Example 1:

Rat throws a baseball with a horizontal speed of 15 m/s from a building that is 35 m tall.

- a.) What is the initial velocity in the *x*-direction?
- What is the initial velocity in the y-direction? b.)
- How much time does it take the ball to hit the c.) ground?
- How far from the base of the building does the d.) ball strike the ground?
- e.) Find the magnitude and direction of the velocity of the ball just before it hits the ground.

Example 1:

a.) What is the initial velocity in the *x*-direction?

$$v_x = v_{x_i} = v_i \cos \theta_i = 15 \frac{\text{m}}{\text{s}} \cos \theta = \boxed{15 \frac{\text{m}}{\text{s}}}$$

 $y_i = 35 \text{ m}, \ \theta_i = 0, \ \text{and} \ v_i = 15 \ \frac{\text{m}}{2}$

b.) What is the initial velocity in the y-direction?

$$v_{y_i} = v_i \sin \theta_i = 15 \frac{\mathrm{m}}{\mathrm{s}} \sin \theta = 0$$

c.) How much time does it take the ball to reach the ground? y = 0 so $\Delta y = y - y_i = 0 - 35$ m = -35 m, t = ?

$$\Delta y = -\frac{1}{2}gt^{2} + v_{y_{i}}t = -\frac{1}{2}gt^{2}$$
$$t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-35 \text{ m})}{9.8 \frac{\text{m}}{\text{s}^{2}}}} = \boxed{2.67 \text{ s}}$$

2

= ?

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 $y_i = 35 \text{ m}, \ \theta_i = 0, \ \text{and} \ v_i = 15 \ \frac{\text{m}}{\text{s}}$ Example 1:

e.) Find the magnitude and direction of the velocity of the ball just before it hits the ground.

$$\Delta y = y - y_i = 0 - 35 \text{ m} = -35 \text{ m}, t = 2.67 \text{ s}, v = ?, 6$$

alternatively $v_y^2 = v_{y_i}^2 - 2g\Delta y$
so $v_y = \pm \sqrt{v_{y_i}^2 - 2g\Delta y}$
 $v_y = -\sqrt{0 - 2(9.8 \frac{\text{m}}{\text{s}^2})(-35 \text{ m})}$
 $v_y = -26.2 \frac{\text{m}}{\text{s}}$

Example 2: $y_i = 60 \text{ m}, \theta_i = 0$, and $\Delta x = 80 \text{ m}$ when y = 0

Example 2:

Larry throws a baseball with a horizontal speed from a building that is 60 m tall. The ball hits the ground a distance of 80 m from the base of the building.

- What is the initial velocity in the *x*-direction? a.)
- What is the initial velocity in the y-direction? b.)
- c.) Find the magnitude and direction of the velocity of the ball just before it hits the ground.

a.) What is the initial velocity in the *x*-direction? $v_{x_i} = v_x = ?$ $\Delta x = v_x t$ so $v_x = \frac{\Delta x}{t}$ where t is the time in the air $\Delta y = y - y_i = 0 - 60 \text{ m} = -60 \text{ m}, t = ?$ $\Delta y = -\frac{1}{2}gt^{2} + v_{y_{i}}t = -\frac{1}{2}gt^{2}$ $v_{y_{i}} = 0$ $t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-60 \text{ m})}{9.8 \frac{\text{m}}{c^2}}} = 3.50 \text{ s}$ $v_x = \frac{\Delta x}{t} = \frac{80 \text{ m}}{3.50 \text{ s}} = \boxed{22.9 \text{ m}}{\text{s}}$

 $y_i = 35 \text{ m}, \ \theta_i = 0, \text{ and } v_i = 15 \frac{\text{m}}{\text{s}}$ Example 1:

d.) How far from the base of the building does the ball strike the ground? y = 0 so $\Delta y = y - y_i = 0 - 35$ m = -35 m, t = 2.67 s, $\Delta x = ?$

$$\Delta x = v_x t = \left(15 \ \frac{\mathrm{m}}{\mathrm{s}}\right) (2.67 \ \mathrm{s}) = \boxed{40 \ \mathrm{m}}$$

e.) Find the magnitude and direction of the velocity of the ball just before it hits the ground.

