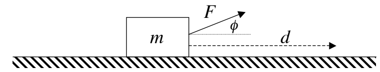
Example 1:

Rat pulls a 50 kg wagon with a force of 60 N. The handle of the wagon makes an angle of 36.87˚ with the horizontal direction of the wagon.

a.) How much work is done if she pulls the wagon 30 m and there is no friction?

b.) If the wagon is initially at rest, what is its speed after being pulled for 30 m?

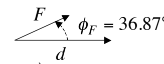
Example 1:



$m = 50$ kg, $F = 60$ N, $d = 30$ m, $\phi = 36.87°$, and $v_1 = 0$

a.) $W_F = ?$

$$W_F = Fd\cos\phi_F$$



$$W_F = (60\text{ N})(30\text{ m})\cos(36.87°)$$

$$\boxed{W_F = 1440\text{ J}}$$

b.) $v_2 = ?$

$$W_{net} = \Delta K = K_2 - K_1$$

$$W_F = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \qquad v_1 = 0$$

$$v_2 = \sqrt{\frac{2W_F}{m}} = \sqrt{\frac{2(1440\text{ J})}{50\text{ kg}}}$$

$$\boxed{v_2 = 7.6\ \frac{\text{m}}{\text{s}}}$$

Example 2:

A single force $F_x(x) = (-2x + 10)$ N is applied to an object with a mass of 2.0 kg, where x is in meters. As the particle moves along the x-axis from $x = 1.0$ m to $x = 2.0$ m, calculate

a.) the work done by the force.

b.) the change in kinetic energy of the particle

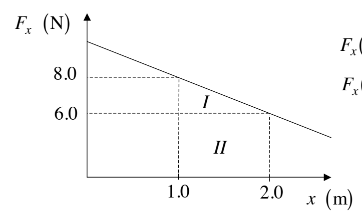
c.) the kinetic energy of the particle at $x = 2.0$ m, if its speed at $x = 1.0$ m is 2.0 m/s.

d.) the speed of the particle when $x = 2.0$ m.

Example 2:

$m = 2.0$ kg, $F_x(x) = (-2x + 10)$ N, $x_1 = 1.0$ m, $x_2 = 2.0$ m

a.) $W_F = ?$



$$F_x(1.0) = (-2(1.0) + 10) = 8.0\text{ N}$$

$$F_x(2.0) = (-2(2.0) + 10) = 6.0\text{ N}$$

$$W_F = Area_I + Area_{II}$$

$$W_F = \frac{1}{2}(1.0\text{ m})(2.0\text{ N}) + (1.0\text{ m})(6.0\text{ N})$$

$$\boxed{W_F = 7.0\text{ J}}$$

Example 2:

$m = 2.0$ kg, $F_x(x) = (-2x + 10)$ N, $x_1 = 1.0$ m, $x_2 = 2.0$ m, $W_F = 7.0$ J

b.) $\Delta K = ?$

$$W_{net} = \Delta K$$

$$\boxed{\Delta K = W_F = 7.0\text{ J}}$$

c.) $v_1 = 2.0\ \frac{\text{m}}{\text{s}}$ @ $x_1 = 1.0$ m, $K_2 = ?$

$$\Delta K = K_2 - K_1$$

$$K_2 = K_1 + \Delta K = \frac{1}{2}mv_1^2 + \Delta K$$

$$K_2 = \frac{1}{2}(2.0\text{ kg})\left(2.0\ \frac{\text{m}}{\text{s}}\right)^2 + 7.0\text{ J}$$

$$\boxed{K_2 = 11.0\text{ J}}$$

d.) $v_2 = ?$

$$K_2 = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(11\text{ J})}{2.0\text{ kg}}}$$

$$\boxed{v_2 = 3.3\ \frac{\text{m}}{\text{s}}}$$

Example 3:

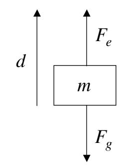
An elevator lifts a 1000 kg load upward a distance of 50 m at a constant velocity.

a.) What is the work done on the load by gravity?

b.) What is the work done on the load by the elevator?

