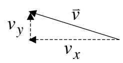
Example 1:

A car is moving at 26 m/s at an angle of 160°. Find the x and y components of the car's velocity.

$$\vec{v} = 26\ \frac{\text{m}}{\text{s}}\ \angle 160°,\ v_x = ?, v_y = ?$$



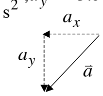
$$v_x = v\cos\theta = \left(26\ \frac{\text{m}}{\text{s}}\right)\cos(160°) = \boxed{-24.4\ \frac{\text{m}}{\text{s}}}$$

$$v_y = v\sin\theta = \left(26\ \frac{\text{m}}{\text{s}}\right)\sin(160°) = \boxed{8.9\ \frac{\text{m}}{\text{s}}}$$

Example 2:

A car is accelerating and its x and y components are a_x = -2.5 m/s^2 and a_y = -3.0 m/s^2. Find the magnitude and direction of the car's acceleration.

$$a_x = -2.5\ \frac{\text{m}}{\text{s}^2}, a_y = -3.0\ \frac{\text{m}}{\text{s}^2},\ a = ?,\ \theta = ?$$



$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(-2.5\ \frac{\text{m}}{\text{s}^2}\right)^2 + \left(-3.0\ \frac{\text{m}}{\text{s}^2}\right)^2} = \boxed{3.9\ \frac{\text{m}}{\text{s}^2}}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-3.0\ \frac{\text{m}}{\text{s}^2}}{-2.5\ \frac{\text{m}}{\text{s}^2}}\right) = 50.2° + 180° = \boxed{230.2°}$$

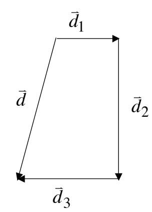
Example 3:

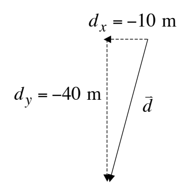
Larry walks 20 m east, 40 m south, then 30 m west. Find the magnitude and direction of the resultant displacement.

Example 3:

$$\vec{d}_1 = 20\text{ m } \angle 0,\ \vec{d}_2 = 40\text{ m } \angle 270°,\ \vec{d}_3 = 30\text{ m } \angle 180°$$

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = ?$$





$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(-10\text{ m})^2 + (-40\text{ m})^2} = \boxed{41.2\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-40\text{ m}}{-10\text{ m}}\right) = 76° + 180° = \boxed{256°}$$

$$\boxed{\vec{d} = 41.2\text{ m } \angle 256°}$$

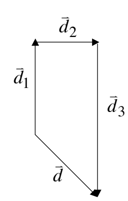
Example 4:

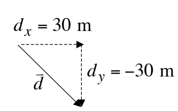
Rat walks 50 m north, 30 m east, then 80 m south. Find the magnitude and direction of the resultant displacement.

Example 4:

$$\vec{d}_1 = 50\text{ m } \angle 90°,\ \vec{d}_2 = 30\text{ m } \angle 0,\ \vec{d}_3 = 80\text{ m } \angle 270°$$

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = ?$$





$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(30\text{ m})^2 + (-30\text{ m})^2} = \boxed{42.4\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-30\text{ m}}{30\text{ m}}\right) = -45° + 360° = \boxed{315°}$$

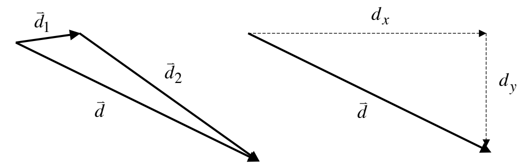
$$\boxed{\vec{d} = 42.4\text{ m } \angle 315°}$$

$$\boxed{\vec{d} = 42.4\text{ m } \angle -45°}$$

Example 5:

Rat walks 20 m at 25° north of east. She then walks 80 m in a direction 50° south of east. Determine the magnitude and direction of the resultant displacement.