> $\Delta y = y - y_i = 0 - 35 \text{ m} = -35 \text{ m}, t = 2.67 \text{ s}, v = ?, \theta = ?$ m (m),

$$v_{x} = 15 \frac{m}{s} \text{ and } v_{y} = -gt + v_{y_{i}} = -\left(9.8 \frac{m}{s^{2}}\right)(2.67 \text{ s}) + 0 = -26.2 \frac{m}{s}$$

$$v_{x} = 15 \frac{m}{s}$$

$$v_{y} = -26.2 \frac{m}{s}$$

$$v = \sqrt{v_{x}^{2} + v_{y}^{2}} = \sqrt{\left(15 \frac{m}{s}\right)^{2} + \left(-26.2 \frac{m}{s}\right)^{2}} = \boxed{30 \frac{m}{s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{y}}{v_{x}}\right) = \tan^{-1}\left(\frac{-26.2 \frac{m}{s}}{15 \frac{m}{s}}\right) = \boxed{-60^{\circ}}$$
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Example 2: $y_i = 60 \text{ m}, \ \theta_i = 0, \text{ and } \Delta x = 80 \text{ m when } y = 0$

b.) What is the initial velocity in the y-direction?

$$v_{y_i} = ?$$
 horizontal projectile $v_{y_i} = 0$ ($\theta_i = 0$ so $v_i \sin \theta_i = 0$)

c.) Find the magnitude and direction of the velocity of the ball just before it hits the ground.

 $\Delta y = y - y_i = 0 - 60 \text{ m} = -60 \text{ m}, t = 3.50 \text{ s}, v = ?, \theta = ?$

$$v_{x} = 22.9 \frac{m}{s} \text{ and } v_{y} = -gt + v_{y_{i}} = -\left(9.8 \frac{m}{s^{2}}\right)(3.50 \text{ s}) + 0 = -34.3 \frac{m}{s}$$

$$v_{x} = 22.9 \frac{m}{s}$$

$$v_{y} = -34.3 \frac{m}{s} \quad v = \sqrt{v_{x}^{2} + v_{y}^{2}} = \sqrt{\left(22.9 \frac{m}{s}\right)^{2} + \left(-34.3 \frac{m}{s}\right)^{2}} = \boxed{41.2 \frac{m}{s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{y}}{v_{x}}\right) = \tan^{-1}\left(\frac{-34.3 \frac{m}{s}}{22.9 \frac{m}{s}}\right) = \boxed{-56.3^{\circ}}$$
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Example 3:

Example 4:

Rat is flying a plane and is traveling horizontally at 40 m/s at a height of 100 m above the ground when she drops a package of emergency supplies to a group on the ground.

- Where does the package strike the ground relative a.) to the point at which it was released?
- b.) Find the magnitude and direction of the velocity of the package just before it hits the ground.

 $\theta_i = 0, \ v_i = 40 \ \frac{m}{s}, \ y_i = 100 \ m, \ and \ y = 0$ Example 3:

b.) Find the magnitude and direction of the velocity of the package just before it hits the ground.

$$\Delta y = -100 \text{ m}, t = 4,52 \text{ s}, v = ?, \theta = ?$$

$$v_x = 40 \frac{\text{m}}{\text{s}} \quad \text{and} \ v_y = -gt + v_{y_i} = -\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(4.52 \text{ s}) + 0 = -44.3 \frac{\text{m}}{\text{s}}$$

$$v_x = 40 \frac{\text{m}}{\text{s}}$$

$$v_y = -44.3 \frac{\text{m}}{\text{s}} \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(40 \frac{\text{m}}{\text{s}}\right)^2 + \left(-44.3 \frac{\text{m}}{\text{s}}\right)^2} = \frac{60 \frac{\text{m}}{\text{s}}}{\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-44.3 \frac{\text{m}}{\text{s}}}{40 \frac{\text{m}}{\text{s}}}\right) = \left[-48^\circ\right]$$

