

Example 1:

A 0.144 kg baseball is pitched horizontally at 45 m/s. After it is hit by a bat, it is hit back towards the pitcher with a speed of 45 m/s.

- What is the final and initial momentum of the baseball?
- What impulse did the bat deliver to the ball?
- If the bat and ball were in contact for 0.60 ms, what was the average force the bat exerted on the ball?

Example 1: $m = 0.144 \text{ kg}$, $v_1 = 45 \frac{\text{m}}{\text{s}}$, and $v_2 = -45 \frac{\text{m}}{\text{s}}$

a.) $p_1 = ?$ and $p_2 = ?$

$$p_1 = mv_1 = (0.144 \text{ kg})\left(45 \frac{\text{m}}{\text{s}}\right) \quad p_2 = mv_2 = (0.144 \text{ kg})\left(-45 \frac{\text{m}}{\text{s}}\right)$$

$$p_1 = 6.48 \frac{\text{kg} \cdot \text{m}}{\text{s}} = 6.48 \text{ N} \cdot \text{s} \quad p_2 = -6.48 \frac{\text{kg} \cdot \text{m}}{\text{s}} = -6.48 \text{ N} \cdot \text{s}$$

b.) $J = ?$

$$J = \Delta p = p_2 - p_1 = -6.48 \text{ N} \cdot \text{s} - 6.48 \text{ N} \cdot \text{s} \quad \text{and} \quad J = -12.96 \text{ N} \cdot \text{s}$$

c.) $\Delta t = 0.6 \times 10^{-3} \text{ s}$, $F = ?$

$$J = F \cdot \Delta t \quad \text{so} \quad F = \frac{J}{\Delta t} = \frac{-12.96 \text{ N} \cdot \text{s}}{0.6 \times 10^{-3} \text{ s}}$$

$$F = -21,600 \text{ N}$$

Example 2:

A tennis racket delivers an average force of 100 N to a tennis ball with a mass of 0.060 kg. Before impact, the ball was traveling with a speed of 20 m/s and is in contact with the tennis racket for 15 ms.

- What is the initial momentum of the tennis ball?
- What is the impulse delivered to the ball by the tennis racket?
- What is the speed of the ball as it leaves the racket?

Example 2: $m = 0.060 \text{ kg}$, $v_1 = 20 \frac{\text{m}}{\text{s}}$, $F = -100 \text{ N}$, and $\Delta t = 15 \times 10^{-3} \text{ s}$

a.) $p_1 = ?$

$$p_1 = mv_1 = (0.060 \text{ kg})\left(20 \frac{\text{m}}{\text{s}}\right)$$

$$p_1 = 1.2 \frac{\text{kg} \cdot \text{m}}{\text{s}} = 1.2 \text{ N} \cdot \text{s}$$

b.) $J = ?$

$$J = F \cdot \Delta t = (-100 \text{ N})\left(15 \times 10^{-3} \text{ s}\right)$$

$$J = -1.5 \text{ N} \cdot \text{s}$$

c.) $v_2 = ?$

$$J = \Delta p = m\Delta v = m(v_2 - v_1)$$

$$v_2 = \frac{J}{m} + v_1 = \frac{-1.5 \text{ N} \cdot \text{s}}{0.06 \text{ kg}} + 20 \frac{\text{m}}{\text{s}} \quad \text{and} \quad v_2 = -5 \frac{\text{m}}{\text{s}}$$

Example 3:

A car with a mass of 5000 kg is moving at 10 m/s towards an identical car at rest. The cars collide and lock together.

- What is the velocity of the coupled cars following the collision?
- Show that the kinetic energy is not conserved.

Example 3:

$m_1 = 5000 \text{ kg}$, $v_{1i} = 10 \frac{\text{m}}{\text{s}}$, $m_2 = 5000 \text{ kg}$, and $v_{2i} = 0$

a.) $v_1 = v_2 = v = ?$

Using conservation of momentum:

$$m_1v_{1i} + m_2v_{2i} = m_1v_1 + m_2v_2 = m_1v + m_2v$$

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v$$

$$v = \frac{m_1v_{1i} + m_2v_{2i}}{m_1 + m_2} = \frac{(5000 \text{ kg})\left(10 \frac{\text{m}}{\text{s}}\right) + (5000 \text{ kg})(0)}{5000 \text{ kg} + 5000 \text{ kg}}$$

$$v = 5 \frac{\text{m}}{\text{s}}$$

Example 3:

$$m_1 = 5000 \text{ kg}, v_{1i} = 10 \frac{\text{m}}{\text{s}}, m_2 = 5000 \text{ kg}, v_{2i} = 0, \text{ and } v_1 = v_2 = v = 5 \frac{\text{m}}{\text{s}}$$

b.) $\Delta KE = ?$

$$\Delta KE = KE_f - KE_i$$

$$KE_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} (5000 \text{ kg}) \left(10 \frac{\text{m}}{\text{s}} \right)^2 + \frac{1}{2} (5000 \text{ kg}) (0)^2 = 250,000 \text{ J}$$

$$KE_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 = \frac{1}{2} (m_1 + m_2) v^2$$

$$KE_f = \frac{1}{2} (5000 \text{ kg} + 5000 \text{ kg}) \left(5 \frac{\text{m}}{\text{s}} \right)^2 = 125,000 \text{ J}$$

$$\Delta KE = KE_f - KE_i = 125,000 \text{ J} - 250,000 \text{ J}$$

$$\boxed{\Delta KE = -125,000 \text{ J}}$$

Impulse and Momentum

8

Example 4:

A car with a mass of 5000 kg is moving at 10 m/s towards an identical car moving at 5 m/s away from the first car. The cars collide and lock together.