Example 5: $\vec{d}_1 = 20 \text{ m} \angle 25°,\ \vec{d}_2 = 80 \text{ m} \angle -50°,\ \vec{d} = \vec{d}_1 + \vec{d}_2 = ?$



$$d_x = d_{1_x} + d_{2_x} = d_1\cos\theta_1 + d_2\cos\theta_2 = (20 \text{ m})\cos(25°) + (80 \text{ m})\cos(-50°) = 69.5 \text{ m}$$

$$d_y = d_{1_y} + d_{2_y} = d_1\sin\theta_1 + d_2\sin\theta_2 = (20 \text{ m})\sin(25°) + (80 \text{ m})\sin(-50°) = -52.9 \text{ m}$$

$$d = \sqrt{{d_x}^2 + {d_y}^2} = \sqrt{(69.5 \text{ m})^2 + (-52.9 \text{ m})^2} = \boxed{87.3 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-52.9 \text{ m}}{69.5 \text{ m}}\right) = \boxed{-37.3°}$$

$$\boxed{\vec{d} = 87.3 \text{ m} \angle -37.3°}$$

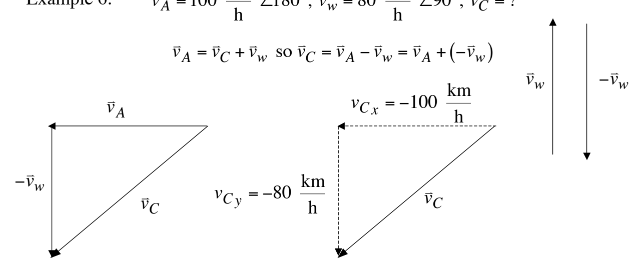
Example 6:

Lily wishes to fly due west with a speed of 100 km/hr. A wind is blowing from the south at 80 km/hr. Find the speed and angle that Lily must choose in order to reach her destination on time.

$$\vec{v}_A = 100 \ \frac{\text{km}}{\text{h}} \angle 180°,\ \vec{v}_w = 80 \ \frac{\text{km}}{\text{h}} \angle 90°,\ \vec{v}_C = ?$$

Example 6: $\vec{v}_A = 100 \ \frac{\text{km}}{\text{h}} \angle 180°,\ \vec{v}_w = 80 \ \frac{\text{km}}{\text{h}} \angle 90°,\ \vec{v}_C = ?$

$\vec{v}_A = \vec{v}_C + \vec{v}_w$ so $\vec{v}_C = \vec{v}_A - \vec{v}_w = \vec{v}_A + (-\vec{v}_w)$



$$v_c = \sqrt{{v_{C_x}}^2 + {v_{C_y}}^2} = \sqrt{\left(-100 \ \frac{\text{km}}{\text{h}}\right)^2 + \left(-80 \ \frac{\text{km}}{\text{h}}\right)^2} = \boxed{128 \ \frac{\text{km}}{\text{h}}}$$

$$\vec{v}_C = 128 \ \frac{\text{km}}{\text{h}} \angle 218.7°$$

$$\theta_C = \tan^{-1}\left(\frac{v_{C_y}}{v_{C_x}}\right) = \tan^{-1}\left(\frac{-80 \ \frac{\text{km}}{\text{h}}}{-100 \ \frac{\text{km}}{\text{h}}}\right) = 38.7° + 180° = \boxed{218.7°}$$

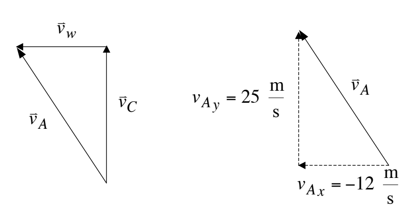
Example 7:

Larry flies north at a speed of 25 m/s with a 12 m/s wind blowing from the east. Find the resultant vector that describes his actual motion.

$$\vec{v}_C = 25 \ \frac{\text{m}}{\text{s}} \angle 90°,\ \vec{v}_w = 12 \ \frac{\text{m}}{\text{s}} \angle 180°,\ \vec{v}_A = ?$$

Example 7: $\vec{v}_C = 25 \ \frac{\text{m}}{\text{s}} \angle 90°,\ \vec{v}_w = 12 \ \frac{\text{m}}{\text{s}} \angle 180°,\ \vec{v}_A = ?$