 $y_i = 0, \ \theta_i = 53.13^\circ$, and $v_i = 20^\circ$ a.) What is the initial velocity in the *x*-direction? $v_x = v_{x_i} = v_i \cos \theta_i = 20 \frac{m}{s} \cos(53.13^\circ) = 12 \frac{m}{s}$ b.) What is the initial velocity in the y-direction? $v_{y_i} = v_i \sin \theta_i = 20 \frac{\text{m}}{\text{s}} \sin(53.13^\circ) = \boxed{16 \frac{\text{m}}{\text{s}}}$ c.) What is the velocity of the ball just before it hits the ground? y = 0 so $\Delta y = y - y_i = 0 - 0 = 0$, v = ?, $\theta = ?$ $v_{x} = 12 \frac{m}{s} \qquad v_{y}^{2} = v_{y_{i}}^{2} - 2g\Delta y \text{ so } v_{y} = \pm \sqrt{v_{y_{i}}^{2} - 2g\Delta y}$ $v_{y} = -\sqrt{\left(16 \frac{m}{s}\right)^{2} - 2\left(9.8 \frac{m}{s^{2}}\right)(0)} = -16 \frac{m}{s}$ $v = \sqrt{v_{x}^{2} + v_{y}^{2}} = \sqrt{\left(12 \frac{m}{s}\right)^{2} + \left(-16 \frac{m}{s}\right)^{2}} = \boxed{20 \frac{m}{s}} \qquad \theta = \tan^{-1}\left(\frac{v_{y}}{v_{x}}\right) = \tan^{-1}\left(\frac{-16 \frac{m}{s}}{12 \frac{m}{s}}\right)$ $\theta = \boxed{-53.13^{\circ}} \qquad 12$

$$\Delta x = ?$$

$$\Delta x = v_x t$$

horizontal projectile ($\theta_i = 0$) so $v_x = v_i = 40 \frac{\text{m}}{\text{s}}$ and $v_{y_i} = 0$

$$\Delta y = y - y_i = 0 - 100 \text{ m} = -100 \text{ m}, t = ?$$

$$\Delta y = -\frac{1}{2}gt^2 + \frac{y_{y_i}t}{v_{y_i}} = 0 - \frac{1}{2}gt^2$$

$$\sqrt{-2\Delta y} = -\frac{1}{2}(-2\sqrt{100}) \text{ m} + c^2$$

 $\theta_i = 0, v_i = 40 \frac{m}{s}, y_i = 100 m, and y = 0$

$$t = \sqrt{\frac{g}{g}} = \sqrt{\frac{9.8 \text{ m}}{\text{s}^2}} = 4.32 \text{ s}$$
$$\Delta x = v_x t = \left(40 \text{ m}{\text{s}}\right)(4.52 \text{ s}) = \boxed{181 \text{ m}}$$

Example 4:

Example 3:

a.)

Rat kicks a soccer ball from ground level at an angle of 53.13° with respect to the horizontal. The initial speed of the ball is 20 m/s.

- a.) What is the initial velocity in the *x*-direction?
- What is the initial velocity in the y-direction? b.)
- What is the velocity of the ball just before it hits c.) the ground?
- How much time does it take the ball to hit the d.) ground?
- e.) How far does the ball travel in the horizontal direction?
- What is the maximum height of the ball? f.)

 $y_i = 0, \ \theta_i = 53.13^\circ$, and $v_i = 20 \ \frac{m}{s}$

d.) How much time does it take the ball to reach the ground? $y = 0 \text{ so } \Delta y = y - y_i = 0 - 0 = 0, v_{y_i} = 16 \frac{\text{m}}{\text{s}}, v_y = -16 \frac{\text{m}}{\text{s}}, t = ?$ $v_y = -gt + v_{y_i} \text{ so } t = \frac{v_y - v_{y_i}}{-g} = \frac{-16 \frac{\text{m}}{\text{s}} - 16 \frac{\text{m}}{\text{s}}}{-9.8 \frac{\text{m}}{\text{s}^2}} = \boxed{3.27 \text{ s}}$ alternatively $\Delta y = -\frac{1}{2}gt^2 + v_{y_i}t = 0$ so $0 = t\left(-\frac{1}{2}gt + v_{y_i}\right)$ solutions are when t = 0 and $-\frac{1}{2}gt + v_{y_i} = 0$ so $t = \frac{2v_{y_i}}{g} = \frac{2\left(16 \frac{\text{m}}{\text{s}}\right)}{9.8 \frac{\text{m}}{\text{s}^2}} = \boxed{3.27 \text{ s}}$ Example 4: $y_i = 0, \ \theta_i = 53.13^\circ$, and $v_i = 20 \ \frac{m}{c}$

