Pluto?

(M_P = 1.22 x 10^{22} kg and R_P = 1.15 x 10^6 m)

b.) Find the gravitational constant on the surface of Pluto.

Example 4: $M_P = 1.2\times10^{22}$ kg, $m = 85$ kg, and $R_P = 1.15\times10^{6}$ m

a.) $F_g = ?$

$$F_g = G\frac{M_P\, m}{{R_P}^2}$$

$$F_g = \left(6.67\times10^{-11}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\frac{\left(1.2\times10^{22}\ \mathrm{kg}\right)\left(85\ \mathrm{kg}\right)}{\left(1.15\times10^{6}\ \mathrm{m}\right)^2}$$

$$\boxed{F_g = 51.4\ \mathrm{N}}$$

b.) $g_P = ?$

$$F_g = G\frac{M_P\, m}{{R_P}^2} = mg_P$$

$$g_P = G\frac{M_P}{{R_P}^2}$$

$$g_P = \left(6.67\times10^{-11}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\frac{\left(1.2\times10^{22}\ \mathrm{kg}\right)}{\left(1.15\times10^{6}\ \mathrm{m}\right)^2}$$

$$\boxed{g_P = 0.61\ \frac{\mathrm{m}}{\mathrm{s^2}}}$$

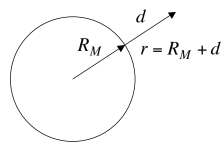
Example 5:

Find the speed of a satellite that would orbit Mars 200 km above its surface.

(M_M = 6.42 x 10^{23} kg and R_M = 3.38 x 10^6 m)

Example 5: $M_M = 6.42\times10^{23}$ kg, $R_M = 3.38\times10^{6}$ m, and $d = 200$ km

$v = ?$



$$F_g = G\frac{M_M\, m}{r^2} = ma_c = m\frac{v^2}{r}$$

$$v = \sqrt{G\frac{M_M}{r}}$$

$$v = \sqrt{G\frac{M_M}{R_M + d}}$$

$$v = \sqrt{\left(6.67\times10^{-11}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\frac{\left(6.42\times10^{23}\ \mathrm{kg}\right)}{\left(3.38\times10^{6}\ \mathrm{m} + 200\times10^{3}\ \mathrm{m}\right)}}$$

$$\boxed{v = 3459\ \frac{\mathrm{m}}{\mathrm{s}}}$$

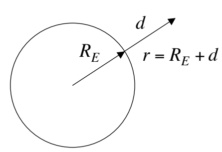
Example 6:

How many minutes would it take a satellite to orbit Earth 150 km above its surface?

(M_E = 5.98 x 10^{24} kg and R_E = 6.37 x 10^6 m)

Example 6: $M_E = 5.98\times10^{24}$ kg , $R_E = 6.37\times10^{6}$ m, and $d = 150$ km

$T = ?$



$$F_g = G\frac{M_E\, m}{r^2} = ma_c = m\frac{v^2}{r}$$

$$v = \sqrt{G\frac{M_E}{r}} = \frac{2\pi r}{T}$$

$$T = 2\pi r\sqrt{\frac{r}{GM_E}} = 2\pi\sqrt{\frac{r^3}{GM_E}}$$

$$T = 2\pi\sqrt{\frac{(R_E + d)^3}{GM_E}}$$

$$T = 2\pi\sqrt{\frac{\left(6.37\times10^{6}\ \mathrm{m} + 150\times10^{3}\ \mathrm{m}\right)^3}{\left(6.67\times10^{-11}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)}}$$

$$\boxed{T = 5238\ \mathrm{s} = 87.3\ \mathrm{min}}$$

Example 7:

Neptune requires 165 years to circle the Sun. Find Neptune's orbital radius using Kepler's third law. The orbital radius for the Earth around the Sun is 1.5 x 10^{11} m.

Example 7:

$$T_E = 1 \text{ yr},\ T_N = 165 \text{ yr},\ r_E = 1.5\times10^{11} \text{ m},\ r_N = ?$$

$$\left(\frac{T_N}{T_E}\right)^2 = \left(\frac{r_N}{r_E}\right)^3$$

$$\left(\frac{T_N}{T_E}\right)^{2/3} = \frac{r_N}{r_E}$$

$$r_N = r_E\left(\frac{T_N}{T_E}\right)^{2/3}$$

$$r_N = \left(1.5\times10^{11} \text{ m}\right)\left(\frac{165 \text{ yr}}{1 \text{ yr}}\right)^{2/3}$$

$$\boxed{r_N = 4.5\times10^{12} \text{ m}}$$

Example 8:

The mean distance from Saturn to the Sun is 9 times greater than the mean distance from Earth to the Sun. How many Earth years is a Saturn year?

