Impulse and Momentum

Momentum and Impulse

***Momentum* (*p*) - the product of mass and velocity of an object.**

***Impulse* (*J*) - the product of the net force and the time interval over which the force acts.**

$$\vec{p} = m\vec{v} \quad \text{units are} \quad \frac{\text{kg}\cdot\text{m}}{\text{s}} = \text{N}\cdot\text{s}$$

$$\vec{J} = \vec{F}\Delta t \quad \text{units are} \quad \text{N}\cdot\text{s}$$

Impulse-Momentum Theorem

Using the equations for momentum and impulse:

$$\vec{p} = m\vec{v} \quad \text{and} \quad \vec{J} = \vec{F}\Delta t$$

And recalling that:

$$\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} \quad \text{or} \quad \vec{F}\Delta t = m\Delta\vec{v}$$

Results in the following which relates impulse to the change in momentum:

$$m\Delta\vec{v} = \Delta\vec{p} \quad \text{or} \quad \boxed{\Delta\vec{p} = \vec{F}\Delta t = \vec{J}}$$

Conservation of Momentum

The momentum of any closed, isolated system does not change.

$$\vec{p}_{1_i} + \vec{p}_{2_i} + \cdots + \vec{p}_{n_i} = \vec{p}_{1_f} + \vec{p}_{2_f} + \cdots + \vec{p}_{n_f}$$

Where:

$\vec{p}_{n_i}$ = **initial momentum vector of object *n***

$\vec{p}_{n_f}$ = **final momentum vector of object *n***

Conservation of Momentum

For 1-D problems involving 2 objects

$$p_{1_i} + p_{2_i} = p_{1_f} + p_{2_f}$$

$$\boxed{m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f}}$$

Inelastic Collisions Between Objects

- **Momentum is *conserved*.**
- **Kinetic Energy is *not conserved*.**
- **When objects stick together the collision is called a *completely inelastic collision*.**

Elastic Collisions Between Objects

- Momentum is *conserved.*
- Kinetic Energy is *also conserved.*

Collisions Between Objects

In any collision, momentum is conserved and the total momentum before equals the total momentum after; in elastic collisions only, the total kinetic energy before equals the total kinetic energy after.

Conservation of Kinetic Energy

For 1-D problems involving 2 objects

$$KE_{1_i} + KE_{2_i} = KE_{1_f} + KE_{2_f}$$

$$\boxed{\frac{1}{2}m_1v_{1_i}^2 + \frac{1}{2}m_2v_{2_i}^2 = \frac{1}{2}m_1v_{1_f}^2 + \frac{1}{2}m_2v_{2_f}^2}$$

1D Elastic Collisions Between Objects

m_1, v_{1_i} $\quad v_{1_f} = ?$ and $v_{2_f} = ?$ $\quad m_2, v_{2_i}$

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$(1)\quad m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f}$$

$$\sum K_i = \sum K_f$$

$$(2)\quad \frac{1}{2}m_1{v_{1_i}}^2 + \frac{1}{2}m_2{v_{2_i}}^2 = \frac{1}{2}m_1{v_{1_f}}^2 + \frac{1}{2}m_2{v_{2_f}}^2$$

1D Elastic Collisions Between Objects

$$(2)\quad \cancel{\frac{1}{2}}m_1{v_{1_i}}^2 + \cancel{\frac{1}{2}}m_2{v_{2_i}}^2 = \cancel{\frac{1}{2}}m_1{v_{1_f}}^2 + \cancel{\frac{1}{2}}m_2{v_{2_f}}^2$$

$$m_1{v_{1_i}}^2 + m_2{v_{2_i}}^2 = m_1{v_{1_f}}^2 + m_2{v_{2_f}}^2$$

$$m_1\left({v_{1_i}}^2 - {v_{1_f}}^2\right) = m_2\left({v_{2_f}}^2 - {v_{2_i}}^2\right)$$

$$(3)\quad m_1\left(v_{1_i} - v_{1_f}\right)\left(v_{1_i} + v_{1_f}\right) = m_2\left(v_{2_f} - v_{2_i}\right)\left(v_{2_f} + v_{2_i}\right)$$

$$(1)\quad m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f}$$

$$m_1v_{1_i} - m_1v_{1_f} = m_2v_{2_f} - m_2v_{2_i}$$

$$(4)\quad m_1\left(v_{1_i} - v_{1_f}\right) = m_2\left(v_{2_f} - v_{2_i}\right)$$

1D Elastic Collisions Between Objects

$$(4)\quad m_1\left(v_{1_i} - v_{1_f}\right) = m_2\left(v_{2_f} - v_{2_i}\right)$$

$$(3)\quad \cancel{m_1\left(v_{1_i} - v_{1_f}\right)}\left(v_{1_i} + v_{1_f}\right) = \cancel{m_2\left(v_{2_f} - v_{2_i}\right)}\left(v_{2_f} + v_{2_i}\right)$$

$$(5)\quad \left(v_{1_i} + v_{1_f}\right) = \left(v_{2_f} + v_{2_i}\right)$$

$$\boxed{(6)\quad \left(v_{1_i} - v_{2_i}\right) = -\left(v_{1_f} - v_{2_f}\right)}$$

The relative speed of the objects before the collision equals the negative of the relative speed after the collision.

$$\boxed{(1)\quad m_1v_{1_i} + m_2v_{2_i} = m_1v_{1_f} + m_2v_{2_f}}$$