e.) How far does the ball travel in the horizontal direction? $y = 12 \frac{m}{2}, t = 3.27 \text{ s. } \Delta x = ?$

$$v_x = 12 \frac{m}{s}, t = 3.27 s, tx = 1$$

 $\Delta x = v_x t = (12 \frac{m}{s})(3.27 s) = 39.2 m$

f.) What is the maximum height of the ball?

$$v_{y_i} = 16 \frac{m}{s}, v_y = 0 \text{ (max height)}, y_i = 0, y = ?$$

$$v_y^2 = v_{y_i}^2 - 2g\Delta y \text{ so } \Delta y = y - y_i = \frac{v_y^2 - v_{y_i}^2}{-2g}$$

$$y = \frac{v_y^2 - v_{y_i}^2}{-2g} + y_i = \frac{0 - \left(16 \frac{m}{s}\right)^2}{-2\left(9.8 \frac{m}{s^2}\right)} + 0 = \boxed{13.1 \text{ m}}$$

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Example 5:

Rat kicks a soccer ball from ground level at an angle of 36.87° with respect to the horizontal. The initial speed of the ball is 16 m/s.

- a.) How much time does it take the ball to reach its maximum height?
- b.) What is the velocity of the ball at its maximum height?
- c.) What is the velocity of the ball when its strikes the ground?
- d.) How far does the ball travel in the horizontal direction?

Example 5:
$$y_i = 0, \ \theta_i = 36.87^\circ, \ \text{and} \ v_i = 16 \ \frac{\text{m}}{\text{s}}$$

 $v_x = v_{x_i} = v_i \cos \theta_i = 16 \ \frac{\text{m}}{\text{s}} \cos(36.87^\circ) = 12.8 \ \frac{\text{m}}{\text{s}}$
 $v_{y_i} = v_i \sin \theta_i = 16 \ \frac{\text{m}}{\text{s}} \sin(36.87^\circ) = 9.6 \ \frac{\text{m}}{\text{s}}$

a.) How much time does it take the ball to reach its maximum height? $y_{ij} = 0$ (max height), t = ?

$$v_y = -gt + v_{y_i}$$
 so $t = \frac{v_y - v_{y_i}}{-g} = \frac{0 - 9.6}{-9.8} \frac{m}{s^2} = 0.98 s$

b.) What is the velocity of the ball at its maximum height?

v

$$v_x = 12.8 \frac{m}{s}$$
 and $v_y = 0, v = ?, \theta = ?$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(12.8 \ \frac{m}{s}\right)^2 + 0} = \boxed{12.8 \ \frac{m}{s}} \qquad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{0}{12.8 \ \frac{m}{s}}\right)$$
$$\theta = \boxed{0}$$

Example 5:

$$y_i = 0, \ \theta_i = 36.87^\circ$$
, and $v_i = 16 \ \frac{\text{m}}{\text{s}}$

c.) What is the velocity of the ball when it strikes the ground? $\Delta y = 0, v_x = 12.8 \frac{m}{2} \text{ and } v_{y_x} = 9.6 \frac{m}{2}, v = ?, \theta = ?$

ground - to - ground is symmetric so
$$v = \boxed{16 \frac{\text{m}}{\text{s}}}$$
 and $\theta = -\theta_i = \boxed{-36.87^\circ}$

d.) How far does the ball travel in the horizontal direction?