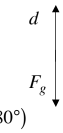
Example 3:

$$m = 1000 \text{ kg},\ d = 50 \text{ m, and } a = 0$$



a.) $W_g = ?$ $W_g = F_g d\cos\phi_g$

$$W_g = mgd\cos\phi_g$$



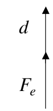
$$\phi_g = 180°$$

$$W_g = (1000 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(50 \text{ m})\cos(180°)$$

$$\boxed{W_g = -490{,}000 \text{ J}}$$

b.) $W_e = ?$ $W_e = F_e d\cos\phi_e$

$$W_e = mgd\cos\phi_e$$



$$\phi_e = 0$$

$a = 0$ so $F_e = F_g = mg$

$$W_e = (1000 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(50 \text{ m})\cos(0)$$

$$\boxed{W_e = 490{,}000 \text{ J}}$$

Example 4:

How much energy is needed to accelerate a 3000 kg truck from 10 m/s to 30 m/s?

Example 4:

$$m = 3000 \text{ kg},\ v_1 = 10 \ \frac{\text{m}}{\text{s}},\ \text{and } v_2 = 30 \ \frac{\text{m}}{\text{s}},\ W = ?$$

$$W_{net} = \Delta K = K_2 - K_1$$

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W = \frac{1}{2}(3000 \text{ kg})\left(30 \ \frac{\text{m}}{\text{s}}\right)^2 - \frac{1}{2}(3000 \text{ kg})\left(10 \ \frac{\text{m}}{\text{s}}\right)^2$$

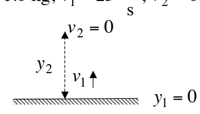
$$\boxed{W = 1.2 \times 10^6 \text{ J}}$$

Example 5:

A 1.0 kg ball is tossed straight up with an initial speed of 25 m/s. What is the maximum height of the ball?

Example 5:

$$m = 1.0 \text{ kg},\ v_1 = 25 \ \frac{\text{m}}{\text{s}},\ v_2 = 0,\ y_2 = ?$$



$$K_1 + \cancel{U_1} + \cancel{W_{other}} = \cancel{K_2} + U_2$$

$y_1 = 0$ $v_2 = 0$

$$K_1 = U_{g2}$$

$$\frac{1}{2}mv_1^2 = mgy_2$$

$$y_2 = \frac{v_1^2}{2g} = \frac{\left(25 \ \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)}$$

$$\boxed{y_2 = 31.9 \text{ m}}$$

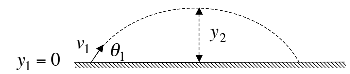
Example 6:

A cannonball is fired with a speed of 100 m/s at an angle of 53.13˚. What is the maximum height of the cannonball?

Example 6:

$v_1 = 100\ \frac{\text{m}}{\text{s}},\ \theta_1 = 53.13°,\ v_2 = v_x,\ y_2 = ?$

$v_2 = v_x$



$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$y_1 = 0$

$$K_1 = K_2 + U_{g_2}$$

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_x^2 + mgy_2$$

$$\frac{1}{2}mv_1^2 = \frac{1}{2}m(v_1\cos\theta_1)^2 + mgy_2$$

$$y_2 = \frac{v_1^2 - (v_1\cos\theta_1)^2}{2g} = \frac{\left(100\ \frac{\text{m}}{\text{s}}\right)^2 - \left(100\ \frac{\text{m}}{\text{s}}\cos(53.13°)\right)^2}{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)}$$

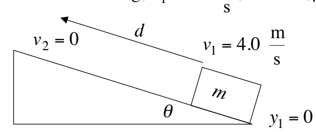
$$\boxed{y_2 = 326\ \text{m}}$$

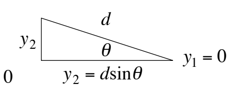
Example 7:

A 15 kg box is sliding up a frictionless 30˚ incline. If the box has an initial speed of 4.0 m/s, how far does it slide along the direction of the incline?

Example 7:

$m = 15\ \text{kg},\ v_1 = 4.0\ \frac{\text{m}}{\text{s}},\ \theta = 30°, \mu = 0,\ d = ?$





$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$y_1 = 0 \qquad v_2 = 0$

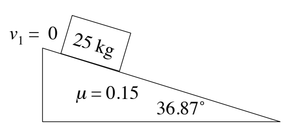
$$K_1 = U_{g_2}$$

$$\frac{1}{2}mv_1^2 = mgy_2$$

$$\frac{1}{2}mv_1^2 = mgd\sin\theta$$

$$d = \frac{v_1^2}{2g\sin\theta} = \frac{\left(4.0\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\sin(30°)}$$

$$\boxed{d = 1.63\ \text{m}}$$



Example 8:

A 25 kg box is released from rest on a rough inclined surface as shown in the figure above. After sliding 2.0 m down the incline find

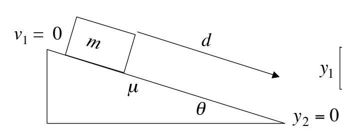
a.) the kinetic energy of the box.