What is the velocity of the coupled cars following the collision?

Example 4:

$$m_1 = 5000 \text{ kg}, v_{1i} = 10 \frac{\text{m}}{\text{s}}, m_2 = 5000 \text{ kg}, \text{ and } v_{2i} = 5 \frac{\text{m}}{\text{s}}$$

$$v_1 = v_2 = v = ?$$

Using conservation of momentum:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_1 + m_2 v_2 = m_1 v + m_2 v$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v$$

$$v = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(5000 \text{ kg}) \left(10 \frac{\text{m}}{\text{s}} \right) + (5000 \text{ kg}) \left(5 \frac{\text{m}}{\text{s}} \right)}{5000 \text{ kg} + 5000 \text{ kg}}$$

$$\boxed{v = 7.5 \frac{\text{m}}{\text{s}}}$$

Impulse and Momentum

10

Example 5:

Two coupled railroad cars, one of mass 10,000 kg and the other of mass 15,000 kg, are coasting along a track at a speed of 5 m/s toward the east. An explosion causes the cars to separate with the 15,000 kg car traveling at 20 m/s toward the east. Find the speed and direction of the 10,000 kg car.

Example 5:

$$m_1 = 10,000 \text{ kg}, m_2 = 15,000 \text{ kg}, v_{1i} = v_{2i} = v_i = 5 \frac{\text{m}}{\text{s}}, \text{ and } v_2 = 20 \frac{\text{m}}{\text{s}}$$

$$v_1 = ?$$

Using conservation of momentum:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_1 + m_2 v_2$$

$$m_1 v_i + m_2 v_i = (m_1 + m_2) v_i = m_1 v_1 + m_2 v_2$$

$$v_1 = \frac{(m_1 + m_2) v_i - m_2 v_2}{m_1} = \frac{(10,000 \text{ kg} + 15,000 \text{ kg}) \left(5 \frac{\text{m}}{\text{s}} \right) - (15,000 \text{ kg}) \left(20 \frac{\text{m}}{\text{s}} \right)}{10,000 \text{ kg}}$$

$$v_1 = -17.5 \frac{\text{m}}{\text{s}}$$

$$\boxed{v_1 = 17.5 \frac{\text{m}}{\text{s}}, \text{ west}}$$

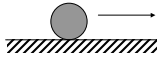
Impulse and Momentum

12

Example 6:

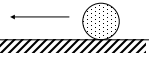
$$m_1 = 6.0 \text{ kg}$$

$$v_{1i} = 1.0 \text{ m/s}$$



$$m_2 = 3.0 \text{ kg}$$

$$v_{2i} = -5.0 \text{ m/s}$$

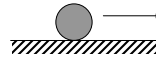


Two balls are approaching each other as shown in the figure above. What are the final velocities of each ball if the collision is perfectly elastic?

Example 6:

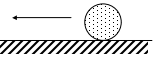
$$m_1 = 6.0 \text{ kg}$$

$$v_{1i} = 1.0 \text{ m/s}$$



$$m_2 = 3.0 \text{ kg}$$

$$v_{2i} = -5.0 \text{ m/s}$$



$$(1) m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(6) (v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

$$(6^*) v_{1i} - v_{2i} = -v_{1f} + v_{2f}$$

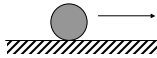
$$(6^*) m_1 v_{1i} - m_1 v_{2i} = -m_1 v_{1f} + m_1 v_{2f}$$

$$(1 + 6^*) 2m_1 v_{1i} + m_2 v_{2i} - m_1 v_{2i} = m_2 v_{2f} + m_1 v_{2f}$$

Example 6:

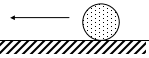
$$m_1 = 6.0 \text{ kg}$$

$$v_{1i} = 1.0 \text{ m/s}$$



$$m_2 = 3.0 \text{ kg}$$

$$v_{2i} = -5.0 \text{ m/s}$$



$$(1 + 6^*) 2m_1 v_{1i} + m_2 v_{2i} - m_1 v_{2i} = m_2 v_{2f} + m_1 v_{2f}$$

$$(1 + 6^*) 2m_1 v_{1i} + (m_2 - m_1) v_{2i} = (m_2 + m_1) v_{2f}$$

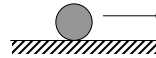
$$(1 + 6^*) v_{2f} = \frac{2m_1 v_{1i} + (m_2 - m_1) v_{2i}}{(m_2 + m_1)}$$

$$v_{2f} = \frac{2(6.0 \text{ kg})\left(1.0 \frac{\text{m}}{\text{s}}\right) + (3.0 \text{ kg} - 6.0 \text{ kg})\left(-5.0 \frac{\text{m}}{\text{s}}\right)}{(3.0 \text{ kg} + 6.0 \text{ kg})} = \boxed{3 \frac{\text{m}}{\text{s}}}$$

Example 6:

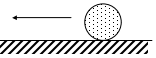
$$m_1 = 6.0 \text{ kg}$$

$$v_{1i} = 1.0 \text{ m/s}$$



$$m_2 = 3.0 \text{ kg}$$

$$v_{2i} = -5.0 \text{ m/s}$$



$$(6) (v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

$$v_{1i} = v_{2f} - v_{1f} + v_{2i}$$

$$v_{1f} = 3.0 \frac{\text{m}}{\text{s}} - 1.0 \frac{\text{m}}{\text{s}} + \left(-5.0 \frac{\text{m}}{\text{s}}\right)$$

$$\boxed{v_{1f} = -3.0 \frac{\text{m}}{\text{s}}}$$