Example 1:

A 0.144 kg baseball is pitched horizontally at 45 m/s. After it is hit by a bat, it is hit back towards the pitcher with a speed of 45 m/s.

a.) What is the final and initial momentum of the baseball?

b.) What impulse did the bat deliver to the ball?

c.) If the bat and ball were in contact for 0.60 ms, what was the average force the bat exerted on the ball?

Example 1: $m = 0.144\text{ kg},\ v_1 = 45\ \frac{\text{m}}{\text{s}}$, and $v_2 = -45\ \frac{\text{m}}{\text{s}}$

a.) $p_1 = ?$ and $p_2 = ?$

$$p_1 = mv_1 = (0.144\text{ kg})\left(45\ \frac{\text{m}}{\text{s}}\right) \qquad p_2 = mv_2 = (0.144\text{ kg})\left(-45\ \frac{\text{m}}{\text{s}}\right)$$

$$\boxed{p_1 = 6.48\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = 6.48\text{ N}\cdot\text{s}} \qquad \boxed{p_2 = -6.48\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = -6.48\text{ N}\cdot\text{s}}$$

b.) $J = ?$

$$J = \Delta p = p_2 - p_1 = -6.48\text{ N}\cdot\text{s} - 6.48\text{ N}\cdot\text{s} \quad \text{and} \quad \boxed{J = -12.96\text{ N}\cdot\text{s}}$$

c.) $\Delta t = 0.6\times10^{-3}\text{ s},\ F = ?$

$$J = F\cdot\Delta t \quad \text{so } F = \frac{J}{\Delta t} = \frac{-12.96\text{ N}\cdot\text{s}}{0.6\text{ x }10^{-3}\text{ s}}$$

$$\boxed{F = -21,600\text{ N}}$$

Example 2:

A tennis racket delivers an average force of 100 N to a tennis ball with a mass of 0.060 kg. Before impact, the ball was traveling with a speed of 20 m/s and is in contact with the tennis racket for 15 ms.

a.) What is the initial momentum of the tennis ball?

b.) What is the impulse delivered to the ball by the tennis racket?

c.) What is the speed of the ball as it leaves the racket?

Example 2: $m = 0.060\text{ kg},\ v_1 = 20\ \frac{\text{m}}{\text{s}},\ F = -100\text{ N}$, and $\Delta t = 15\text{ x }10^{-3}\text{ s}$

a.) $p_1 = ?$

$$p_1 = mv_1 = (0.060\text{ kg})\left(20\ \frac{\text{m}}{\text{s}}\right)$$

$$\boxed{p_1 = 1.2\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = 1.2\text{ N}\cdot\text{s}}$$

b.) $J = ?$

$$J = F\cdot\Delta t = (-100\text{ N})(15\text{ x }10^{-3}\text{ s})$$

$$\boxed{J = -1.5\text{ N}\cdot\text{s}}$$

c.) $v_2 = ?$

$$J = \Delta p = m\Delta v = m(v_2 - v_1)$$

$$v_2 = \frac{J}{m} + v_1 = \frac{-1.5\text{ N}\cdot\text{s}}{0.06\text{ kg}} + 20\ \frac{\text{m}}{\text{s}} \quad \text{and} \quad \boxed{v_2 = -5\ \frac{\text{m}}{\text{s}}}$$

Example 3:

A car with a mass of 5000 kg is moving at 10 m/s towards an identical car at rest. The cars collide and lock together.

a.) What is the velocity of the coupled cars following the collision?

b.) Show that the kinetic energy is not conserved.

Example 3:

$m_1 = 5000\text{ kg},\ v_{1_i} = 10\ \frac{\text{m}}{\text{s}},\ m_2 = 5000\text{ kg}$, and $v_{2i} = 0$

a.) $v_1 = v_2 = v = ?$

Using conservation of momentum:

$$m_1v_{1_i} + m_2v_{2_i} = m_1v_1 + m_2v_2 = m_1v + m_2v$$

$$m_1v_{1_i} + m_2v_{2_i} = (m_1 + m_2)v$$

$$v = \frac{m_1v_{1_i} + m_2v_{2_i}}{m_1 + m_2} = \frac{(5000\text{ kg})\left(10\ \frac{\text{m}}{\text{s}}\right) + (5000\text{ kg})(0)}{5000\text{ kg} + 5000\text{ kg}}$$

$$\boxed{v = 5\ \frac{\text{m}}{\text{s}}}$$

Example 3:

$$m_1 = 5000 \text{ kg},\ v_{1_i} = 10\ \frac{\text{m}}{\text{s}},\ m_2 = 5000 \text{ kg},\ v_{2i} = 0, \text{ and } v_1 = v_2 = v = 5\ \frac{\text{m}}{\text{s}}$$

b.) $\Delta KE = ?$

$$\Delta KE = KE_f - KE_i$$

$$KE_i = \frac{1}{2}m_1 v_{1_i}^2 + \frac{1}{2}m_2 v_{2_i}^2 = \frac{1}{2}(5000 \text{ kg})\left(10\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(5000 \text{ kg})(0)^2 = 250{,}000 \text{ J}$$

$$KE_f = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 = \frac{1}{2}(m_1 + m_2)v^2$$

$$KE_f = \frac{1}{2}(5000 \text{ kg} + 5000 \text{ kg})\left(5\ \frac{\text{m}}{\text{s}}\right)^2 = 125{,}000 \text{ J}$$

$$\Delta KE = KE_f - KE_i = 125{,}000 \text{ J} - 250{,}000 \text{ J}$$

$$\boxed{\Delta KE = -125{,}000 \text{ J}}$$

Example 4:

A car with a mass of 5000 kg is moving at 10 m/s towards an identical car moving at 5 m/s away from the first car. The cars collide and lock together.

