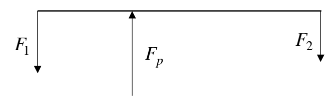
Example 1:

A 300 N child and a 200 N child sit on either end of a 3.0 m seesaw with negligible mass. The pivot is positioned so that the seesaw is balanced.

a.) What is the force exerted on the seesaw at the pivot point?

b.) Where along the seesaw should the pivot be placed?

Example 1a: $\ell = 3.0\text{ m},\ F_1 = 300\text{ N},\ F_2 = 200\text{ N},\ F_p = ?$



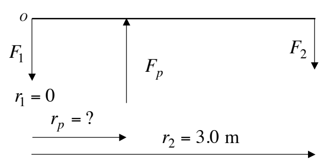
$$\sum F_y = 0 \quad \text{so} \quad F_p - F_1 - F_2 = 0$$

$$F_p = F_1 + F_2$$

$$F_p = 300\text{ N} + 200\text{ N}$$

$$\boxed{F_p = 500\text{ N}}$$

Example 1b: $\ell = 3.0\text{ m},\ F_1 = 300\text{ N},\ F_2 = 200\text{ N},\ F_p = 500\text{ N},\ r_p = ?$

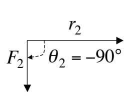


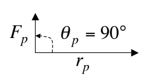
$$\sum \tau = 0 \quad \text{so} \quad \tau_1 + \tau_p + \tau_2 = 0$$

using point O as a reference $\tau_1 = 0\ (r_1 = 0)$

$$\tau_p + \tau_2 = 0$$

$$r_p F_p \sin\theta_p + r_2 F_2 \sin\theta_2 = 0$$

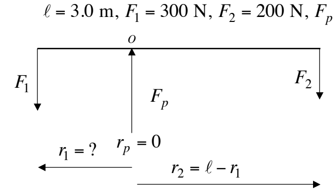




$$r_p = \frac{-r_2 F_2 \sin\theta_2}{F_p \sin\theta_p} = \frac{-(3\text{ m})(200\text{ N})\sin(-90°)}{(500\text{ N})\sin(90°)}$$

$$\boxed{r_p = 1.2\text{ m}}$$

Example 1b: $\ell = 3.0\text{ m},\ F_1 = 300\text{ N},\ F_2 = 200\text{ N},\ F_p = 500\text{ N}$



$$\sum \tau = 0 \quad \text{so} \quad \tau_1 + \tau_p + \tau_2 = 0$$

using point O as a reference $\tau_p = 0\ (r_p = 0)$

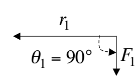
$$\tau_1 + \tau_2 = 0$$

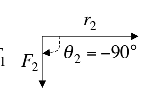
$$r_1 F_1 \sin\theta_1 + r_2 F_2 \sin\theta_2 = 0$$

$$r_1 F_1 \sin\theta_1 + (\ell - r_1) F_2 \sin\theta_2 = 0$$

$$r_1 F_1 \sin\theta_1 + \ell F_2 \sin\theta_2 - r_1 F_2 \sin\theta_2 = 0$$

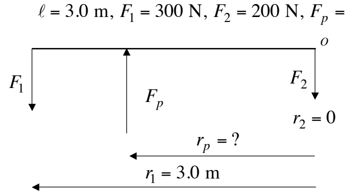
$$r_1 (F_1 \sin\theta_1 - F_2 \sin\theta_2) + \ell F_2 \sin\theta_2 = 0$$





$$r_1 = \frac{-\ell F_2 \sin\theta_2}{F_1 \sin\theta_1 - F_2 \sin\theta_2} = \frac{-(3\text{ m})(200\text{ N})\sin(-90°)}{(300\text{ N})\sin(90°) - (200\text{ N})\sin(-90°)} = \boxed{1.2\text{ m}}$$

Example 1b: $\ell = 3.0\text{ m},\ F_1 = 300\text{ N},\ F_2 = 200\text{ N},\ F_p = 500\text{ N}$