$$\Delta y = 0, \ v_x = 12.8 \ \frac{m}{s} \text{ and } v_{y_i} = 9.6 \ \frac{m}{s}, \ \Delta x = ?$$
$$v_y^2 = v_{y_i}^2 - 2g\Delta y \text{ so } v_y = \pm \sqrt{v_{y_i}^2 - 2g\Delta y} = -\sqrt{\left(9.6 \ \frac{m}{s}\right)^2 - 2\left(9.8 \ \frac{m}{s^2}\right)(0)} = -9.6 \ \frac{m}{s}$$

$$v_y = -gt + v_{y_i}$$
 so $t = \frac{v_y - v_{y_i}}{-g} = \frac{-9.6 - 9.6 \frac{\text{m}}{\text{s}}}{-9.8 \frac{\text{m}}{\text{s}^2}} = 1.96 \text{ s}$ $\Delta x = v_x t = \left(12.8 \frac{\text{m}}{\text{s}}\right)(1.96 \text{ s})$
 $\Delta x = \left(25.1 \text{ m}\right)$

Example 6:

Larry kicks a soccer ball with a speed of 15 m/s and angle of 36.87° above the horizontal from a building that is 35 m tall.

- a.) What is the magnitude and direction of the velocity of the ball 0.5 s after it is kicked?
- b.) What is the maximum height of the ball?
- c.) What is the magnitude and direction of the velocity of the ball when it hits the ground?
- d.) How much time does it take the ball to hit the ground?
- e.) How far from the base of the building does the ball strike the ground?

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Example 6:

$$y_i = 35 \text{ m}, \ \theta_i = 36.87^\circ, \text{ and } v_i = 15 \frac{\text{m}}{\text{s}}$$
$$v_x = v_{x_i} = v_i \cos \theta_i = 15 \frac{\text{m}}{\text{s}} \cos(36.87^\circ) = 12 \frac{\text{m}}{\text{s}}$$
$$v_{y_i} = v_i \sin \theta_i = 15 \frac{\text{m}}{\text{s}} \sin(36.87^\circ) = 9 \frac{\text{m}}{\text{s}}$$

a.) What is the magnitude and direction of the velocity of the ball 0.5 s after the ball is kicked? t = 0.5 s, v = ?, $\theta = ?$

$$v_{x} = 12 \frac{m}{s} \text{ and } v_{y} = -gt + v_{y_{i}} = -9.8 \frac{m}{s^{2}}(0.5 s) + 9 \frac{m}{s} = 4.1 \frac{m}{s}$$

$$v_{x} = 12 \frac{m}{s} \quad v_{y} = 4.1 \frac{m}{s} \quad v = \sqrt{v_{x}^{2} + v_{y}^{2}} = \sqrt{\left(12 \frac{m}{s}\right)^{2} + \left(4.1 \frac{m}{s}\right)^{2}} = \boxed{12.7 \frac{m}{s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{y}}{v_{x}}\right) = \tan^{-1}\left(\frac{4.1 \frac{m}{s}}{12 \frac{m}{s}}\right) = \boxed{18.9^{\circ}}$$
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Example 6: $y_i = 35 \text{ m}, \ \theta_i = 36.87^\circ, \text{ and } v_i = 15 \frac{\text{m}}{\text{s}}$ b.) What is the maximum height of the ball?

$$v_{y_i} = 9 \frac{m}{s}, v_y = 0 \text{ (max height)}, y = ?$$

 $v_y^2 = v_{y_i}^2 - 2g\Delta y \text{ so } \Delta y = y - y_i = \frac{v_y^2 - v_{y_i}^2}{-2g}$
 $y = \frac{v_y^2 - v_{y_i}^2}{-2g} + y_i = \frac{0 - \left(9 \frac{m}{s}\right)^2}{-2\left(9.8 \frac{m}{s^2}\right)} + 35 \text{ m} = \boxed{39.1 \text{ m}}$

c.) What is the magnitude and direction of the velocity of the ball when it strikes the ground?

$$v_x = 12 \frac{\text{m}}{\text{s}}, \ v_{y_i} = 9 \frac{\text{m}}{\text{s}}, \ y_i = 35 \text{ m}, \ y = 0, \ \Delta y = y - y_i = -35 \text{ m}, \ v = ?, \ \theta = ?$$
$$v_y^2 = v_{y_i}^2 - 2g\Delta y \text{ so } v_y = \pm \sqrt{v_{y_i}^2 - 2g\Delta y} = -\sqrt{\left(12 \frac{\text{m}}{\text{s}}\right)^2 - 2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(-35 \text{ m})} = -27.7 \frac{\text{m}}{\text{s}^2}$$

