Gravitation

Newton's Law of Gravitation

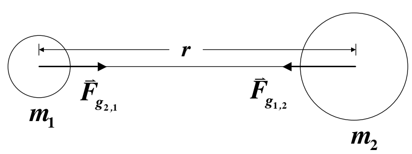
Newton proposed that any two masses were attracted by a gravitational force (inverse square law).

$$\left|F_g\right| = G\frac{m_1 m_2}{r^2}$$

Where:

- F = gravitational force (N)
- m = mass of body (kg)
- r = distance between the center of m_1 and m_2 (m)
- G = 6.67 x 10^{-11} $\left(\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{kg}^2}\right)$

Newton's Law of Gravitation

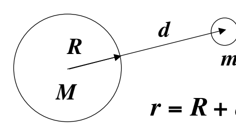


$$F_{g_{1,2}} = F_{g_{2,1}} = G\frac{m_1 m_2}{r^2}$$

Weight

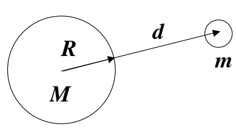
For a planet or moon of mass M, the weight of a body with mass m at a distance r from the center of the planet or moon is:

$$F_g = G\frac{Mm}{r^2}$$



$$r = R + d$$

Acceleration due to Gravity



$$r = R + d$$

$$F_g = G\frac{Mm}{r^2} = mg$$

At a point above the surface a distance r from the center of mass, the acceleration due to gravity g is:

$$g = G\frac{M}{r^2}$$

Motion of Satellites

A satellite in an orbit that is always the same height above a planet moves with uniform circular motion. Using Newton's second law:

$$F_g = ma_c$$

$$G\frac{Mm}{r^2} = \frac{mv^2}{r}$$

The *orbital speed* is therefore:

$$v = \sqrt{\frac{GM}{r}}$$

Motion of Satellites

For circular orbits, the period *T* (the time to complete one complete orbit) of the satellite is related to speed *v*.

$$v = \frac{2\pi r}{T}$$

Therefore the period for circular orbits is:

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}}$$

$$\boxed{T = 2\pi\sqrt{\frac{r^3}{GM}}}$$

Kepler's Laws of Planetary Motion

1.) **The paths of the planets are ellipses with the center of the Sun at one focus.**

2.) **An imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals. Thus, planets move fastest when closest the Sun, slowest when farthest away.**

3.) **The ratio of the squares of the periods of any two planets revolving about the Sun is equal to the cubes of their average distances from the Sun.**

$$\boxed{\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3}$$

Kepler's Third Law (Newton's version)

Recall for circular orbits:

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Squaring both sides:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

This equation is Kepler's third law.

Kepler's Third Law (Newton's version)

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

$$T_1^{\,2} = \left(\frac{4\pi^2}{GM}\right)r_1^{\,3} \quad \text{and} \quad T_2^{\,2} = \left(\frac{4\pi^2}{GM}\right)r_2^{\,3}$$

$$\frac{T_1^{\,2}}{T_2^{\,2}} = \frac{\left(\frac{4\pi^2}{GM}\right)r_1^{\,3}}{\left(\frac{4\pi^2}{GM}\right)r_2^{\,3}} \quad \text{so} \quad \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

Gravitational Potential Energy

The gravitational force is conservative, therefore a potential energy function can be defined. The *gravitational potential energy* between two masses m_1 and m_2 separated by a distance *r* is:

$$\boxed{U_G = -G\frac{m_1 m_2}{r}}$$

U_G is zero when the masses are an infinite distance apart. The potential energy decreases (gets more negative) as the masses get closer together.

Mechanical Energy of Satellites

The total mechanical energy *E* of a satellite in a circular orbit of radius *r* around mass *M* is:

$$E = K + U \quad = \frac{1}{2}mv^2 + \left(-G\frac{Mm}{r}\right)$$

$$E = \frac{1}{2}m\left(\frac{GM}{r}\right) + \left(-G\frac{Mm}{r}\right)$$

$$\boxed{E = -G\frac{Mm}{2r}}$$