What is the velocity of the coupled cars following the collision?

Example 4:

$$m_1 = 5000 \text{ kg},\ v_{1i} = 10\ \frac{\text{m}}{\text{s}},\ m_2 = 5000 \text{ kg}, \text{ and } v_{2i} = 5\ \frac{\text{m}}{\text{s}}$$

$$v_1 = v_2 = v = ?$$

Using conservation of momentum:

$$m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_1 + m_2 v_2 = m_1 v + m_2 v$$

$$m_1 v_{1_i} + m_2 v_{2_i} = (m_1 + m_2)v$$

$$v = \frac{m_1 v_{1_i} + m_2 v_{2_i}}{m_1 + m_2} = \frac{(5000 \text{ kg})\left(10\ \frac{\text{m}}{\text{s}}\right) + (5000 \text{ kg})\left(5\ \frac{\text{m}}{\text{s}}\right)}{5000 \text{ kg} + 5000 \text{ kg}}$$

$$\boxed{v = 7.5\ \frac{\text{m}}{\text{s}}}$$

Example 5:

Two coupled railroad cars, one of mass 10,000 kg and the other of mass 15,000 kg, are coasting along a track at a speed of 5 m/s toward the east. An explosion causes the cars to separate with the 15,000 kg car traveling at 20 m/s toward the east. Find the speed and direction of the 10,000 kg car.

Example 5:

$$m_1 = 10{,}000 \text{ kg},\ m_2 = 15{,}000 \text{ kg},\ v_{1_i} = v_{2i} = v_i = 5\ \frac{\text{m}}{\text{s}}, \text{ and } v_2 = 20\ \frac{\text{m}}{\text{s}}$$

$$v_1 = ?$$

Using conservation of momentum:

$$m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_1 + m_2 v_2$$

$$m_1 v_i + m_2 v_i = (m_1 + m_2)v_i = m_1 v_1 + m_2 v_2$$

$$v_1 = \frac{(m_1 + m_2)v_i - m_2 v_2}{m_1} = \frac{(10{,}000 \text{ kg} + 15{,}000 \text{ kg})\left(5\ \frac{\text{m}}{\text{s}}\right) - (15{,}000 \text{ kg})\left(20\ \frac{\text{m}}{\text{s}}\right)}{10{,}000 \text{ kg}}$$

$$v_1 = -17.5\ \frac{\text{m}}{\text{s}}$$

$$\boxed{v_1 = 17.5\ \frac{\text{m}}{\text{s}}, \text{ west}}$$

Example 6:

$m_1 = 6.0\text{ kg}$ $\quad$ $m_2 = 3.0\text{ kg}$

$v_{1_i} = 1.0\text{ m/s}$ $\quad$ $v_{2_i} = -5.0\text{ m/s}$

Two balls are approaching each other as shown in the figure above. What are the final velocities of each ball if the collision is perfectly elastic?

Example 6:

$m_1 = 6.0\text{ kg}$ $\quad$ $m_2 = 3.0\text{ kg}$

$v_{1_i} = 1.0\text{ m/s}$ $\quad$ $v_{2_i} = -5.0\text{ m/s}$

$$(1)\ m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_1 + m_2 v_2$$

$$(6)\ \left(v_{1_i} - v_{2_i}\right) = -\left(v_1 - v_2\right)$$

$$\left(6^*\right)\ v_{1_i} - v_{2_i} = -v_1 + v_2$$

$$\left(6^*\right)\ m_1 v_{1_i} - m_1 v_{2_i} = -m_1 v_1 + m_1 v_2$$

$$\left(1 + 6^*\right)\ 2m_1 v_{1_i} + m_2 v_{2_i} - m_1 v_{2_i} = m_2 v_2 + m_1 v_2$$

Example 6:

$m_1 = 6.0\text{ kg}$ $\quad$ $m_2 = 3.0\text{ kg}$

$v_{1_i} = 1.0\text{ m/s}$ $\quad$ $v_{2_i} = -5.0\text{ m/s}$

$$\left(1 + 6^*\right)\ 2m_1 v_{1_i} + m_2 v_{2_i} - m_1 v_{2_i} = m_2 v_2 + m_1 v_2$$

$$\left(1 + 6^*\right)\ 2m_1 v_{1_i} + \left(m_2 - m_1\right) v_{2_i} = \left(m_2 + m_1\right) v_2$$

$$\left(1 + 6^*\right)\ v_2 = \frac{2m_1 v_{1_i} + \left(m_2 - m_1\right) v_{2_i}}{\left(m_2 + m_1\right)}$$

$$v_2 = \frac{2(6.0\text{ kg})\left(1.0\ \frac{\text{m}}{\text{s}}\right) + (3.0\text{ kg} - 6.0\text{ kg})\left(-5.0\ \frac{\text{m}}{\text{s}}\right)}{(3.0\text{ kg} + 6.0\text{ kg})} = \boxed{3\ \frac{\text{m}}{\text{s}}}$$

Example 6:

$m_1 = 6.0\text{ kg}$ $\quad$ $m_2 = 3.0\text{ kg}$

$v_{1_i} = 1.0\text{ m/s}$ $\quad$ $v_{2_i} = -5.0\text{ m/s}$

$$(6)\ \left(v_{1_i} - v_{2_i}\right) = -\left(v_1 - v_2\right)$$

$$v_1 = v_2 - v_{1_i} + v_{2_i}$$

$$v_1 = 3.0\ \frac{\text{m}}{\text{s}} - 1.0\ \frac{\text{m}}{\text{s}} + \left(-5.0\ \frac{\text{m}}{\text{s}}\right)$$

$$\boxed{v_1 = -3.0\ \frac{\text{m}}{\text{s}}}$$