Example 6: $y_i = 35 \text{ m}, \ \theta_i = 36.87^\circ, \text{ and } v_i = 15 \frac{\text{m}}{\text{s}}$

d.) How much time does it take the ball to hit the ground?

$$v_{y_i} = 9 \frac{m}{s}, v_y = -27.7 \frac{m}{s}, t = ?$$

 $v_y = -gt + v_{y_i}$ so $t = \frac{v_y - v_{y_i}}{-g} = \frac{-27.7 \frac{m}{s} - 9 \frac{m}{s}}{-9.8 \frac{m}{s^2}} = \boxed{3.74 \text{ s}}$

e.) How far from the base of the building does the ball strike the ground?

$$\Delta x = ?$$

$$\Delta x = v_x t = \left(12 \frac{\text{m}}{\text{s}}\right)(3.74 \text{ s})$$

$$\Delta x = \boxed{44.9 \text{ m}}$$

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Example 7: $\Delta x = 48 \text{ m}, \ \theta_i = 53.13^\circ, \text{ and } t = 4.0 \text{ s}$

a.) What is the magnitude of the initial velocity of the ball? $v_i = ?$

$$\Delta x = v_x t \quad \text{so} \quad v_x = \frac{\Delta x}{t} = v_i \cos \theta_i$$
$$v_i = \frac{\Delta x}{t \cos \theta_i} = \frac{48 \text{ m}}{(4 \text{ s}) \cos(53.13^\circ)} = \boxed{20 \frac{\text{m}}{\text{s}}}$$

b.) From what initial height was the ball kicked?

$$\begin{aligned} v_i &= 20 \ \frac{\mathrm{m}}{\mathrm{s}}, \ \theta_i &= 53.13^\circ, \ t = 4.0 \ \mathrm{s}, \ \mathrm{and} \ y = 0, \ y_i = ? \\ \Delta y &= -\frac{1}{2} g t^2 + v_{y_i} t = y - y_i \\ y_i &= \frac{1}{2} g t^2 - v_{y_i} t + y \ = \frac{1}{2} g t^2 - v_i \mathrm{sin} \theta_i t + y \\ y_i &= \frac{1}{2} \left(9.8 \ \frac{\mathrm{m}}{\mathrm{s}^2} \right) (4 \ \mathrm{s})^2 - \left(20 \ \frac{\mathrm{m}}{\mathrm{s}} \right) \mathrm{sin} (4 \ \mathrm{s}) (53.13^\circ) + 0 = \boxed{14.4 \ \mathrm{m}} \end{aligned}$$

Example 6: $y_i = 35 \text{ m}, \ \theta_i = 36.87^\circ, \text{ and } v_i = 15 \frac{\text{m}}{\text{s}}$

c.) What is the magnitude and direction of the velocity of the ball when it strikes the ground?

 $v_x = 12 \frac{m}{s}, v_{y_i} = 9 \frac{m}{s}, y_i = 35 \text{ m}, y = 0, \Delta y = y - y_i = 0 - 35 \text{ m} = -35 \text{ m}, v = ?, \theta = ?$

$$v_{x} = 12 \frac{m}{s}$$

$$v = \sqrt{v_{x}^{2} + v_{y}^{2}} = \sqrt{\left(12 \frac{m}{s}\right)^{2} + \left(-27.7 \frac{m}{s}\right)^{2}} = 30.2 \frac{m}{s}$$

$$\theta = \tan^{-1}\left(\frac{v_{y}}{v_{x}}\right) = \tan^{-1}\left(\frac{-27.7 \frac{m}{s}}{12 \frac{m}{s}}\right) = -66.6^{\circ}$$

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Example 7:

Tiger kicks a ball with a launch angle of 53.13° . The ball hits the ground after traveling 48 m in 4.0 s.

- a.) What is the magnitude of the initial velocity of the ball?
- b.) From what initial height was the ball kicked?