

# Impulse and Momentum

## Momentum and Impulse

**Momentum ( $p$ )** - the product of mass and velocity of an object.

**Impulse ( $J$ )** - the product of the net force and the time interval over which the force acts.

$$\bar{p} = m\bar{v} \quad \text{units are} \quad \frac{\text{kg} \cdot \text{m}}{\text{s}} = \text{N} \cdot \text{s}$$

$$\bar{J} = \bar{F}\Delta t \quad \text{units are} \quad \text{N} \cdot \text{s}$$

## Impulse-Momentum Theorem

Using the equations for momentum and impulse:

$$p = m\bar{v} \quad \text{and} \quad J = F\Delta t$$

And recalling that:

$$\bar{F} = m\bar{a} = m\frac{\Delta\bar{v}}{\Delta t} \quad \text{or} \quad \bar{F}\Delta t = m\Delta\bar{v}$$

Results in the following which relates impulse to the change in momentum:

$$m\Delta\bar{v} = \Delta\bar{p} \quad \text{or} \quad \boxed{\Delta\bar{p} = \bar{F}\Delta t = \bar{J}}$$

## Conservation of Momentum

*The momentum of any closed, isolated system does not change.*

$$\bar{p}_{1i} + \bar{p}_{2i} + \dots + \bar{p}_{ni} = \bar{p}_{1f} + \bar{p}_{2f} + \dots + \bar{p}_{nf}$$

Where:

$\bar{p}_{ni}$  = initial momentum vector of object  $n$

$\bar{p}_{nf}$  = final momentum vector of object  $n$

## Conservation of Momentum

For 1-D problems involving 2 objects

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$\boxed{m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}}$$

## Inelastic Collisions Between Objects

- Momentum is *conserved*.
- Kinetic Energy is *not conserved*.
- When objects stick together the collision is called a *completely inelastic collision*.

## Elastic Collisions Between Objects

- **Momentum is conserved.**
- **Kinetic Energy is also conserved.**

### Conservation of Kinetic Energy

For 1-D problems involving 2 objects

$$KE_{1_i} + KE_{2_i} = KE_{1_f} + KE_{2_f}$$

$$\frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2 = \frac{1}{2}m_1v_{1_f}^2 + \frac{1}{2}m_2v_{2_f}^2$$

### 1D Elastic Collisions Between Objects

$$(2) \frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2 = \frac{1}{2}m_1v_{1_f}^2 + \frac{1}{2}m_2v_{2_f}^2$$

$$m_1v_{1_i}^2 + m_2v_{2_i}^2 = m_1v_{1_f}^2 + m_2v_{2_f}^2$$

$$m_1(v_{1_i}^2 - v_{1_f}^2) = m_2(v_{2_f}^2 - v_{2_i}^2)$$

$$(3) m_1(v_{1_i} - v_{1_f})(v_{1_i} + v_{1_f}) = m_2(v_{2_f} - v_{2_i})(v_{2_f} + v_{2_i})$$

$$(1) m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f}$$

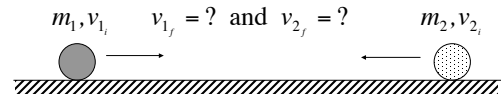
$$m_1v_{1_i} - m_1v_{1_f} = m_2v_{2_f} - m_2v_{2_i}$$

$$(4) m_1(v_{1_i} - v_{1_f}) = m_2(v_{2_f} - v_{2_i})$$

## Collisions Between Objects

In any collision, momentum is conserved and the total momentum before equals the total momentum after; in elastic collisions only, the total kinetic energy before equals the total kinetic energy after.

### 1D Elastic Collisions Between Objects



$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$(1) m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f}$$

$$\sum K_i = \sum K_f$$

$$(2) \frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2 = \frac{1}{2}m_1v_{1_f}^2 + \frac{1}{2}m_2v_{2_f}^2$$

### 1D Elastic Collisions Between Objects

$$(4) m_1(v_{1_i} - v_{1_f}) = m_2(v_{2_f} - v_{2_i})$$

$$(3) m_1(v_{1_i} - v_{1_f})(v_{1_i} + v_{1_f}) = m_2(v_{2_f} - v_{2_i})(v_{2_f} + v_{2_i})$$

$$(5) (v_{1_i} + v_{1_f}) = (v_{2_f} + v_{2_i})$$

$$(6) (v_{1_i} - v_{2_i}) = -(v_{1_f} - v_{2_f})$$

The relative speed of the objects before the collision equals the negative of the relative speed after the collision.

$$(1) m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f}$$