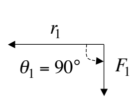


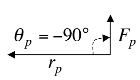
$$\sum \tau = 0 \quad \text{so} \quad \tau_1 + \tau_p + \tau_2 = 0$$

using point O as a reference $\tau_2 = 0\ (r_2 = 0)$

$$\tau_1 + \tau_p = 0$$

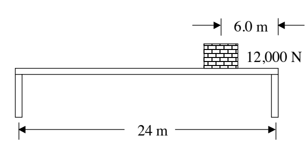
$$r_1 F_1 \sin\theta_1 + r_p F_p \sin\theta_p = 0$$





$$r_p = \frac{-r_1 F_1 \sin\theta_1}{F_p \sin\theta_p} = \frac{-(3\text{ m})(300\text{ N})\sin(90°)}{(500\text{ N})\sin(-90°)}$$

$$\boxed{r_p = 1.8\text{ m}}$$

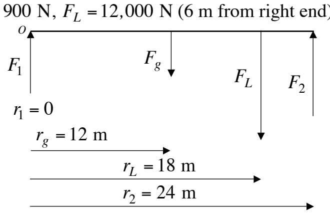


Example 2:

A uniform 900 N beam, 24 m long, supports a 12,000 N load 6.0 m from the right support column. Calculate the force on each vertical support column.

Example 2:

$\ell = 24\text{ m},\ F_g = 900\text{ N},\ F_L = 12{,}000\text{ N (6 m from right end)},\ F_1 = \ ?\text{ and } F_2 = \ ?$

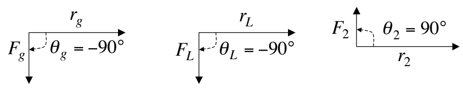


$$\sum \tau = 0 \quad \text{so} \quad \tau_1 + \tau_g + \tau_L + \tau_2 = 0$$

using point O as a reference $\tau_1 = 0\ \ (r_1 = 0)$

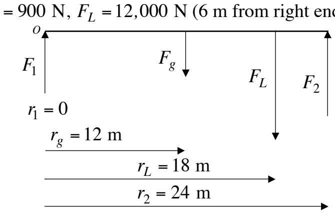
$$\tau_g + \tau_L + \tau_2 = 0$$

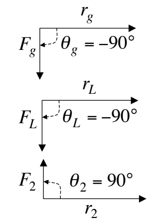
$$r_g F_g \sin\theta_g + r_L F_L \sin\theta_L + r_2 F_2 \sin\theta_2 = 0$$



Example 2:

$\ell = 24\text{ m},\ F_g = 900\text{ N},\ F_L = 12{,}000\text{ N (6 m from right end)},\ F_1 = \ ?\text{ and } F_2 = \ ?$





$$r_g F_g \sin\theta_g + r_L F_L \sin\theta_L + r_2 F_2 \sin\theta_2 = 0$$

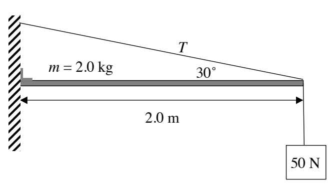
$$F_2 = \frac{-\left(r_g F_g \sin\theta_g + r_L F_L \sin\theta_L\right)}{r_2 \sin\theta_2} = \frac{-\left((12\text{ m})(900\text{ N})\sin(-90°) + (18\text{ m})(12{,}000\text{ N})\sin(-90°)\right)}{(24\text{ m})\sin(90°)}$$

$$\boxed{F_2 = 9450\text{ N}}$$

$$\sum F_y = 0 \quad \text{so} \quad F_1 - F_g - F_L + F_2 = 0$$

$$F_1 = F_g + F_L - F_2 = 900\text{ N} + 12{,}000\text{ N} - 9450\text{ N}$$

$$\boxed{F_1 = 3450\text{ N}}$$



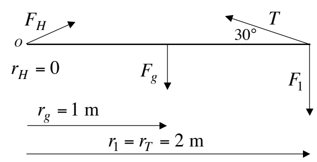
Example 3:

A 50 N sign is attached to the end of a 2 m horizontal beam that is hinged to a vertical support. The beam is uniform with a mass of 2.0 kg. A cable that makes an angle of 30° with respect to the beam is attached to the wall to help support the sign. Find the tension in the cable and the force exerted on the beam by the vertical support.