$\vec{v}_A = \vec{v}_C + \vec{v}_w$



$$v_A = \sqrt{{v_{A_x}}^2 + {v_{A_y}}^2} = \sqrt{\left(-12 \ \frac{\text{m}}{\text{s}}\right)^2 + \left(25 \ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{27.7 \ \frac{\text{m}}{\text{s}}}$$

$$\vec{v}_A = 27.7 \ \frac{\text{m}}{\text{s}} \angle 115.6°$$

$$\theta_A = \tan^{-1}\left(\frac{v_{A_y}}{v_{A_x}}\right) = \tan^{-1}\left(\frac{25 \ \frac{\text{m}}{\text{s}}}{-12 \ \frac{\text{m}}{\text{s}}}\right) = -64.4° + 180° = \boxed{115.6°}$$

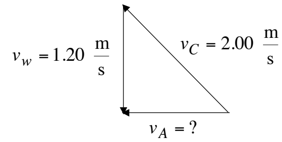
Example 8:

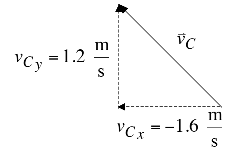
A boat's speed in still water is 2.00 m/s. A river has a current of 1.20 m/s towards the south. What course should the boat head so that it reaches a point directly across and on the west bank of the river?

$$\vec{v}_C = 2.00\ \frac{\text{m}}{\text{s}}\ \angle(\theta_C = ?),\ \vec{v}_w = 1.20\ \frac{\text{m}}{\text{s}}\ \angle 270°,\ \vec{v}_A = (v_A = ?)\ \angle 180°$$

Example 8: $\vec{v}_C = 2.00\ \frac{\text{m}}{\text{s}}\ \angle(\theta_C = ?),\ \vec{v}_w = 1.20\ \frac{\text{m}}{\text{s}}\ \angle 270°,\ \vec{v}_A = (v_A = ?)\ \angle 180°$

$$\vec{v}_A = \vec{v}_C + \vec{v}_w$$





$$v_C^2 = v_A^2 + v_w^2$$

$$v_A = \sqrt{v_C^2 - v_w^2}$$

$$v_A = \sqrt{\left(2.00\ \frac{\text{m}}{\text{s}}\right)^2 - \left(1.20\ \frac{\text{m}}{\text{s}}\right)^2}$$

$$v_A = 1.6\ \frac{\text{m}}{\text{s}}$$

$$\theta_C = \tan^{-1}\left(\frac{v_{C_y}}{v_{C_x}}\right)$$

$$\theta_C = \tan^{-1}\left(\frac{1.2\ \frac{\text{m}}{\text{s}}}{-1.6\ \frac{\text{m}}{\text{s}}}\right) = -36.87° + 180°$$

$$\theta_C = 143.13°$$

$$\boxed{\vec{v}_C = 2.00\ \frac{\text{m}}{\text{s}}\ \angle 143.13°}$$

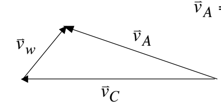
Example 9:

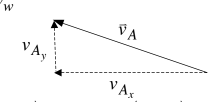
Tiger flies due west at a speed of 50 m/s with a 15 m/s wind blowing from the southwest towards an angle of 75˚. Find the resultant vector that describes his actual motion.

$$\vec{v}_C = 50\ \frac{\text{m}}{\text{s}}\ \angle 180°,\ \vec{v}_w = 15\ \frac{\text{m}}{\text{s}}\ \angle 75°,\ \vec{v}_A = ?$$