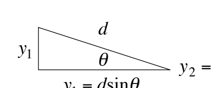
b.) the velocity of the box.

c.) the work done by gravity on the box.

Example 8:

$m = 25\ \text{kg},\ d = 2.0\ \text{m},\ \theta = 36.87°,\ \mu = 0.15,\ \text{and}\ v_1 = 0$

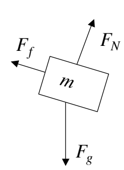


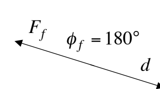


a.) $K_2 = ?$

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$v_1 = 0 \qquad y_2 = 0$





$$U_{g_1} + W_f = K_2$$

$$K_2 = U_{g_1} + W_f$$

$$K_2 = mgy_1 + F_f d\cos\phi_f$$

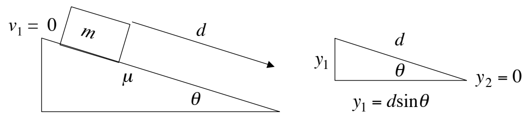
$$K_2 = mgy_1 + \mu F_N d\cos\phi_f = mgy_1 + \mu F_\perp d\cos\phi_f$$

$$K_2 = mgd\sin\theta + \mu mg\cos\theta d\cos\phi_f$$

$$K_2 = (25\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.0\ \text{m})\sin 36.87° + 0.15(25\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\cos(36.87°)(2.0\ \text{m})\cos(180°)$$

$$\boxed{K_2 = 235.2\ \text{J}}$$

Example 8: $m = 25\text{ kg},\ d = 2.0\text{ m},\ \theta = 36.87°,\ \mu = 0.15,\text{ and } v_1 = 0$



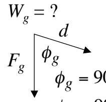
b.) $v_2 = ?$

$$K_2 = \frac{1}{2} m v_2^2$$

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(235.2\text{ J})}{25\text{ kg}}}$$

$$\boxed{v_2 = 4.34\ \frac{\text{m}}{\text{s}}}$$

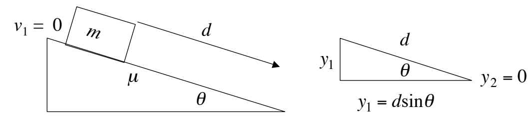
c.) $W_g = ?$



$$\phi_g = 90° - \theta$$
$$\phi_g = 90° - 36.87°$$
$$\phi_g = 53.13°$$

$$W_g = F_g d\cos\phi_g$$
$$W_g = mgd\cos\phi_g$$
$$W_g = (25\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.0\text{ m})\cos(53.13°)$$
$$\boxed{W_g = 294\text{ J}}$$

Example 8: $m = 25\text{ kg},\ d = 2.0\text{ m},\ \theta = 36.87°,\ \mu = 0.15,\text{ and } v_1 = 0$



c.) $W_g = ?$

$$W_g = -\Delta U_g$$
$$W_g = -\left(U_{g_2} - U_{g_1}\right)$$
$$W_g = -(mgy_2 - mgy_1)$$
$$W_g = -(0 - mgd\sin\theta)$$
$$W_g = mgd\sin\theta$$
$$W_g = (25\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.0\text{ m})\sin(36.87°)$$
$$\boxed{W_g = 294\text{ J}}$$

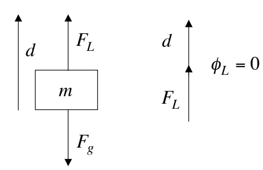
Example 9:

Larry climbs a 10.0 m tree at a constant speed in 8.0 s. If Larry has a mass of 7.0 kg, what is his power output?

Example 9:

$m = 7.0\text{ kg},\ d = 10.0\text{ m},\ a = 0,\ \Delta t = 8.0\text{ s},\ P_L = ?$

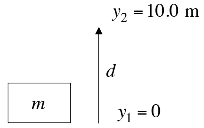
$a = 0$ so $F_L = F_g = mg$



$$P_L = \frac{W_L}{\Delta t}$$
$$W_L = F_L d\cos\phi_L = mgd\cos\phi_L$$
$$P_L = \frac{mgd\cos\phi_L}{\Delta t} = \frac{(7.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(10.0\text{ m})\cos(0)}{8.0\text{ s}}$$
$$\boxed{P_L = 86\text{ W}}$$

Example 9: (Alternative solution)

$m = 7.0\text{ kg},\ d = 10.0\text{ m},\ a = 0,\ \Delta t = 8.0\text{ s},\ P_L = ?$