Example 3: $\ell = 2\text{ m},\ m = 2.0\text{ kg},\ F_1 = 50\text{ N},\ T = \ ?\text{ and } F_H = \ ?$

$$F_g = mg = (2.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)$$

$$F_g = 19.6\text{ N}$$

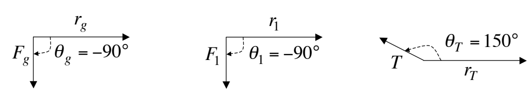


$$\sum \tau = 0 \quad \text{so} \quad \tau_H + \tau_g + \tau_1 + \tau_T = 0$$

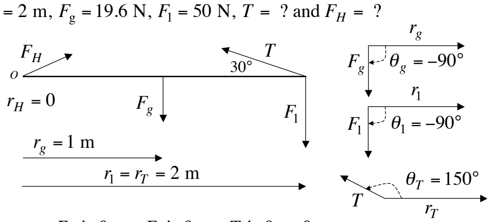
using point O as a reference $\tau_H = 0\ \ (r_H = 0)$

$$\tau_g + \tau_1 + \tau_T = 0$$

$$r_g F_g \sin\theta_g + r_1 F_1 \sin\theta_1 + r_T T \sin\theta_T = 0$$



Example 3: $\ell = 2\text{ m},\ F_g = 19.6\text{ N},\ F_1 = 50\text{ N},\ T = \ ?\text{ and } F_H = \ ?$



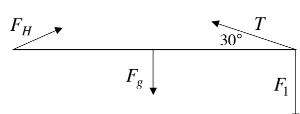
$$r_g F_g \sin\theta_g + r_1 F_1 \sin\theta_1 + r_T T \sin\theta_T = 0$$

$$T = \frac{-\left(r_g F_g \sin\theta_g + r_1 F_1 \sin\theta_1\right)}{r_T \sin\theta_T}$$

$$T = \frac{-\left((1\text{ m})(19.6\text{ N})\sin(-90°) + (2\text{ m})(50\text{ N})\sin(-90°)\right)}{(2\text{ m})\sin(150°)}$$

$$\boxed{T = 119.6\text{ N}}$$

Example 3: $\ell = 2\text{ m},\ F_g = 19.6\text{ N},\ F_1 = 50\text{ N},\ T = 119.6\text{ N},\ F_H = \ ?$



$$\sum F_x = 0 \quad \text{so} \quad F_{H_x} + T_x = 0$$

$$F_{H_x} = -T_x$$

$$F_{H_x} = -T\cos\theta_T$$

$$F_{H_x} = -(119.6\text{ N})\cos(150°)$$

$$F_{H_x} = 103.6\text{ N}$$

$$\sum F_y = 0 \quad \text{so} \quad F_{H_y} + T_y - F_g - F_1 = 0$$

$$F_{H_y} = F_g + F_1 - T_y$$

$$F_{H_y} = F_g + F_1 - T\sin\theta_T$$

$$F_{H_y} = 19.6\text{ N} + 50\text{ N} - (119.6\text{ N})\sin(150°)$$

$$F_{H_y} = 9.8\text{ N}$$

$$F_H = \sqrt{{F_{H_x}}^2 + {F_{H_y}}^2} = \sqrt{(103.6\text{ N})^2 + (9.8\text{ N})^2}$$

$$\boxed{F_H = 104.1\text{ N}}$$

$$\theta_H = \tan^{-1}\left(\frac{F_{H_y}}{F_{H_x}}\right) = \tan^{-1}\left(\frac{9.8\text{ N}}{103.6\text{ N}}\right) \quad \text{and} \quad \boxed{\theta_H = 5.4°}$$