Example 9: $\vec{v}_C = 50\ \frac{\text{m}}{\text{s}}\ \angle 180°,\ \vec{v}_w = 15\ \frac{\text{m}}{\text{s}}\ \angle 75°,\ \vec{v}_A = ?$

$$\vec{v}_A = \vec{v}_C + \vec{v}_w$$





$$v_{A_x} = v_{C_x} + v_{w_x} = v_C\cos\theta_C + v_w\cos\theta_w = \left(50\ \frac{\text{m}}{\text{s}}\right)\cos(180°) + \left(15\ \frac{\text{m}}{\text{s}}\right)\cos(75°)$$

$$v_{A_x} = -46.1\ \frac{\text{m}}{\text{s}}$$

$$v_{A_y} = v_{C_y} + v_{w_y} = v_C\sin\theta_C + v_w\sin\theta_w = \left(50\ \frac{\text{m}}{\text{s}}\right)\sin(180°) + \left(15\ \frac{\text{m}}{\text{s}}\right)\sin(75°)$$

$$v_{A_y} = 14.5\ \frac{\text{m}}{\text{s}}$$

$$v_A = \sqrt{v_{A_x}^2 + v_{A_y}^2} = \sqrt{\left(-46.1\ \frac{\text{m}}{\text{s}}\right)^2 + \left(14.5\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{48.3\ \frac{\text{m}}{\text{s}}}$$

$$\theta_A = \tan^{-1}\left(\frac{v_{A_y}}{v_{A_x}}\right) = \tan^{-1}\left(\frac{14.5\ \frac{\text{m}}{\text{s}}}{-46.1\ \frac{\text{m}}{\text{s}}}\right) = -17.5° + 180° = \boxed{162.5°}$$

$$\vec{v}_A = 48.3\ \frac{\text{m}}{\text{s}}\ \angle 162.5°$$

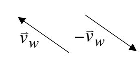
Example 10:

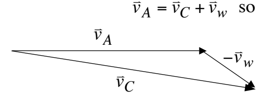
Lily wishes to fly east at a speed of 70 m/s. There is a 30 m/s wind blowing from the southeast towards 140˚. Find the vector that describes the course she should take.

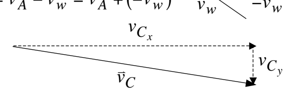
$$\vec{v}_A = 70\ \frac{\text{m}}{\text{s}}\ \angle 0,\ \vec{v}_w = 30\ \frac{\text{m}}{\text{s}}\ \angle 140°,\ \vec{v}_C = ?$$

Example 10: $\vec{v}_A = 70\ \frac{\text{m}}{\text{s}}\ \angle 0,\ \vec{v}_w = 30\ \frac{\text{m}}{\text{s}}\ \angle 140°,\ \vec{v}_C = ?$

$$\vec{v}_A = \vec{v}_C + \vec{v}_w \text{ so } \vec{v}_C = \vec{v}_A - \vec{v}_w = \vec{v}_A + (-\vec{v}_w)$$







$$v_{C_x} = v_{A_x} - v_{w_x} = v_A\cos\theta_A - v_w\cos\theta_w = \left(70\ \frac{\text{m}}{\text{s}}\right)\cos(0) - \left(30\ \frac{\text{m}}{\text{s}}\right)\cos(140°)$$

$$v_{C_x} = 93.0\ \frac{\text{m}}{\text{s}}$$

$$v_{C_y} = v_{A_y} - v_{w_y} = v_A\sin\theta_A - v_w\sin\theta_w = \left(70\ \frac{\text{m}}{\text{s}}\right)\sin(0) - \left(30\ \frac{\text{m}}{\text{s}}\right)\sin(140°)$$

$$v_{C_y} = -19.3\ \frac{\text{m}}{\text{s}}$$

$$v_C = \sqrt{v_{C_x}^2 + v_{C_y}^2} = \sqrt{\left(93.0\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-19.3\ \frac{\text{m}}{\text{s}}\right)^2} = \boxed{95.0\ \frac{\text{m}}{\text{s}}}$$

$$\theta_C = \tan^{-1}\left(\frac{v_{C_y}}{v_{C_x}}\right) = \tan^{-1}\left(\frac{-19.3\ \frac{\text{m}}{\text{s}}}{93.0\ \frac{\text{m}}{\text{s}}}\right) = \boxed{-11.7°}$$

$$\vec{v}_C = 95.0\ \frac{\text{m}}{\text{s}}\ \angle -11.7° \text{ or } 348.3°$$