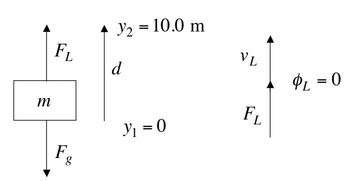


$$P_L = \frac{W_L}{\Delta t}$$
$$W_L = -W_g$$
$$W_g = -\Delta U_g = -\left(U_{g_2} - U_{g_1}\right)$$
$$W_g = -(mgy_2 - mgy_1) = -\left((7.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(10.0\text{ m}) - 0\right) = -686\text{ J}$$
$$W_L = -W_g = 686\text{ J}$$
$$P_L = \frac{W_L}{\Delta t} = \frac{686\text{ J}}{8.0\text{ s}} \qquad \boxed{P_L = 86\text{ W}}$$

Example 9: (Alternative solution)

$m = 7.0\text{ kg},\ d = 10.0\text{ m},\ a = 0,\ \Delta t = 8.0\text{ s},\ P_L = ?$

$a = 0$ so $F_L = F_g = mg$



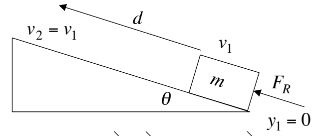
$$P_L = F_L v_L\cos\phi_L = mg\frac{\Delta y}{\Delta t}\cos\phi_L$$
$$P_L = (7.0\text{ kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\frac{(10.0\text{ m})}{(8.0\text{ s})}\cos 0$$
$$\boxed{P_L = 86\text{ W}}$$

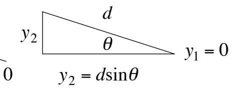
Example 10:

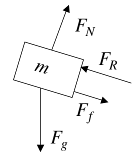
Rat pushes a 25 kg crate up a 36.87° incline at a constant velocity. The coefficient of kinetic friction between the crate and the inclined surface is 0.10. If she pushes the crate a distance of 2.0 m along the incline in 5.0 s, what is Rat's power output?

Example 10:

$m = 25\ \text{kg},\ v_1 = v_2,\ \theta = 36.87°, \mu = 0.10,\ d = 2.0\ \text{m},\ \Delta t = 5.0\ \text{s},\ P_R = ?$

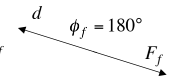






$$\cancel{K_1} + \cancel{U_1} + W_{other} = \cancel{K_2} + U_2$$

$y_1 = 0 \qquad v_2 = v_1$



$$W_R + W_f = U_{g2} \quad \text{so } W_R = U_{g2} - W_f$$

$$W_R = mgy_2 - F_f d\cos\phi_f$$

$$W_R = mgd\sin\theta - \mu F_N d\cos\phi_f = mgd\sin\theta - \mu F_\perp d\cos\phi_f$$

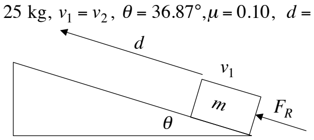
$$W_R = mgd\sin\theta - \mu mg\cos\theta d\cos\phi_f$$

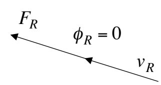
$$W_R = (25\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.0\ \text{m})\sin 36.87° - 0.10(25\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\cos(36.87°)(2.0\ \text{m})\cos(180°)$$

$$W_R = 333.2\ \text{J} \qquad P_R = \frac{W_R}{\Delta t} = \frac{333.2\ \text{J}}{5\ \text{s}} \qquad \boxed{P_R = 67\ \text{W}}$$

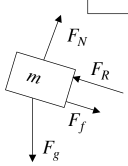
Example 10: (Alternative solution)

$m = 25\ \text{kg},\ v_1 = v_2,\ \theta = 36.87°, \mu = 0.10,\ d = 2.0\ \text{m},\ \Delta t = 5.0\ \text{s},\ P_R = ?$





$$P_R = F_R v_R \cos\phi_R$$



$$\sum F_x = ma$$

$$F_R - F_{//} - F_f = 0$$

$$F_R = F_{//} + F_f = mg\sin\theta + \mu F_N = mg\sin\theta + \mu F_\perp$$

$$F_R = mg\sin\theta + \mu mg\cos\theta$$

$$F_R = (25\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\sin 36.87° + 0.10(25\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\cos(36.87°) = 166.6\ \text{N}$$

$$P_R = F_R v_R \cos\phi_R = F_R \frac{\Delta x}{\Delta t}\cos\phi_R = (166.6\ \text{N})\frac{(2.0\ \text{m})}{(5.0\ \text{s})}\cos 0 \qquad \boxed{P_R = 67\ \text{W}}$$

Example 11:

A spring has a spring constant of 600 N/m.

a.) What force is needed to stretch the spring 2.0 cm?

b.) How much energy is stored in the spring?