Example 8:

$$r_S = 9r_E \text{ and } T_E = 1 \text{ yr},\ T_S = ?$$

$$\left(\frac{T_S}{T_E}\right)^2 = \left(\frac{r_S}{r_E}\right)^3$$

$$\frac{T_S}{T_E} = \left(\frac{r_S}{r_E}\right)^{3/2}$$

$$T_S = T_E\left(\frac{r_S}{r_E}\right)^{3/2}$$

$$T_S = T_E\left(\frac{9r_E}{r_E}\right)^{3/2} = (1 \text{ yr})(9)^{3/2}$$

$$\boxed{T_S = 27 \text{ yrs}}$$

Example 9:

A satellite orbits a planet in a radius R at a speed of 6000 m/s. What is the speed of the satellite if the radius is increased to $2R$?

Example 9:

$$r_1 = R,\ v_1 = 6000 \ \frac{\text{m}}{\text{s}},\ r_2 = 2R,\ v_2 = ?$$

$$v = \sqrt{G\frac{M}{r}}$$

$$v_1 = \sqrt{G\frac{M}{R}} = 6000 \ \frac{\text{m}}{\text{s}}$$

$$v_2 = \sqrt{G\frac{M}{2R}} = \frac{1}{\sqrt{2}}\sqrt{G\frac{M}{R}}$$

$$v_2 = \frac{1}{\sqrt{2}}\left(6000 \ \frac{\text{m}}{\text{s}}\right)$$

$$\boxed{v_2 = 4243 \ \frac{\text{m}}{\text{s}}}$$

(M_S = 1.99 x 10^{30} kg, M_E = 5.98 x 10^{24} kg and r = 1.5 x 10^{11} m)

Example 10:

a.) What is the gravitational potential energy between the Earth and the Sun?

b.) What is the total mechanical energy of the Earth as it orbits the Sun?

Example 10: $M_E = 5.98\times10^{24}$ kg, $M_S = 1.99\times10^{30}$ kg, $r = 1.5\times10^{11}$ m

a.) $U_g = ?$ $$U_g = -G\frac{M_S M_E}{r}$$

$$U_g = -\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(1.99\times10^{30}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(1.5\times10^{11}\ \text{m}\right)}$$

$$\boxed{U_g = -5.29\times10^{33}\ \text{J}}$$

b.) $E = ?$ $$E = -G\frac{M_S M_E}{2r}$$

$$E = -G\frac{Mm}{2r} = -\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(1.99\times10^{30}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{2\left(1.5\times10^{11}\ \text{m}\right)}$$

$$\boxed{E = -2.65\times10^{33}\ \text{J}}$$

Example 11:

How much work is done by the Moon's gravitational field as a 1000 kg meteor comes in from outer space and impacts on the Moon's surface?

(M_M = 7.36 x 10^{22} kg and R_M = 1.74 x 10^6 m)

Example 11:

$M_M = 7.36\times10^{22}$ kg, $m = 1000$ kg, $r_1 = \infty$, $r_2 = R_M = 1.74\times10^6$ m, $W = ?$

$$W = -\Delta U$$

$$W = -(U_2 - U_1)$$

$$W = -\left(-G\frac{M_M m}{r_2} - \left(-G\frac{M_M m}{r_1}\right)\right)$$

$$r_1 = \infty$$

$$W = G\frac{M_M m}{r_2}$$

$$W = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(7.36\times10^{22}\ \text{kg}\right)(1000\ \text{kg})}{\left(1.74\times10^6\ \text{m}\right)}$$

$$\boxed{W = 2.82\times10^9\ \text{J}}$$

Example 12:

A spaceship is fired from the Earth's surface with an initial speed of 2.00 x 10^4 m/s. What will be its speed when it is very far away from the Earth? (Neglect friction)

(M_E = 5.98 x 10^{24} kg and R_E = 6.37 x 10^6 m)

Example 12:

$M = M_E = 5.98\times10^{24}$ kg, $v_1 = 2.00\times10^4\ \frac{\text{m}}{\text{s}}$, $r_1 = R_E = 6.37\times10^6$ m, $r_2 = \infty$, $v_2 = ?$

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 - G\frac{Mm}{r_1} = \frac{1}{2}mv_2^2 - G\frac{Mm}{r_2}$$

$$r_2 = \infty$$

$$v_2 = \sqrt{v_1^2 - 2G\frac{M}{r_1}}$$

$$v_2 = \sqrt{\left(2.00\times10^4\ \frac{\text{m}}{\text{s}}\right)^2 - 2\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^6\ \text{m}\right)}}$$

$$\boxed{v_2 = 1.66\times10^4\ \frac{\text{m}}{\text{s}}}$$