Example 11:

$$k = 600\ \frac{\text{N}}{\text{m}},\ x = 0.02\ \text{m}$$

a.) $F = ?$

$$F = kx$$

$$F = \left(600\ \frac{\text{N}}{\text{m}}\right)(0.02\ \text{m})$$

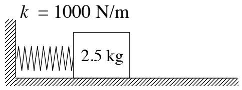
$$\boxed{F = 12\ \text{N}}$$

b.) $U_e = ?$

$$U_e = \frac{1}{2}kx^2$$

$$U_e = \frac{1}{2}\left(600\ \frac{\text{N}}{\text{m}}\right)(0.02\ \text{m})^2$$

$$\boxed{U_e = 0.12\ \text{J}}$$



Example 12:

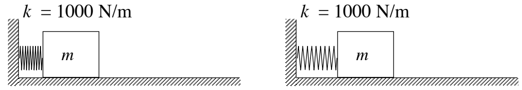
A 2.5 kg block pushed against a spring with a spring constant of 1000 N/m, and the spring is compressed 5.0 cm and then released. The surface is flat and frictionless.

a.) What is the speed when the spring is still compressed 2.5 cm?

b.) What is the speed of the block when it leaves the spring?

Example 12: $m = 2.5\text{ kg},\ x_1 = 0.05\text{ m},\ v_1 = 0,\ k = 1000\ \frac{\text{N}}{\text{m}}$

a.) $x_2 = 0.025\text{ m},\ v_2 = ?$



$x_1 = 0.05\text{ m},\ v_1 = 0$ $\qquad$ $x_2 = 0.025\text{ m},\ v_2 = ?$

$$\cancel{K_1} + U_1 + \cancel{W_{other}} = K_2 + U_2$$

$v_1 = 0$

$$U_{e_1} = K_2 + U_{e_2}$$

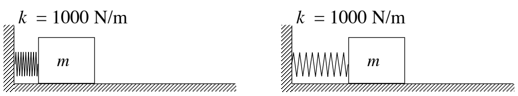
$$\frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

$$v_2 = \sqrt{\frac{k\left(x_1^2 - x_2^2\right)}{m}} = \sqrt{\frac{\left(1000\ \frac{\text{N}}{\text{m}}\right)\left((0.05\text{ m})^2 - (0.025\text{ m})^2\right)}{2.5\text{ kg}}}$$

$$\boxed{v_2 = 0.87\ \frac{\text{m}}{\text{s}}}$$

Example 12: $m = 2.5\text{ kg},\ x_1 = 0.05\text{ m},\ v_1 = 0,\ k = 1000\ \frac{\text{N}}{\text{m}}$

b.) $x_2 = 0,\ v_2 = ?$



$x_1 = 0.05\text{ m},\ v_1 = 0$ $\qquad$ $x_2 = 0,\ v_2 = ?$

$$\cancel{K_1} + U_1 + \cancel{W_{other}} = K_2 + \cancel{U_2}$$

$v_1 = 0 \qquad x_2 = 0$

$$U_{e_1} = K_2$$

$$\frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2$$

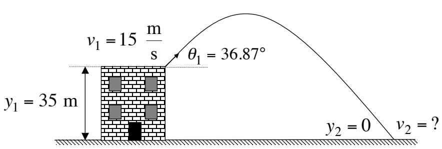
$$v_2 = \sqrt{\frac{kx_1^2}{m}} = \sqrt{\frac{\left(1000\ \frac{\text{N}}{\text{m}}\right)(0.05\text{ m})^2}{2.5\text{ kg}}}$$

$$\boxed{v_2 = 1.0\ \frac{\text{m}}{\text{s}}}$$

Example 13:

Larry kicks a soccer ball with a speed of 15 m/s and angle of 36.87° above the horizontal from a building that is 35 m tall. What is the speed of the ball just before it hits the ground?

Example 13:



$$K_1 + U_1 + \cancel{W_{other}} = K_2 + \cancel{U_2}$$

$y_2 = 0$

$$K_1 + U_1 = K_2$$

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{v_1^2 + 2gy_1}$$

$$v_2 = \sqrt{\left(15\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(35\text{ m})}$$

$$\boxed{v_2 = 30.2\ \frac{\text{m}}{\text{s}}}$$